Lattice Study of Quarkonia at Finite Temperature

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and HotQCD Collaboration

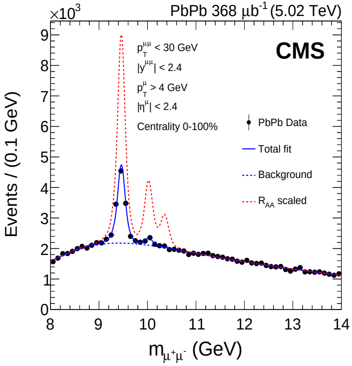
- Quarkonia (bound states of heavy $q\bar{q}$), are an important probe to study QGP.

  - Experimental data show suppression of quarkonia yields.

  - Di-lepton rate

$$\frac{d\Gamma_{\mu+\mu-}}{d^4Q} \sim \frac{e^2}{Q^2}\, n_b\, \rho_V(Q)$$

  - For current,
$J_\Gamma(\vec{x},t) = \bar{\psi}(\vec{x},t)\Gamma\psi(\vec{x},t)$



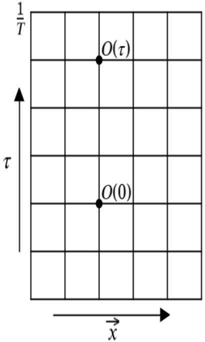
CMS Collaboration, PLB 790 (2019) 270

$$\rho_\Gamma(\omega,\vec{k}) = \int dt\, d^3\vec{x} \exp\left[i(\vec{k}.\vec{x}-\omega t)\right] \langle [J_\Gamma(\vec{x},t), J_\Gamma(0,0)]\rangle_T$$

- Euclidean-correlation function $G^E(\tau, \vec{k}) = \int \exp\left(i\vec{k}.\vec{x}\right)\langle J_\Gamma(\vec{x},\tau)J_\Gamma(0,0)\rangle$ where $J(\vec{x},t) = \bar{\psi}(\vec{x},\tau)\Gamma\psi(\vec{x},\tau)$

$$G^E_\Gamma(\tau,\vec{k}) = \int_0^\infty \frac{d\omega}{\pi}\rho_\Gamma(\omega,\vec{k})\frac{\cosh[\omega(\tau-\frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

- Numerically ill-posed problem. Small number of data points and statistical errors.



We computed for charm and bottom correlation function on the lattice with $m_\pi \sim 315$ MeV.

We consider $\boldsymbol{\Gamma = \gamma_5, \gamma_i}$ **and** $\boldsymbol{\vec{k} = \vec{0}}$.

$T \sim 1.2T_c(N_\tau = 32), 1.3T_c(N_\tau = 28), 1.62T_c(N_\tau = 24)$ $T_c = 180$ MeV.

$L = 2.7$ fm

- $\omega \gg 2M$
  Thermal effects are suppressed. Vacuum perturbation theory will work.
- $\omega \sim 2M$
  Thermal effects are important. Spectral function needs to be calculated using thermal potential.
- $\omega \ll 2M$
  For the pseudoscalar channel, the spectral weights are exponentially suppressed.
  For the vector channel, there is a contribution around $\omega \sim 0$ due to transport.

$$C_>(t;\vec{r},\vec{r}') = \int d^3\vec{x}\,\langle\bar{\psi}(t,x+\frac{\vec{r}}{2})\gamma_5\,U\,\psi(t,x-\frac{\vec{r}}{2})\bar{\psi}(0,-\frac{\vec{r}'}{2})\gamma_5\,U\,\psi(0,-\frac{\vec{r}'}{2})\rangle_T$$

$M_Q \gg \Lambda_{QCD}, T$. Expansion in leading order inverse quark mass leads to, M.Laine et al, JHEP 0703:054

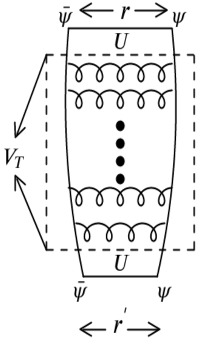
$$\left\{i\partial_t - \left[2M + V_T(r) - \frac{\nabla^2_{\vec{r}}}{M}\right]\right\} C_>(t;\vec{r},\vec{r'}) = 0$$

where V_T is defined in static limit,

$$V_T(r) = i\lim_{t\to\infty}\frac{\partial \log W(r,t)}{\partial t} = V_{re}(r) - iV_{im}(r)$$

with $C_>(0;\vec{r},\vec{r'}) = \delta^3(\vec{r}-\vec{r}')$

$$\rho_p(\omega) \propto \lim_{r\to 0, r'\to 0}\int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, C_>(t;\vec{r},\vec{r'})$$



- Non-perturbative formulation, A. Rothkopf et al., PRL. 108 (2012) 162001

$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega\, \rho(r,\omega,T) \exp(-\omega\,\tau)$$

$$W(r,t) = \int_{-\infty}^{\infty} d\omega\, \rho(r,\omega,T) \exp(-i\omega\,t)$$

- $\rho(\omega, T)$ should have a form which is consistent with potential, $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t}$ should exist.
- **Gaussian spectral function does not have this limit.** PRD 105, 054513
  Simple Lorentzian does have this limit. But $\rho(r,\omega,T)$ depends on the cut-off. PRD 109 ,074504
  Bayesian analysis has a higher systematic error. PRL 114, 082001

- Leading order results: M.Laine et al, JHEP 0703:054

$$\log W(r,\tau) = g^2 \left[\tau C_F \int_0^\infty \frac{q^2}{2\pi^2(q^2+m_d^2)} \left(\frac{\sin(qr)}{qr} - 1 \right) dq \right.$$
$$\left. + \int_{-\infty}^{\infty} \sigma(q_0, r) \left(e^{\beta q_0} + e^{(\beta-\tau) q_0} \right) dq_0 \right]$$
$$\sigma(q_0) \sim \frac{1}{q_0^2}$$

- The thermal potential:

$$V_T(r) = i \lim_{t\to\infty} \frac{\partial \log W(r,it)}{\partial t} = V_{re}(r) - iV_{im}(r)$$

For $r \sim \frac{1}{m_d}$,

$$V_T^{re}(r) = -\frac{g^2}{4\pi} C_F \left[m_d + \frac{\exp(-m_d r)}{r} \right] \Rightarrow \textit{Color Screening}$$

$$V_T^{im}(r) = \frac{g^2}{4\pi} C_F T \int_0^\infty \frac{z dz}{(z^2+1)^2} \left[1 - \frac{\sin(z m_d r)}{z m_d r} \right] \Rightarrow \textit{Landau Damping}$$

$$\log(W(r,\tau)) = -V_{re}(r)\tau - \int_{-\infty}^{\infty} du\, \sigma(r,u)\left[\exp(u\,\tau) + \exp(u\,(\beta-\tau))\right] + \ldots$$

- $i \lim_{t\to\infty} \frac{\partial \log W(r,t)}{\partial t} = \text{finie} \implies \lim_{u\to 0} \sigma(r,u) \sim \frac{1}{u^2}$
- $\sigma(r,u) = n_B(u)\left[\frac{V_{im}}{u} + c_1\, u + c_3\, u^3 + \ldots\right]$
- Parametrization

$$W(r,\tau) \;=\; A \exp\left[-V_{re}(r)\tau - \frac{\beta V_{im}(r)}{\pi} \log\left(\sin\left(\frac{\pi\tau}{\beta}\right)\right) + \ldots\right]$$

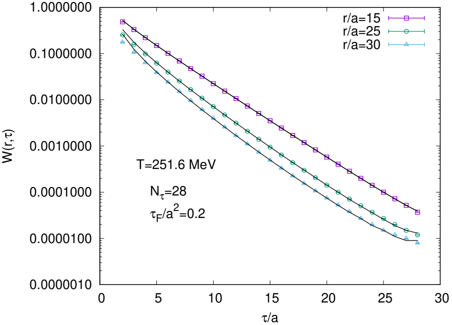
DB and S. Datta, PRD 101, 034507

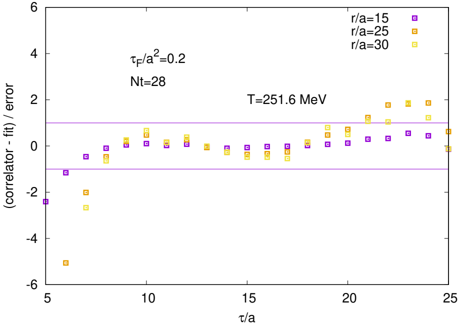
DB and S. Datta, PRD 103, 014512

DB, O. Kaczmarek, et al., PRD 105, 054513

$$W(r,\tau) = A\exp\left[-V_{re}(r)\tau - \frac{\beta V_{im}(r)}{\pi}\log\left(\sin\left(\frac{\pi\tau}{\beta}\right)\right) + \ldots\right]$$

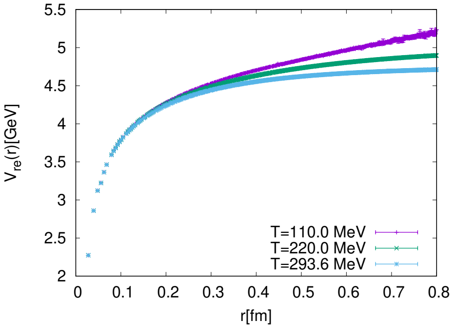
- Three parameter fit of Wilson line correlator for different distances.
- Flowed Wilson line correlator has been used for better S/N .

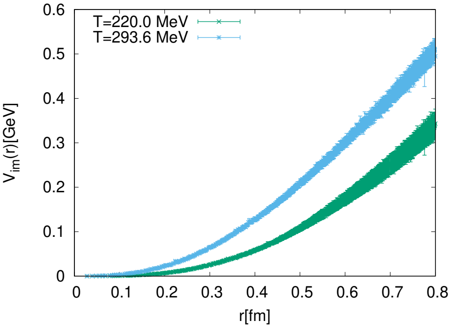




- $\chi^2/ndf \sim 1$

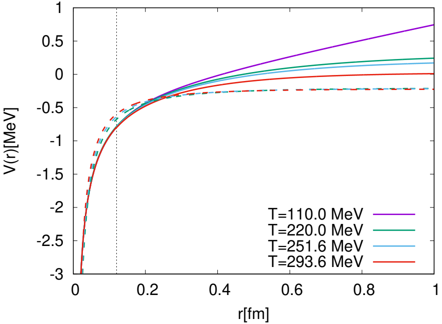
- Lattice data are consistent with Color Screening

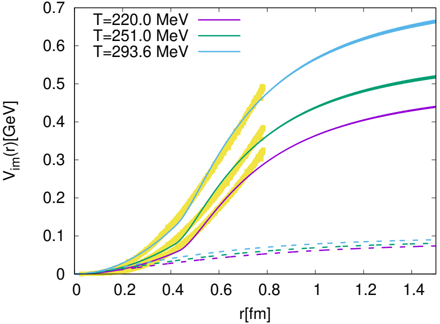




$$V_{re}(r) = \frac{\sigma}{m_d}(1 - \exp(-m_d r)) - \frac{\alpha}{r}\exp(-m_d r) + c$$

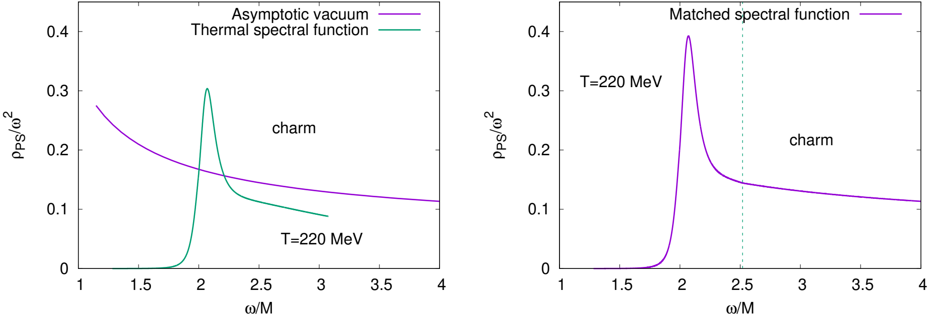
$$V_{im}(r) = \begin{cases} \frac{1}{2}br^2 & \text{for } r < r_0 \\ a_0 - \frac{a_1}{2r^2} - \frac{a_2}{4r^4} & \text{for } r \geq r_0 \end{cases}$$





- Non-perturbative thermal potential is very much different from the perturbative potential.

$$\rho_{PS}(\omega) = A_0\, \rho_{PS}^{T}(\omega)\theta(\omega_0 - \omega) + \rho_{PS}^{T=0}(\omega)\theta(\omega - \omega_0)$$



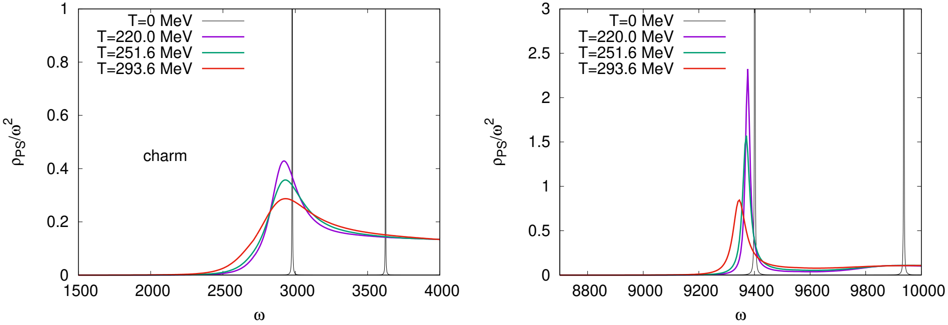
- $A_0 \sim 0.88\, M - 1.2\, M$
- $\omega_0 \sim 2.5\, M$

Similar spectral function using perturbative potential.

$N_f = 0$ M. Laine et al, JHEP11 (2017) 206

$N_f = 2 + 1$ S. Ali, DB , O.Kaczmarek et al, Few-Body Syst 64, 52 (2023)

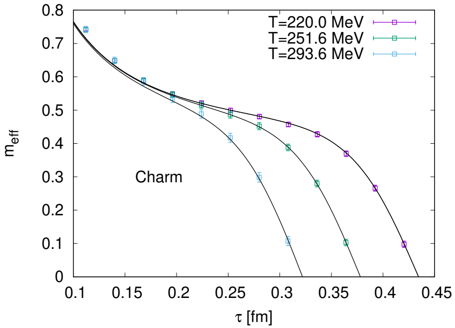
Pole mass: $m_c = 1.35 GeV$ and $m_b = 4.78 GeV$

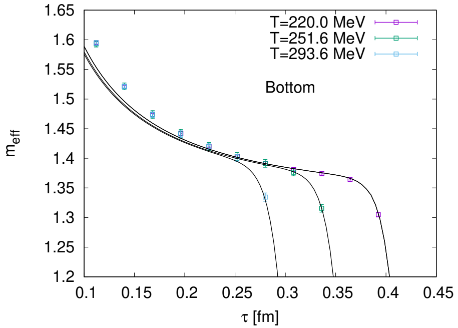


- (1S) state for bottom disappear much after T_c ($T_c = 180 MeV$)
- Significant thermal effects on charmonium state.
- The spectral function is not Gaussian near the peak . contradiction with R. Larsen et al, Phys.Rev.D 100 (2019) 7, 074506

$$G^E_{PS}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{PS}(\omega) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$

$$m_{eff}(\tau_i) = \log\left(\frac{G^E_{PS}(\tau_i)}{G^E_{PS}(\tau_{i+1})}\right)$$

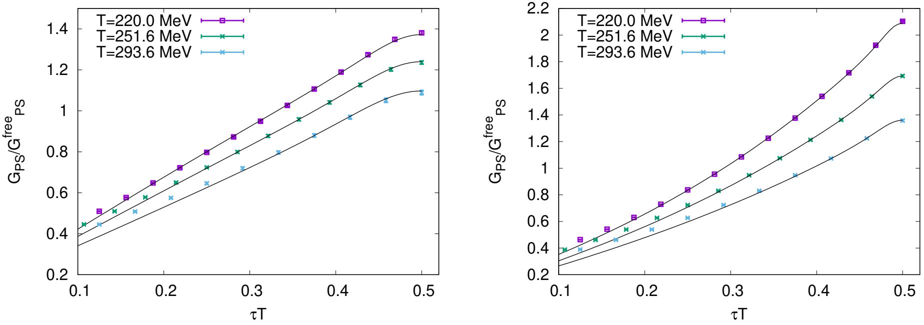




Consistent with lattice data.

$$\rho_{PS}^{model}(\omega, A) = A\,\rho_{PS}(\omega)$$

$$G_{PS}^{E}(\tau, A) = \int_0^{\infty} \frac{d\omega}{\pi} \rho_{PS}^{model}(\omega, A) \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh[\frac{\omega}{2T}]}$$



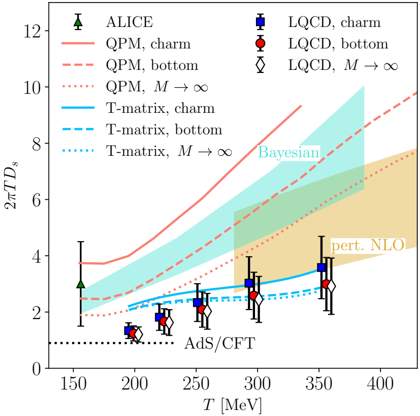
- These spectral functions indeed describe the lattice correlator .

- Transport peak in the vector current correlator:

$$\rho_{transport} = 3\chi_q \frac{T}{M} \frac{\omega \eta_d}{\omega^2 + \eta_d^2}$$

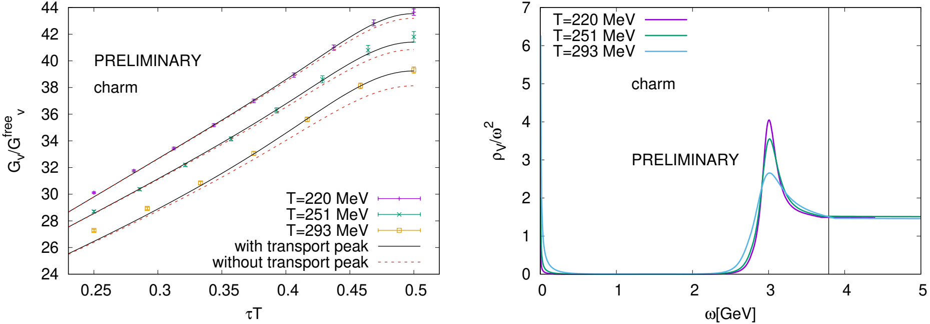
$$\eta_d = \frac{T}{M\,D_s}$$

- Extremely narrow transport contribution $\omega \sim \frac{g^4 T^2}{M}$. Bound state contribution $\omega \sim 2M$.



HotQCD, PRL 132 (2024) 5, 051902

$$\rho_V(\omega) = A\,\rho_{transport}(\omega) + \rho_{boundstate}(\omega)$$



- A small transport contribution is required to fit the lattice data .

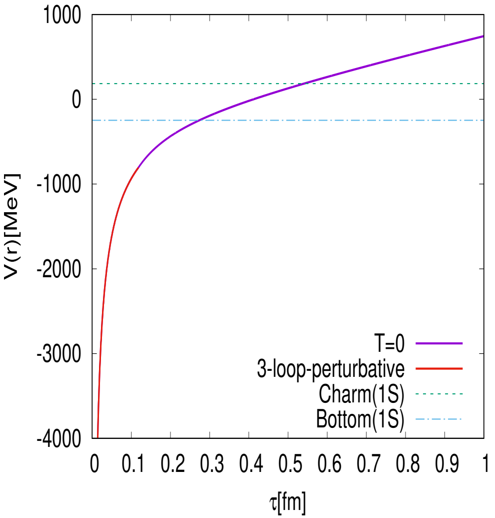
- We calculate the pseudoscalar/vector channel spectral function from 2+1 flavor lattice QCD correlation functions.
- Lattice Data supports Color screening of the non-perturbative thermal potential.
- We observed a small thermal mass shift for the in-medium $\eta_b(1S)$ and $\eta_c(1S)$ channels and a large thermal width ($\Gamma_c(1S) \gg \Gamma_b(1S)$).
- The pseudoscalar channel correlator function can be described by the spectral function obtained from the thermal potential.
- For the vector channel, the spectral function needs a small transport contribution in addition to the bound state contribution to describe the lattice data.

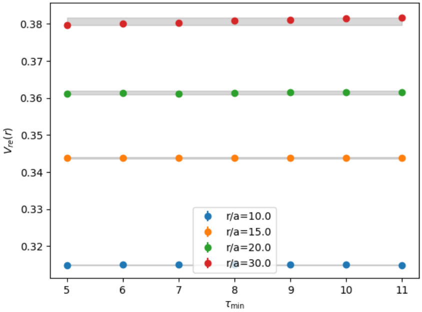
- Cornell fit of $T = 0$ lattice potential.
- Short distance matched renormalon subtracted peruturbative potential.

$$\left[-\frac{\nabla^2}{M} + V(r)\right] \psi_n(r) = E_n \psi_n(r)$$

$$M^{1S} = 2\,M + E_0$$

- $M^b = 4.78$ GeV
- $M^c = 1.35$ GeV





0.38
0.37
0.36
0.35
0.34
0.33
0.32
$V_{re}(r)$
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τ_{min}
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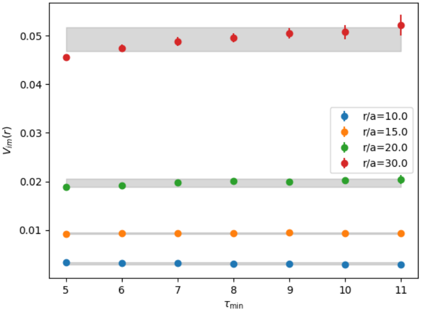
r/a=15.0
r/a=20.0
r/a=30.0



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$V_{im}(r)$
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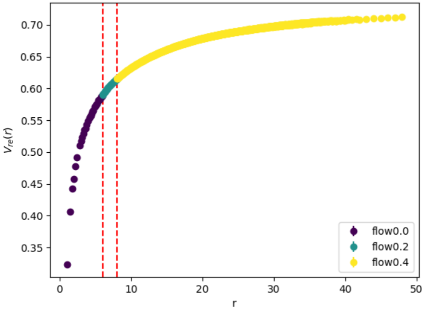
r/a=20.0
r/a=30.0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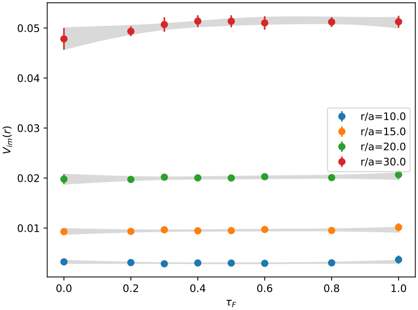
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$V_{im}(r)$
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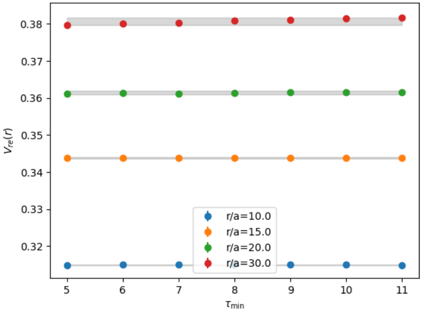
r/a=10.0
r/a=15.0
r/a=20.0
r/a=30.0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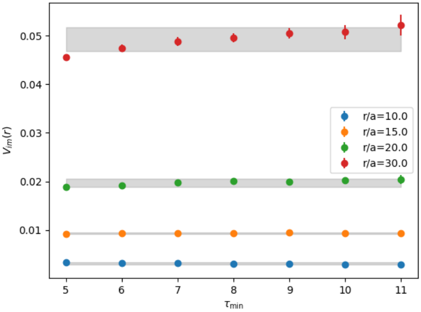
r/a=20.0
r/a=30.0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$V_{im}(r)$
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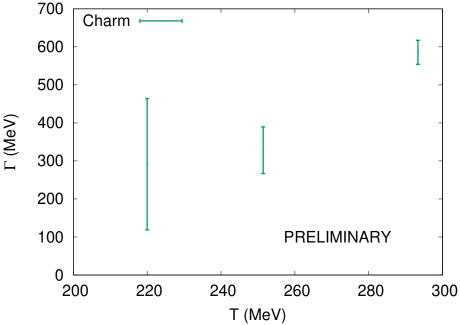
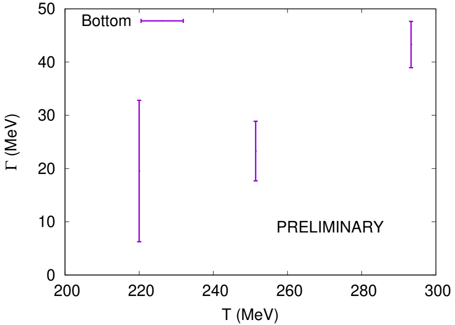
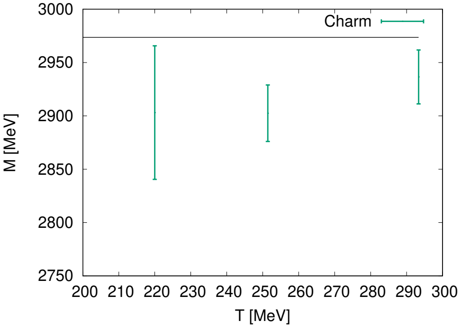
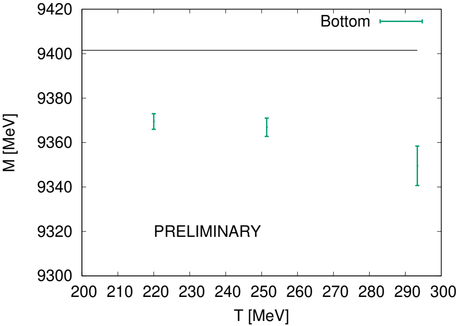
r/a=30.0





- We performed skewed Lorentzian fit near the peak.
- $\Gamma_c(1S) \gg \Gamma_b(1S)$





- Mass is identified with peak position of the spectral function.
- Finite mass shift is observed