2024/9/25
G-J Seminar Mainz

Finite temperature critical point in heavy-quark QCD

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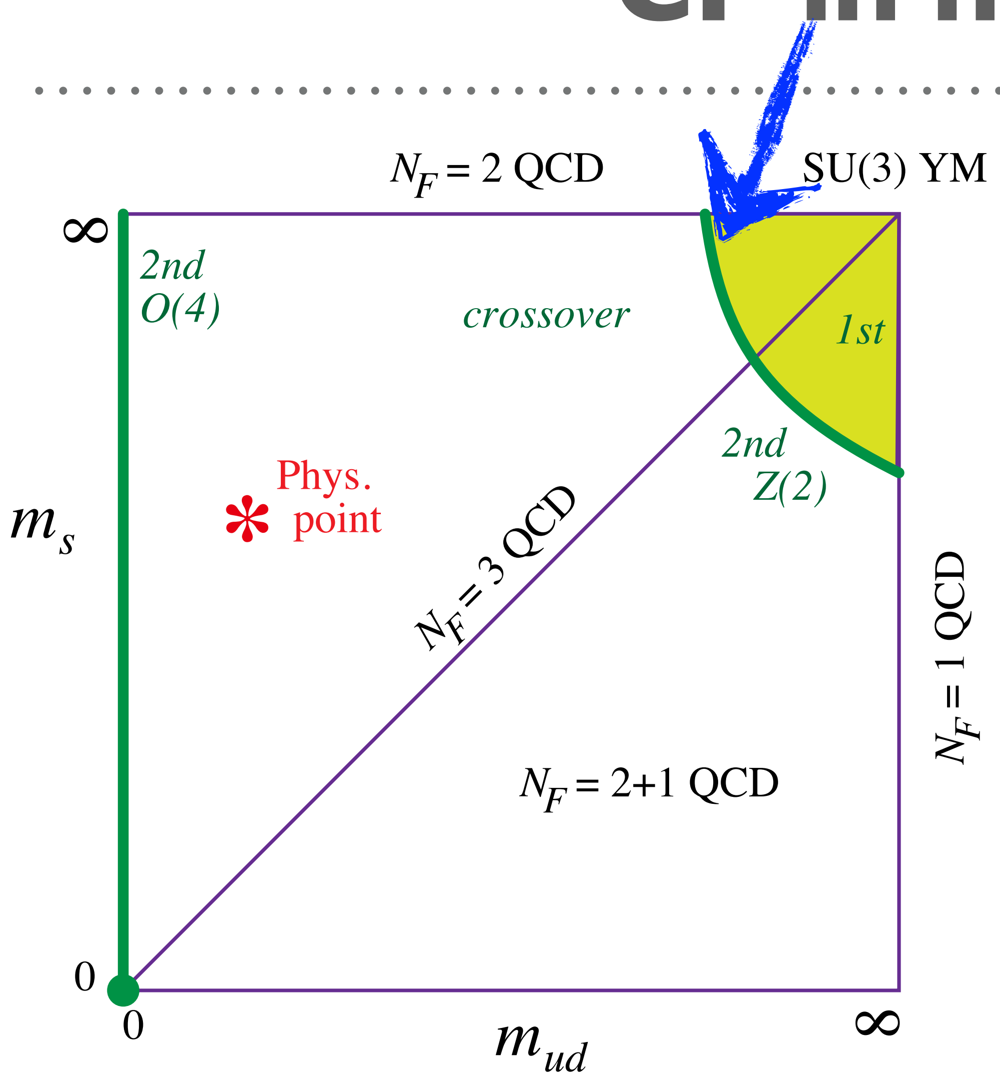
with R. Ashikawa[2)], **S. Ejiri**[3)], **M. Kitazawa**[4,5)], **H. Sugawara**[3)]

(WHOT-QCD Collaboration)

1) *Univ. Tsukuba,* 2) *Osaka Univ.,* 3) *Niigata Univ.,* 4) *Kyoto Univ.,* 5) *KEK J-PARC branch*

CP in heavy-quark QCD



Finite-T QCD transition is important in understanding various phenomena: the early evolution of the Universe, neutron stars, quark matter in relativistic heavy-ion collisions, etc.

<= Columbia plot summarizing the nature of the QCD trans. as function of m_q's.

We focus on the CP near the upper-right corner (hq-QCD).

A powerful way to determine CP:
Binder cumulant analysis based on the expected Z(2) FSS,
assuming (approx.) dominance of the leading sing. in $L \to \infty$ lim.

- *Large spatial lattices & high statistics to identify the leading FSS clearly.*
- *Reweighting to vary coupling parameters continuously as required by Binder cumulant analyses.*

Lattice setup

- Lattice action: plaquette gauge + standard Wilson quarks
- Wilson quark kernel:

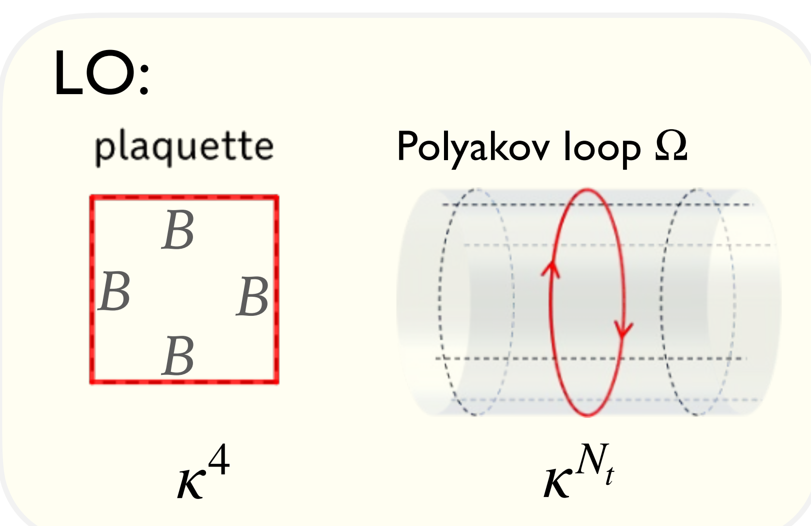
$$M_{xy}(\kappa) = \delta_{xy} - \kappa \sum_\mu \left[(1-\gamma_\mu) U_{x,\mu} \delta_{y,x+\hat\mu} + (1+\gamma_\mu) U^\dagger_{y,\mu} \delta_{y,x-\hat\mu} \right]$$

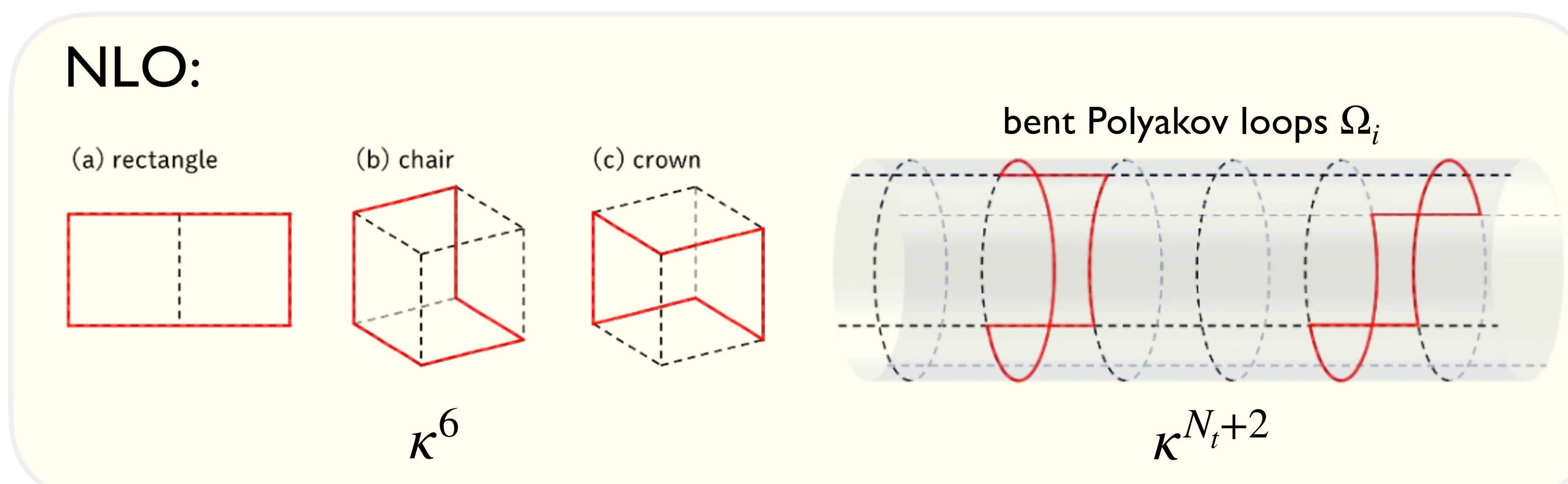
$$= \delta_{xy} - \kappa B_{xy}$$

where $\kappa = 1/(2am_q + 8)$

- Quark contribution to the effective action: $\ln\det M(\kappa) = -\frac{1}{N_{\rm site} n} \sum_{n=1}^{\infty} \mathrm{Tr}[B^n] \kappa^n$
  - each term given by closed loops of B with $\kappa^{\text{[loop length]}}$

- Hopping Parameter Expansion





Wilson loops Polyakov-loop type loops

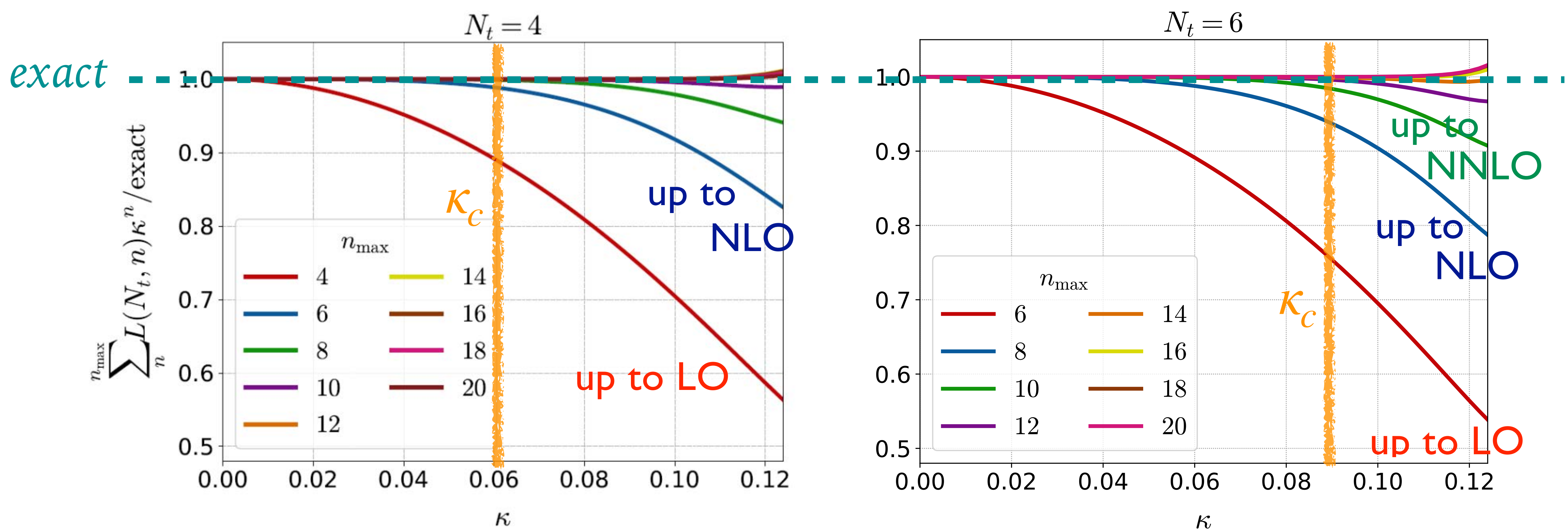
  - HPE ≈ $1/(am_q)$ expansion
  - HPE worsens with $a \to 0$ $(N_t \to \infty)$ => higher orders required.

Convergence of HPE

Wakabayashi, Ejiri, KK, Kitazawa, PTEP 2022, 033B05 (2022)

Deviation from true value due to truncation of HPE in the worst convergent case:

Polyakov-loop type contribution to the eff. action: (Wilson-loop type contribution similar.)

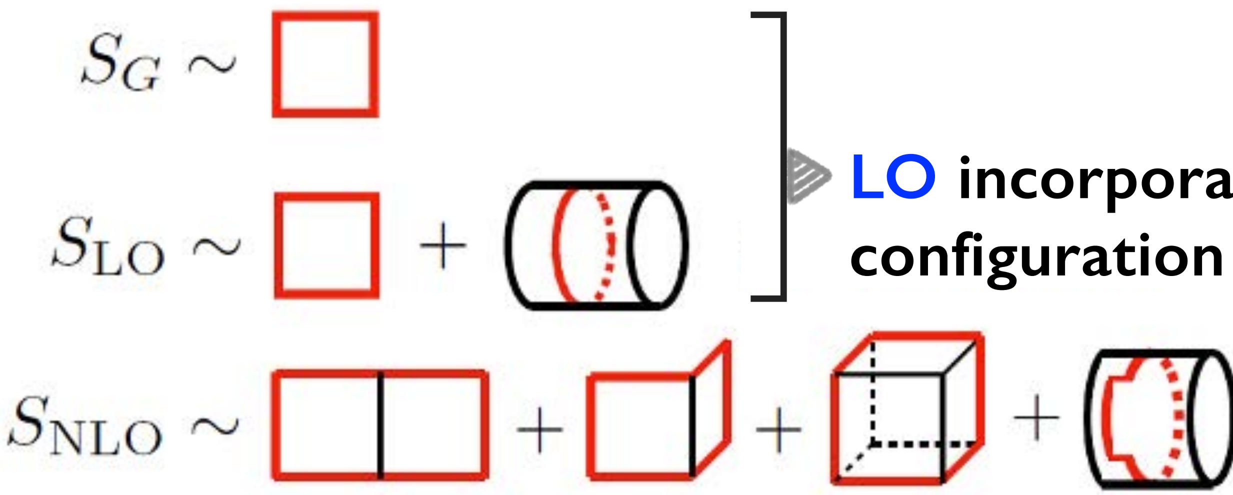


For studies around CP:

Ashikawa+, arXiv:2407.09156 (2024)

- Nt=4: $\kappa_c = 0.0603(4)$ [Kiyohara+ ('21)]
  => LO may have at worst 10% error in the eff. action, NLO is ≥98% accurate.
- Nt=6: $\kappa_c = 0.08769(7)(^{+11}_{-0})$ [Ashikawa+ ('24)], $0.0877(9)$ [Cuteri+ ('22)]
  => NLO is ≥93% accurate, NNLO is ≥97% accurate.
- Nt=8: $\kappa_c = 0.09024(46)$ [Sugawara+ ('24)], $0.1135(8)$ [Cuteri+ ('22)] => NNLO for >95%

Simulation up to NLO exactly



- **LO incorporated in the configuration generation**
- **NLO incorporated exactly by reweighting**

$$\beta \;\rightarrow\; \beta^* = \beta + 48 N_f \kappa^4$$

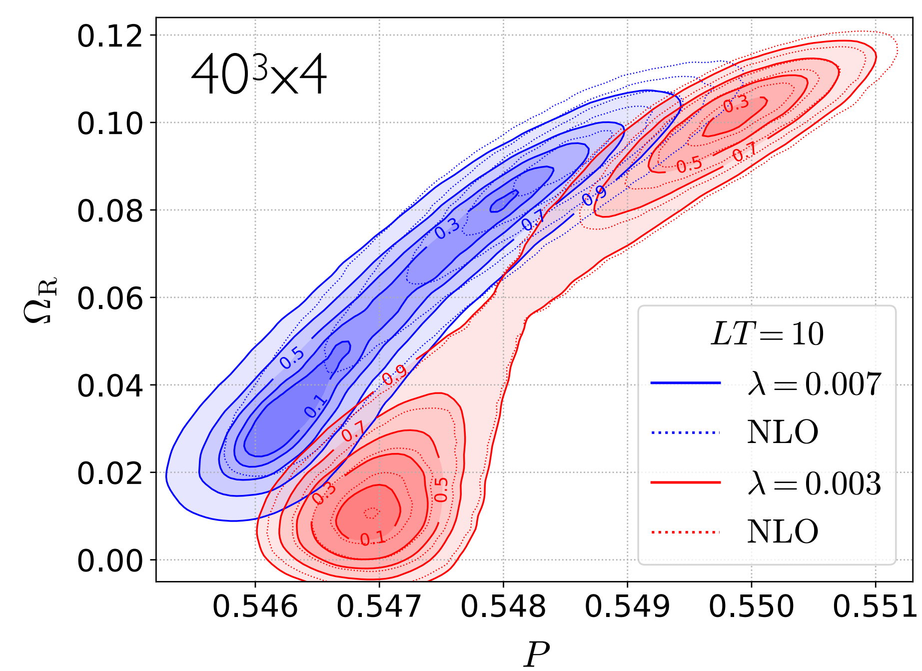
$\lambda \sum_{\mathbf{x}} \Omega(\mathbf{x})$ term can be incorporated in PHB+OR algorithm *à la* pure YM simulations

Kiyohara, Kitazawa, Ejiri, KK PRD 104, 114509 (2021)

- Simulation cost << f-QCD simulations
- Overlap problem of reweighting resolved by the LO in configuration generation

=> 1st order transition on large lattices

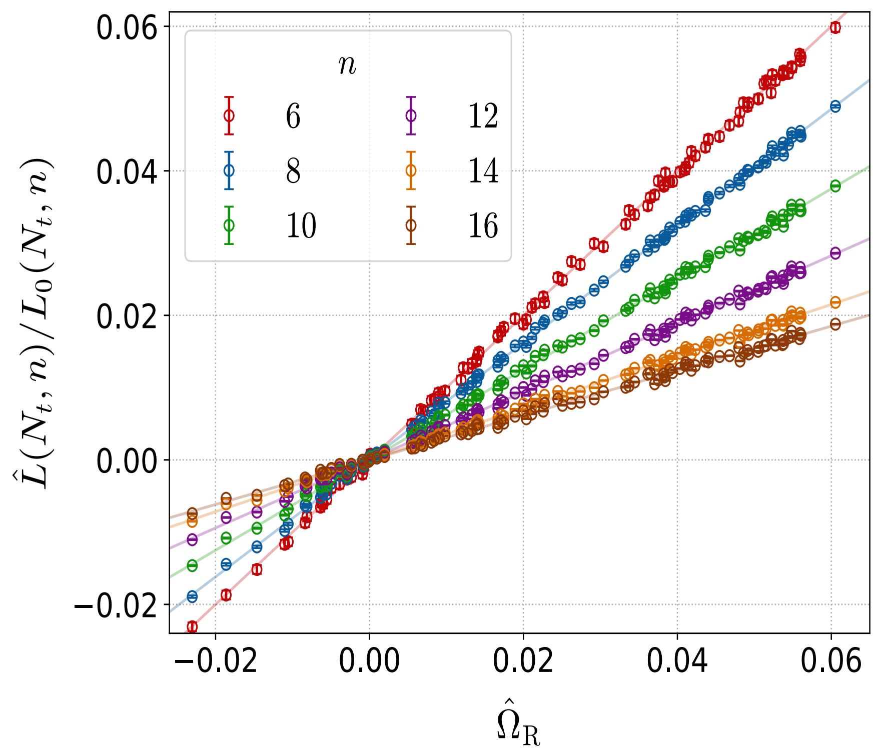
Kiyohara+ (2021)



Effective incorporation of higher orders

Basic observation:

strong linear correlation among different order terms of the HPE.



Scatter plot of n-th order Polyakov-loop type terms of HPE vs LO Polyakov-loop, observed on an Nt=6 lattice near the CP.

Ashikawa,+ (2024)

We may approximate

measured from the slopes

n-th order term $\approx C_n \times$ low-order term

Wilson-loop type terms show weaker but similar correlation.

eff.[LO] method

Exact up to LO.

Effctively incorporate NLO and higher orders by shifting the couplings in $S_{G+\mathrm{LO}}$.

Wakabayashi, Ejiri, KK, Kitazawa, PTEP 2022, 033B05 (2022)

eff.[NLO] method

Exact up to NLO.

Effectively incorporate NNLO and highers by shifting the NLO couplings in S_{NLO}.

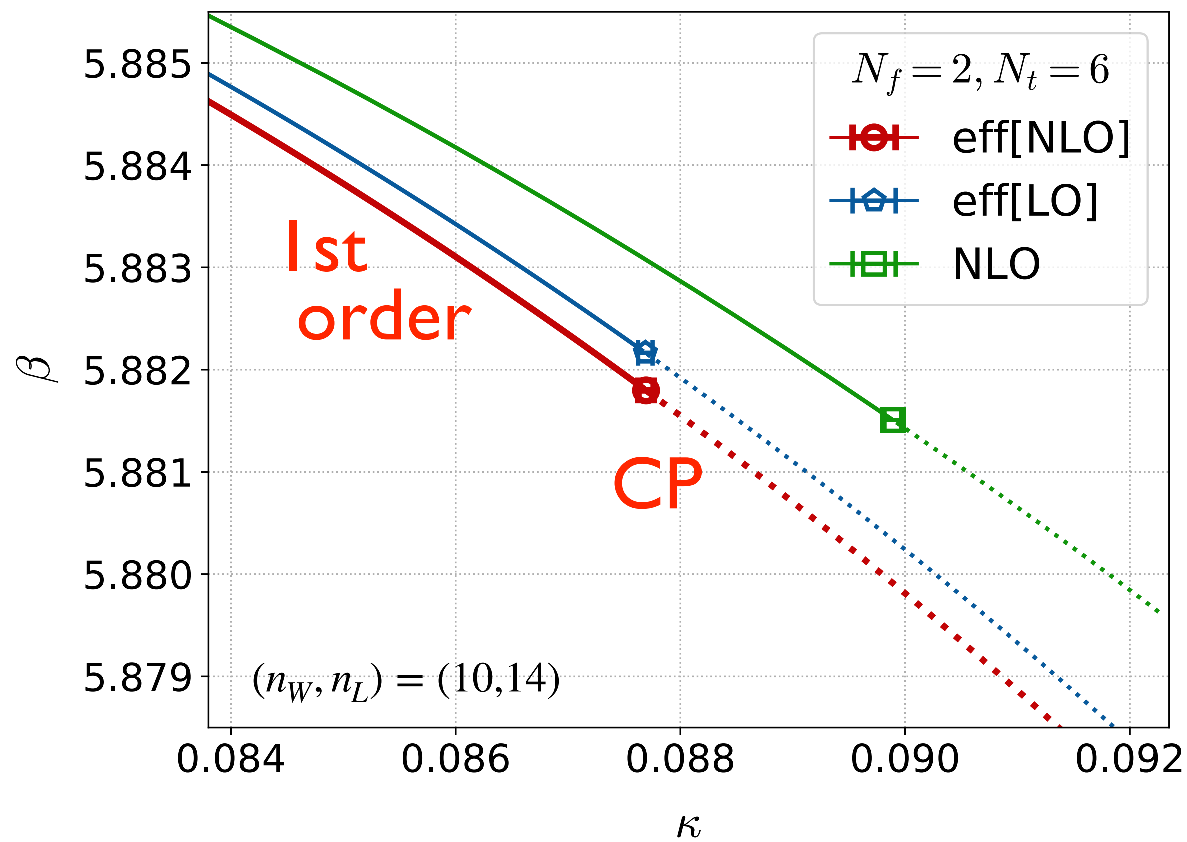
Better because NLO is exact and correlation is stronger with smaller order-differences.

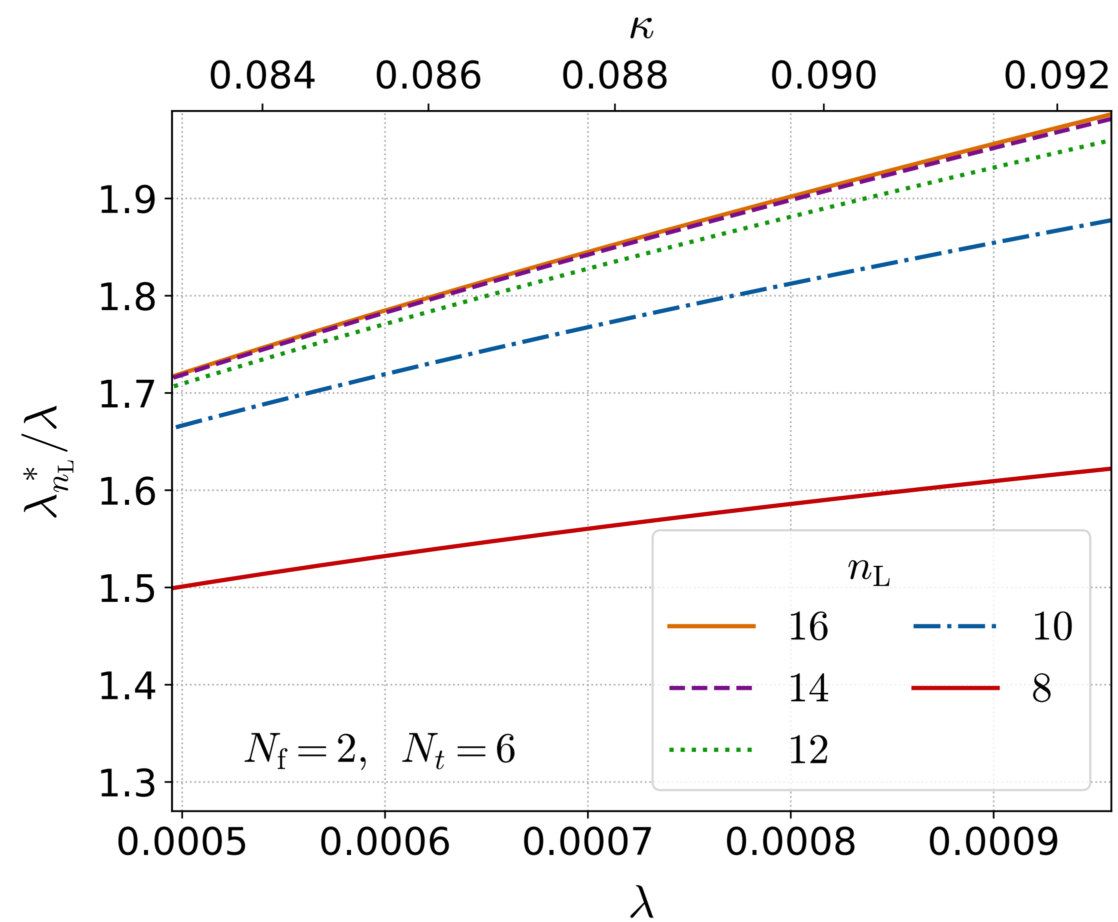
Ashikawa, Kitazawa, Ejiri, KK, arXiv:2407.09156 (2024)

Test of eff. methods using final observables

Results of the phase diagram at *Nt* = 6 (discussed later)

Ashikawa, Kitazawa, Ejiri, KK, arXiv:2407.09156 (2024)





- Transition line and CP with eff.[LO/NLO] shift from NLO

⇒ **NNLO and highers important at *Nt*≥6.**

- eff.[LO] ≈ eff.[NLO]

⇒ **effective incorporation of NLO works**

- Dependence on the truncation order in eff.[LO/NLO]

⇒ **convergent** if $(n_W, n_L) \geq (10,14)$ for *Nt*=6.

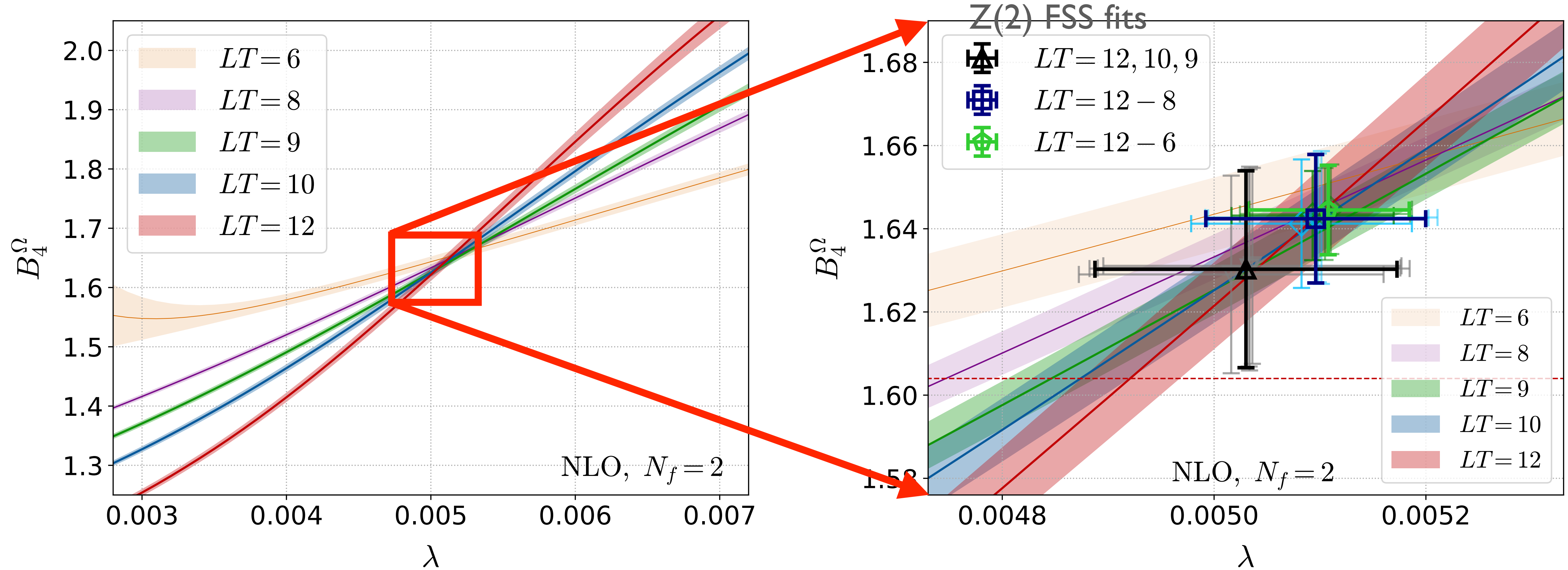
We adopt $(n_W, n_L) = (10,14)$ at *Nt*=6.

Study at $N_t = 4$

Kiyohara, Kitazawa, Ejiri, KK, PRD 104 (2021)

Binder cumulant of the Polyakov loop $B_4^\Omega = \frac{\langle \Omega_R^4 \rangle_c}{\langle \Omega_R^2 \rangle_c^2} + 3$ along the transition line

- Results at *Nt* = 4 with HPE up to NLO $\lambda = 48 N_f N_t \kappa^4$ for *Nt*=4



☆ large lattices with *Ns*/*Nt* = *LT* ≥ 9 required for Z(2) FSS

☆ $\nu = 0.614(48)(3)$, $b_4 = 1.630(24)(2)$ using *Ns*/*Nt* = 9–12

consistent with the Z(2) values 0.630, 1.604 within 1σ

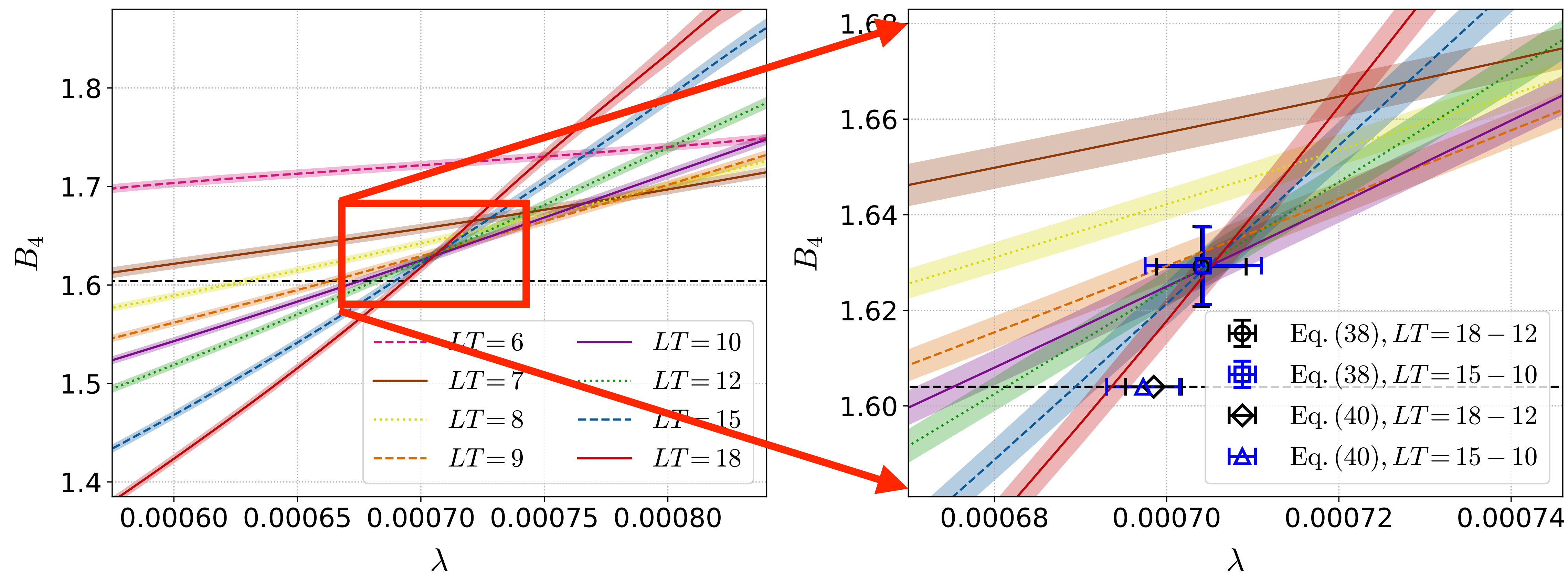
☆ $\lambda_c = 0.00503(14)(2)$ [$\kappa_c = 0.0603(4)$] for *Nt*=4, *Nf*=2

Study at $N_t = 6$

Ashikawa, Kitazawa, Ejiri, KK, arXiv:2407.09156 (2024)

- *Nt*=6, *Ns*/*Nt* = *LT* = 6, 7, 8, 9, 10, 12, 15, 18 (*Ns* = 36–108)
- B_4^{Ω}

$\lambda = 128 N_f N_t \kappa^6$ for *Nt*=6



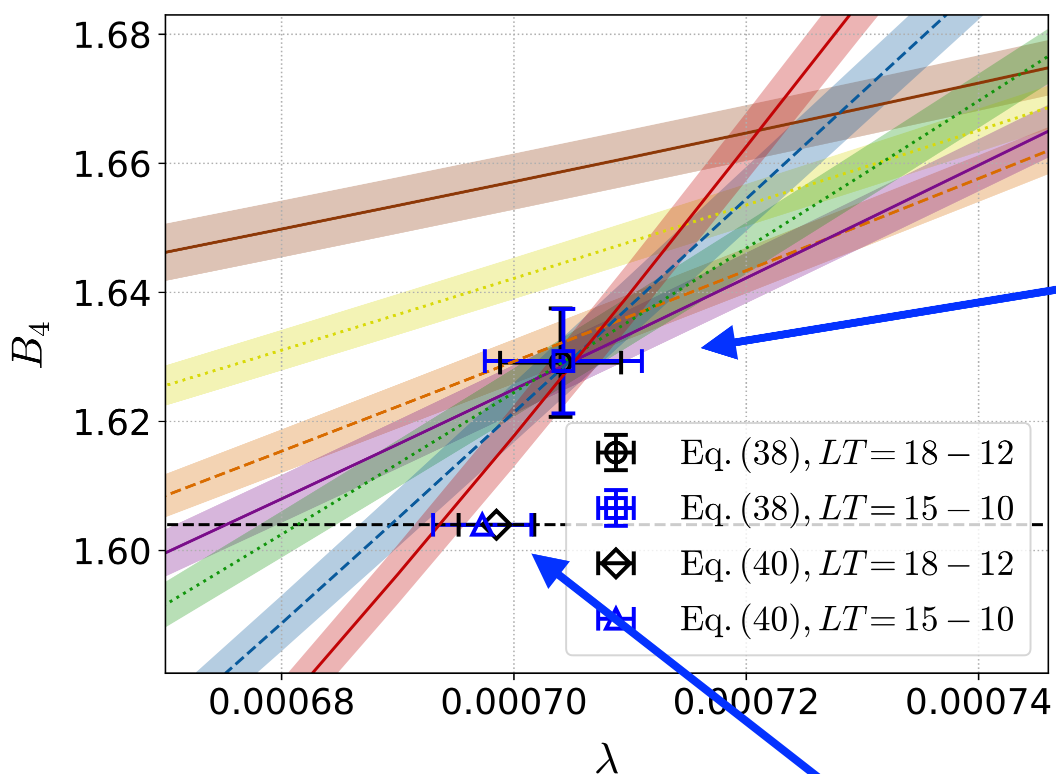
☆ Violation of FSS larger on finer lattice => larger *LT* (≥ 10) required

☆ $\nu = 0.627(19)(5)$, $b_4 = 1.6297(84)(6)$ with *LT* = 12–18

(cf.) Z(2) values: 0.630, 1.604

Study at $N_t = 6$

Ashikawa, Kitazawa, Ejiri, KK, arXiv:2407.09156 (2024)



LT	$\lambda_c \times 10^4$	χ^2/dof
18, 15, 12	6.986(32)(9)	0.51
15, 12, 10	6.973(42)(5)	0.83
18 – 10	6.984(25)(8)	1.07
18 – 9	6.998(21)(5)	4.23
18 – 8	7.035(19)(3)	36.4

FSS fit using standard ansatz for mag. op.:

$$B_4 = (b_4 + c(\lambda - \lambda_c)(LT)^{-1/\nu})$$

☆ $\nu = 0.627(19)(5),\ b_4 = 1.6297(84)(6)$

[cf. Z(2) values: 0.630, 1.604]

FSS fit taking mixing with energy-like op.:

$$B_4 = (b_4 + c(\lambda - \lambda_c)(LT)^{-1/\nu})(1 + d(LT)^{Y})$$

Y = – 0.896 for Z(2)

Full 6 parameter fits were unstable

=> fit fixing b_4, ν, Y to Z(2) values

☆ fits acceptable when $LT \geq 10$.

note: w/ mixing, the crossing pt. moves with LT.

☆ $\kappa_c = 0.08769(7)(^{+11}_{-0})$

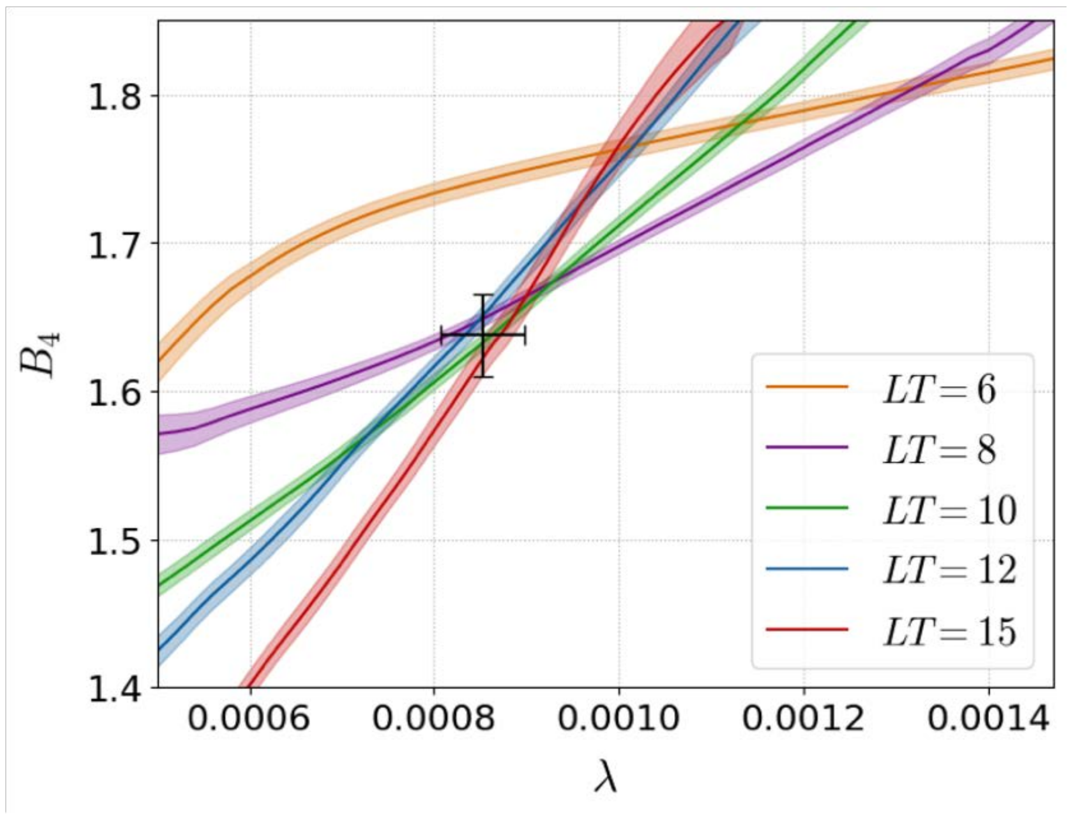
LT=12–18, eff.[NLO], N_F=2

[cf. 0.0877(9) Cuteri+ ('21) fQCD, LT=4–7]

Study at $N_t = 8$ *(in preparation)*

Sugawara+, in preparation

- *Nt*=8, *Ns*/*Nt* = *LT* = 6, 8, 10, 12, 15 (*Ns* = 48 – 120)
- B_4^{Ω}



$\lambda = 384 N_f N_t \kappa^8$ for *Nt*=8

☆ $\nu = 0.72(27),\ b_4 = 1.638(28)$ $[\chi^2/\mathrm{dof} = 1.474]$ with *LT* = 10–15

☆ $b_4 = 1.637(24)$ fixing $\nu = 0.630$ $[\chi^2/\mathrm{dof} = 1.343]$ with *LT* = 10–15

consistent with Z(2) within 1σ

☆ $\kappa_c = 0.09024(46)$ for N_F=2 with eff.[LO] up to 20th order

[cf. 1.1135(8) Cuteri+ ('22) fQCD, *LT*=4-6(7,10)]

Conclusions

We determined CP in heavy-quark QCD by the Binder cumulant:

- **Large spatial lattices with *LT* = *Ns/Nt* $\gtrapprox$ 10 required.**
- **HPE + eff. method to incorporate high orders.**
  - *Nt* = 4: $\kappa_c = 0.0603(4)$ for N_F=2 using *LT*=9–12
  - *Nt* = 6: $\kappa_c = 0.08769(7)(^{+11}_{-0})$ for N_F=2 using *LT*=10–18
  - *Nt* = 8: $\kappa_c = 0.09024(46)$ for N_F=2 using *LT*=10–15
  - N_F-dep. known analytically in HPE => easy to translate them to 2+1 flavors etc.
- **CP in physical units using $m_{\rm PS}$ at *T*=0.** [Cuteri+ ('21), Itagaki+('19) unpublished]
  - *Nt* = 4: $m_{\rm PS}^{\rm (CP)}/T_c = 16.30(3)$ for N_F=2 using *LT*=9–12
  - *Nt* = 6: $m_{\rm PS}^{\rm (CP)}/T_c = 18.07(2)(^{+0}_{-2})$ for N_F=2 using *LT*=10–18
  - *Nt* = 8: $m_{\rm PS}^{\rm (CP)}/T_c = 17.2(2)$ for N_F=2 using *LT*=10–15
  - *Nt*-dep. (*a*-dep.) looks small in this combination.

- The method should work at least up to *Nt* ~ 10.
- Application to finite-density QCD.

We miss

Yusuke Taniguchi

07.04.1968 — 22.07.2022

who was joining the German-Japanese Seminars since 2010, and was the main coordinator of the Seminars in the Japanese side since 2018 until he went into a coma in Nov. 2019.
His application of the Seminar for FY 2020 at Mainz was successful, but the Seminar was postponed and finally withdrawn due to the COVIV-19 pandemic.


15.03.2019, Fukuoka, Japan

brief history of YT

1997	PhD, Kyoto University.
1997-1999	JSPS fellow (Univ. Tsukuba)
1999-2008	Assistant, Univ. Tsukuba
2003-2004	JSPS Visiting Scientist (CERN)
2008-2018	Assistant Professor (Lecturer), Univ. Tsukuba
2010	Japanese-German Seminar, Mishima, Japan
2013	German-Japanese Seminar, Regensburg, Germany
2016	Japanese-German Seminar, Niigata, Japan
2018-	Associate Professor, Univ. Tsukuba


Nov. 2010, J-G Seminar Mishima

Sept. 2016, J-G Seminar Niigata

31.07.2016, Plymouth, UK

18.09.2019, Yamagata, Japan


09.09.2019, Wuhan, China


15.07.2011, Squaw Valley, USA


28.07.2016, Stone Henge, UK


20.06.2017, Granada, Spain