Conserved charge fluctuations in (2+1)-flavor QCD with Domain Wall fermions

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In collaboration with
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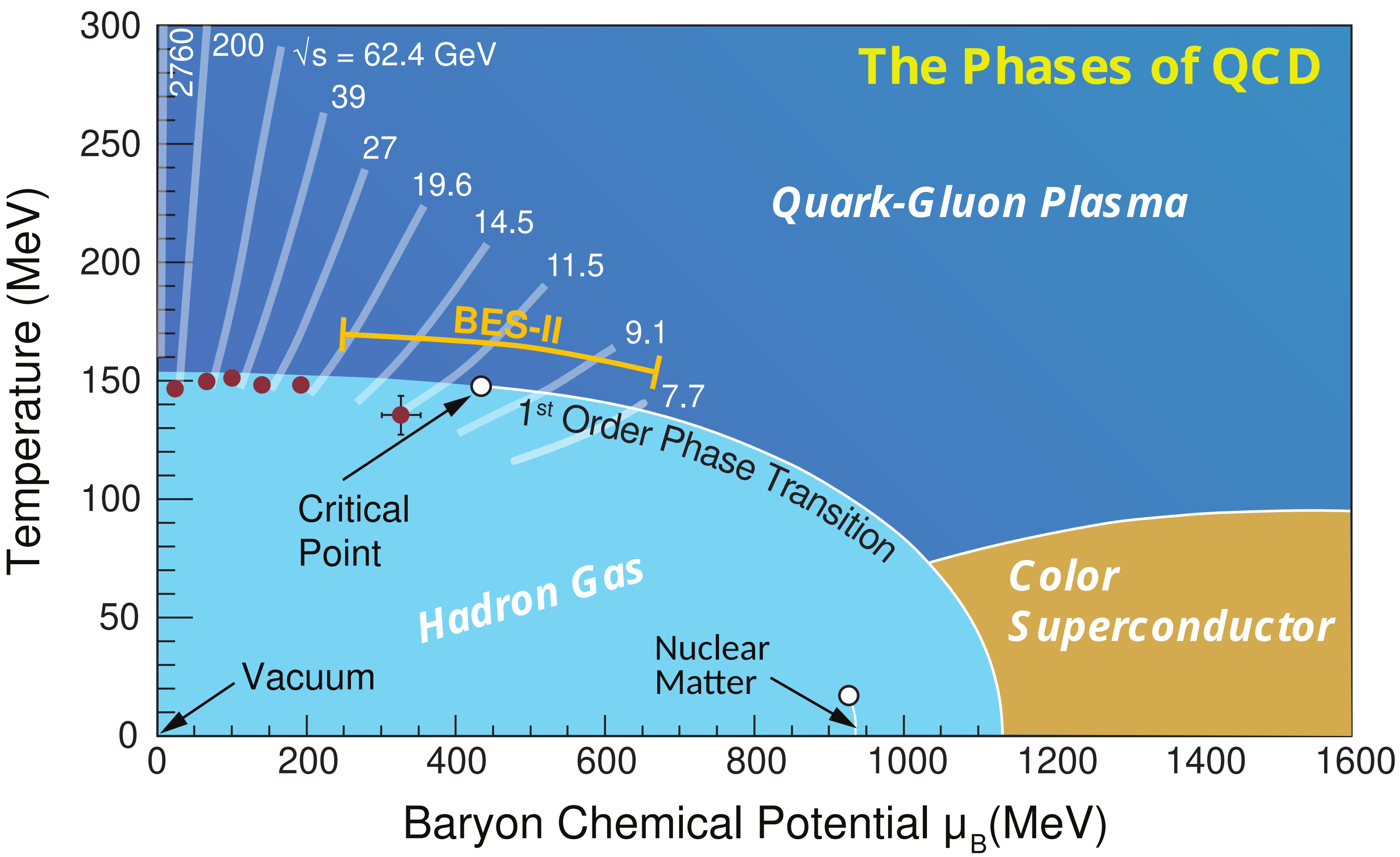
RIKEN
RIKEN Center for Computational Science

German-Japanese seminar:2024

QCD phase diagram

"Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan",
Bzdaket al., Phys. Rept. '20



Bigger Picture: Understand the thermodynamics at the QCD chiral transition and exploration of the QCD phase diagram with lattice chiral fermions i.e. Möbius Domain Wall fermions.

GER-JAP Worskhop:

Chiral phase transtion and topological susc. : Y. Aoki

Conserved charge fluctuations : this talk

3 flavor chiral phase transition : Y. Zhang

Acknowledgments

1. Computational resource:

- Supercomputer Fugaku (hp230207, hp220174, hp210165, hp200130, ra000001).

2. Funding sources :

- MEXT as "Program for Promoting Researches on the Supercomputer Fugaku",*Simulation for basic science: from fundamental laws of particles to creation of nuclei*, JPMXP1020200105; "Simulation for basic science: approaching the quantum era" (JPMXP1020230411).
- JICFuS.
- JPS KAKENHI(JP20K0396, I. Kanamori).

And to all the JLQCD members for regular meetings and discussions.

Code bases

Configuration generation: Grid (https://github.com/paboyle/Grid)

Measurements : (i) Hadrons (https://github.com/aportelli/Hadrons)

(ii) Bridge++ (https://bridge.kek.jp/Lattice-code/)

Data Analysis : https://github.com/LatticeQCD/AnalysisToolbox

A “Recap” on Chemical potential on the lattice

Continuum prescription, Divergence for the free fermion case in QNS: $\chi_2 \sim 1/a^2$

The prescription for chemical potential on the lattice,

$$(1 \pm \gamma_4)U_{\pm 4}(x) \rightarrow (1 \pm \gamma_4)e^{\pm\hat{\mu}}U_{\pm 4}(x)$$

P. Hasenfratz, F. Karsch, Phys.Lett.B 125 (1983) 308-310
R. V. Gavai,Phys. Rev. D 32, 519

No additional divergences appear in the interacting theory.

Steven Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. Lett. 59, 2247.
Rajiv V. Gavai, Sayantan Sharma, *Phys.Lett.B* 749 (2015) 8-13

For Domain Wall fermions :

$$(1 \pm \gamma_4)U_{\pm 4}(x)((1 \pm \gamma_4)U_{\pm 4}(x) \rightarrow (1 \pm \gamma_4)e^{\pm\hat{\mu}}U_{\pm 4}(x)$$

J. C. R. Bloch and T. Wettig, Phys. Rev. D 76, 114511

$$Z = \int DU \prod_{f=u,d,s} detM(m_f)\exp[-S_g], \quad \det M(m_f, \hat{\mu}_f) = \left[\frac{\det D(m_f, \hat{\mu}_f)^{DWF}}{\det D(m_{PV}, \hat{\mu}_f)^{DWF}}\right]$$

Sign problem for, $\mu_f \neq 0$. We use Taylor expansions.

Quark number susceptibility and conserved charge fluctuations in (2+1)-flavor QCD

In QCD with two light (u, d) and one strange flavor (s), pressure is expressed via a Taylor expansion in quark chemical potentials (μ_f).

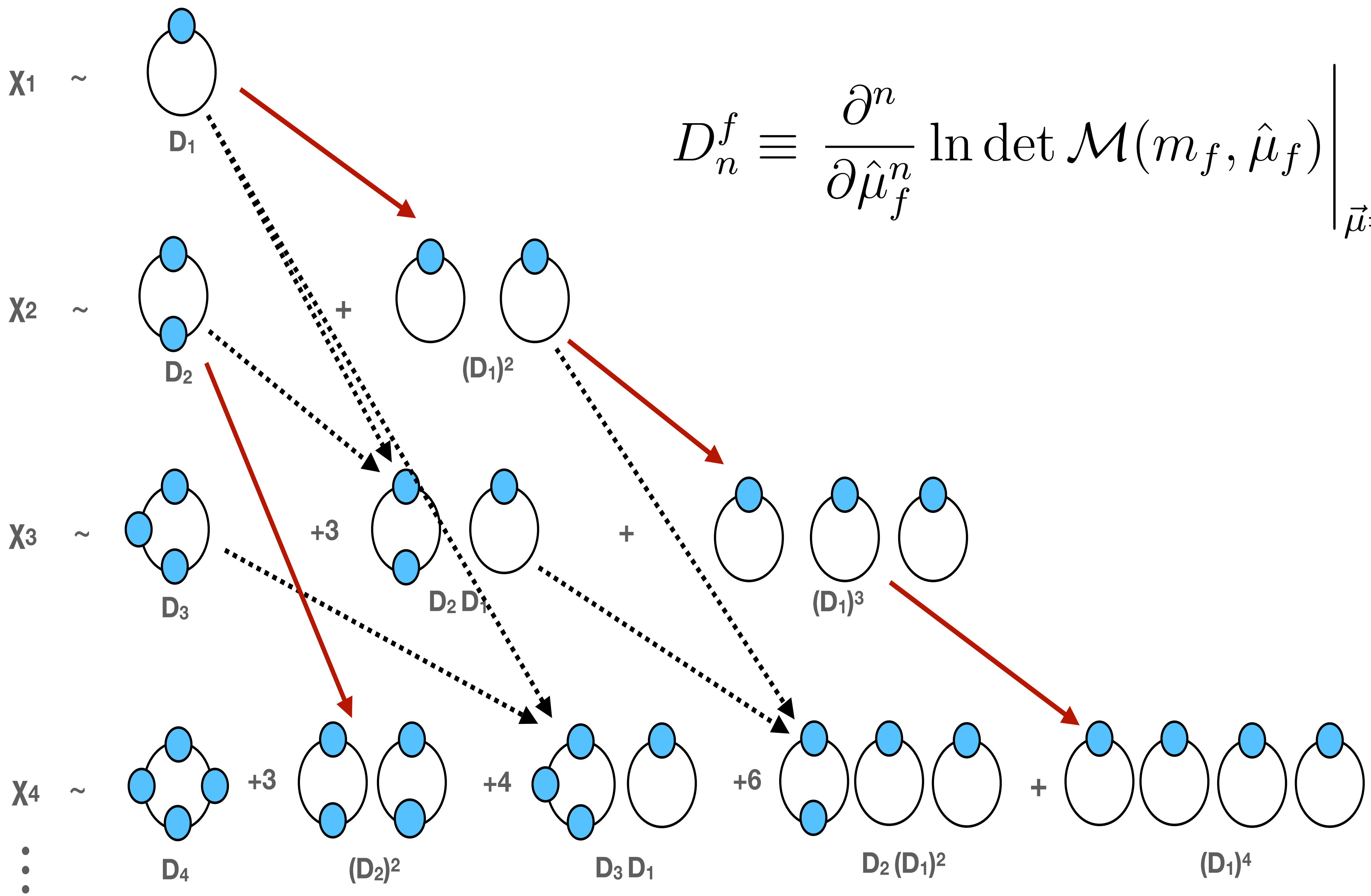
$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \vec{\mu}) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{uds}}{i!j!k!} \hat{\mu}_u^i \hat{\mu}_d^j \hat{\mu}_s^k$$

In the context of heavy ion collision experiments there are 3 **conserved charges**, **B, Q and S** that couples to μ_B, μ_Q, μ_S,

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \quad \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \quad \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S\,.$$

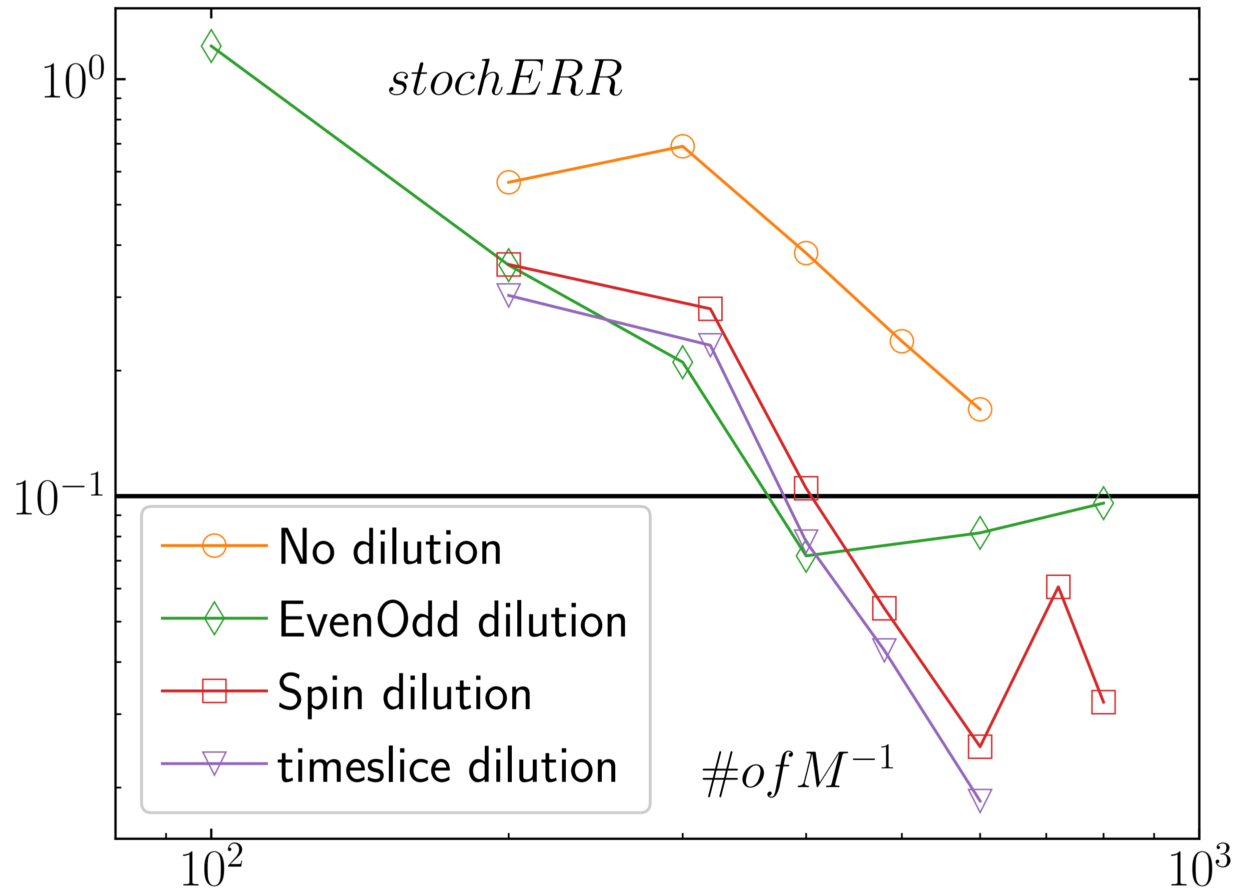
$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{uds}}{i!j!k!} \hat{\mu}_u^i \hat{\mu}_d^j \hat{\mu}_s^k = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k\,.$$

Quark number susceptibilities for Domain Wall fermions



$$D_n^f \equiv \frac{\partial^n}{\partial \hat{\mu}_f^n} \ln \det \mathcal{M}(m_f, \hat{\mu}_f)\Big|_{\vec{\mu}=0} ,$$

Stochastic error reduction



$$D_1^f = \mathrm{Tr}\left[D(m_f)^{-1}\frac{dD}{d\mu_f}\right] - \mathrm{Tr}\left[D(m_{\mathrm{PV}})^{-1}\frac{dD}{d\mu_f}\right]$$
$$\simeq \frac{1}{N_n}\sum_{j=1}^{N_n}\left[\eta_j^\dagger D(m_f)^{-1}\frac{dD}{d\mu_f}\eta_j - \eta_j^\dagger D(m_{\mathrm{PV}})^{-1}\frac{dD}{d\mu_f}\eta_j\right]$$

Diluted noise vector,

$$D_1^f \simeq \frac{1}{N_n}\sum_{j=1}^{N_n}\left[\sum_{a=1}^{N}\eta_{aj}^\dagger D(m_f)^{-1}\frac{dD(m_f)}{d\mu_f}\eta_{aj} - \sum_{a=1}^{N}\eta_{aj}^\dagger D(m_{\mathrm{PV}})^{-1}\frac{dD(m_{\mathrm{PV}})}{d\mu_f}\eta_{aj}\right]$$

We see 2-3 times error reduction using Spin and time slice dilution.

Lattice setup and Outline

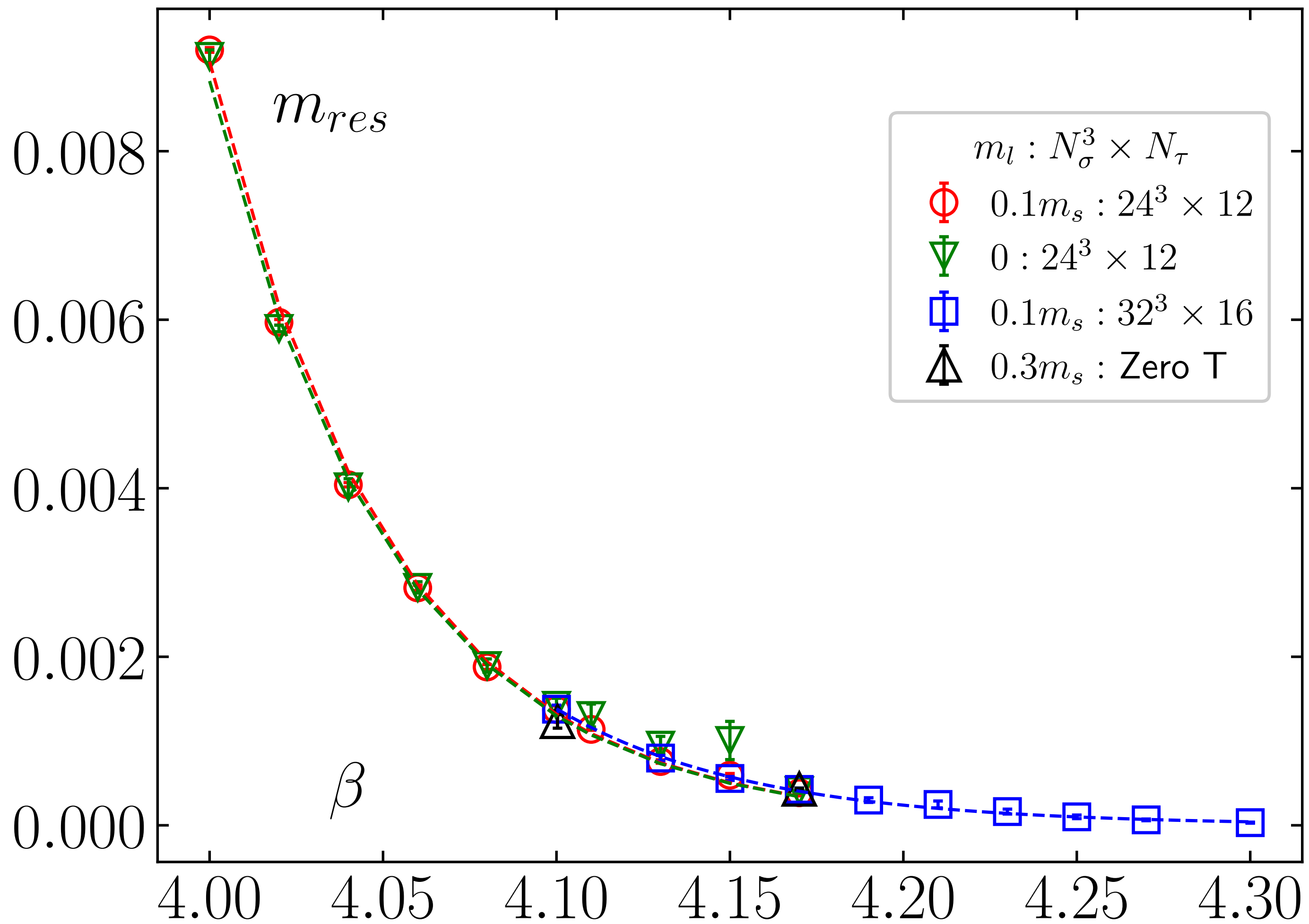
- m_{res} and tuning of the input quark masses.

- Quark number susceptibilities and conserved charge fluctuations for $m_l = 0.0036 m_s (m_\pi \sim 135 \text{ MeV})$ for $36^3 \times 12$.

- Sensitivity of the fluctuations on the pion masses.

- Fourth-order conserved charge fluctuations for physical quark masses.

m_{res} correction for $L_S = 12$

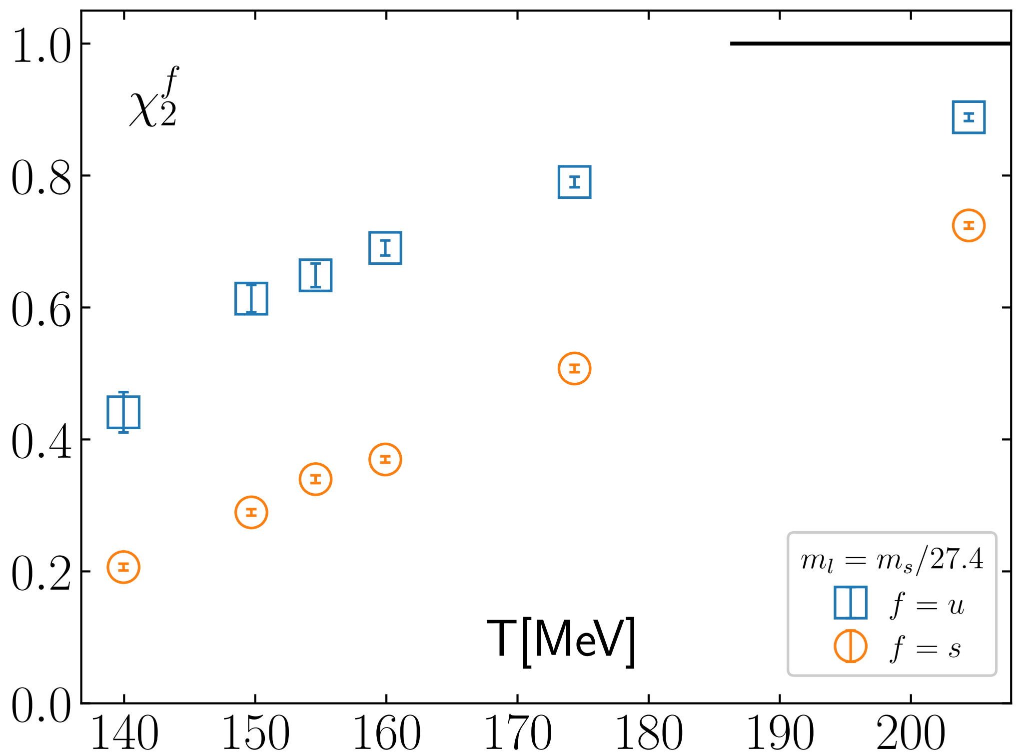


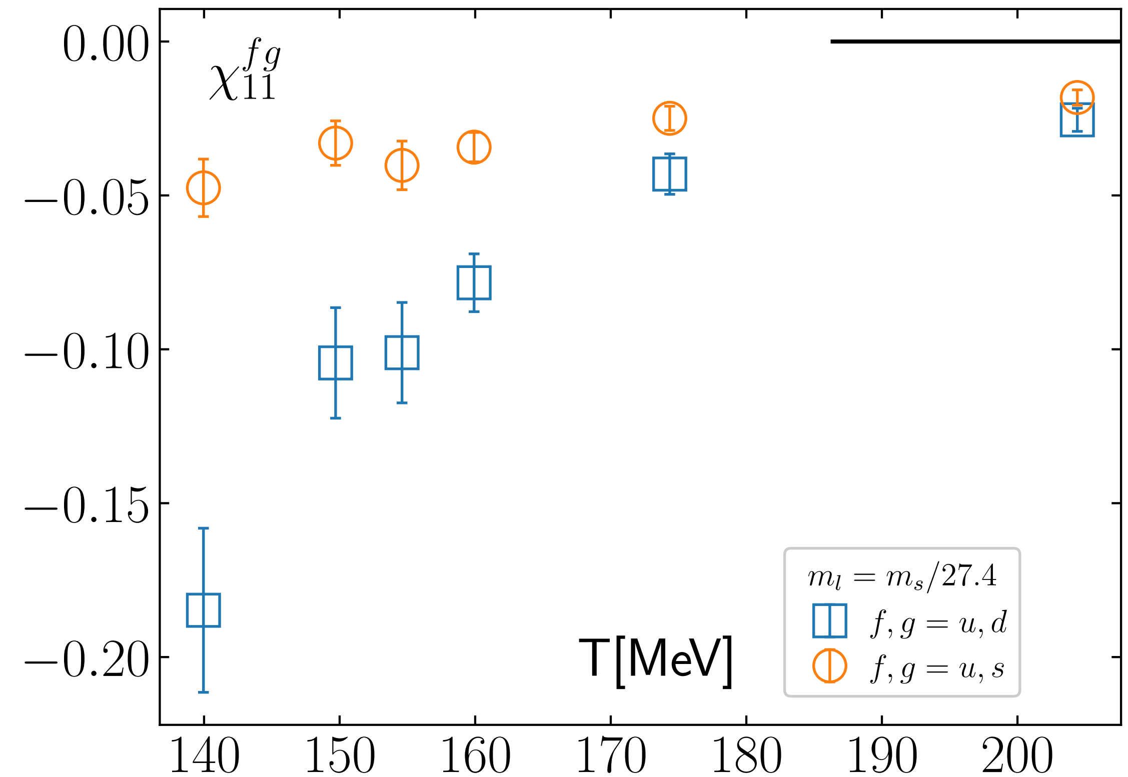
Lattice action : Symanzik with stout smear + Möbius Domainwall fermion.

Tuning of input light quark masses for measurement : $m_f = m_f^{latt} - m_{res}\ ;\ f = \{u, s\}$

- Performed calculations on the Line of constant physics (LCP) (m_l/m_s fixed).
- m_{res} is almost independent of m_l.
- We use m_{res} calculated on the LCP, $m_l/m_s = 0.1$ to tune the input quark masses for configuration generation and measurements for $m_l/m_s = 0.036$

Quark number susceptibility with Möbius Domain Wall Fermions in (2+1)-flavor QCD





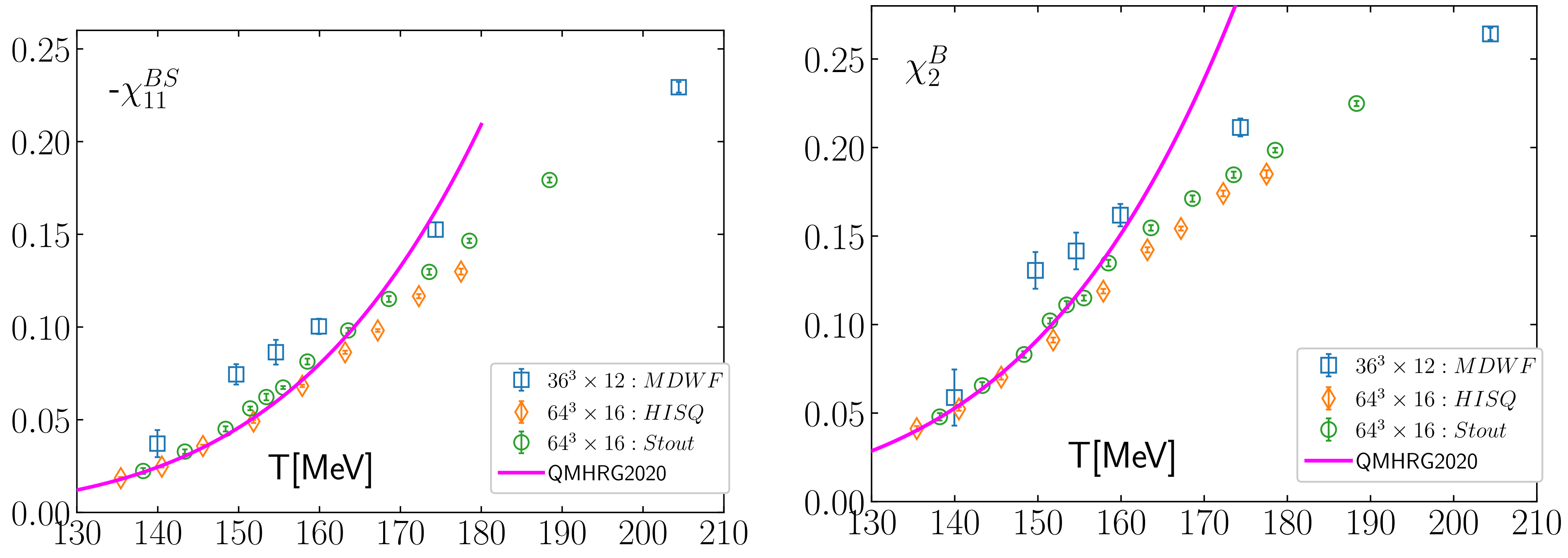
χ_2^f's rise rapidly in the vicinity of the T_{pc}.

At high T: χ_2^f's are smaller than the Ideal gas limit.

χ_{11}^{fg} reaches closer to Ideal gas limit.

In high T PT : $\chi_2^f \sim \chi_2^{f,ideal} + O(g^2)$, $\chi_{11}^{fg} \sim O(g^6 lng)$ **A. Vuorinen, PRD68, 054017 (2003)**

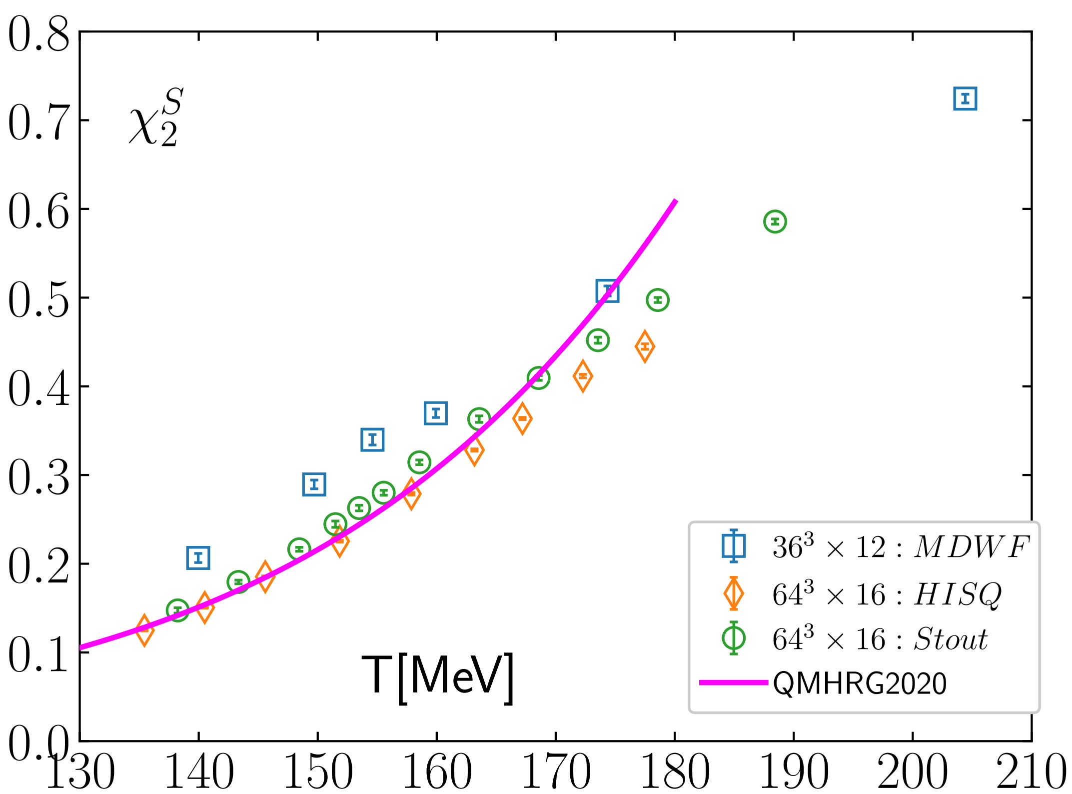
Comparison of χ_2^B calculations with Möbius Domain Wall Fermions and Staggered fermions

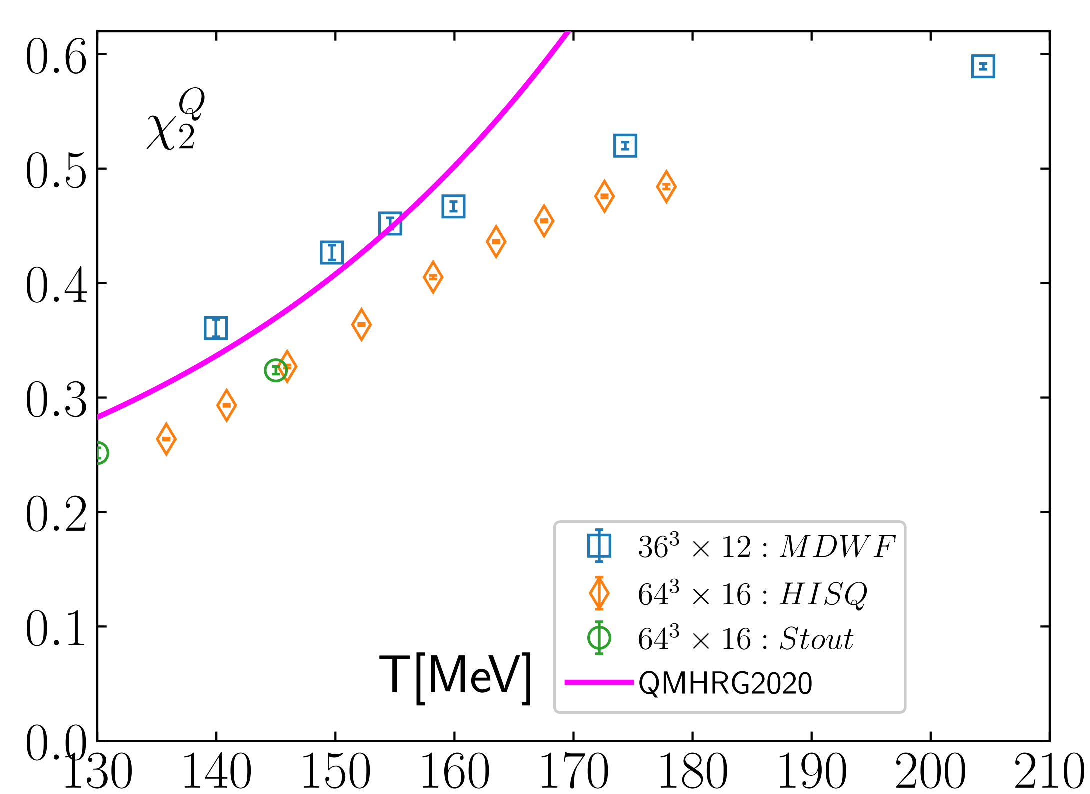


- **Data Comparison**: Our lattice data are systematically higher than those from HISQ and stout smeared staggered quarks near the pseudo-critical temperature.
- **Measurements**: Performed on 150 gauge configurations per temperature, with 100 trajectory separations.
- **Further Analysis**: Additional lattice spacing and additional volume is required to better understand this discrepancy.

Refs: HiSQ : (HotQCD) D. Bollweg et al, arXiv:2107.10011 [hep-lat].
Stout : (WB) R. Bellwied et al, arXiv:1910.14592 [hep-lat]

Comparison of χ_2^Q, χ_2^S calculations with Möbius Domain Wall Fermions and Staggered fermions

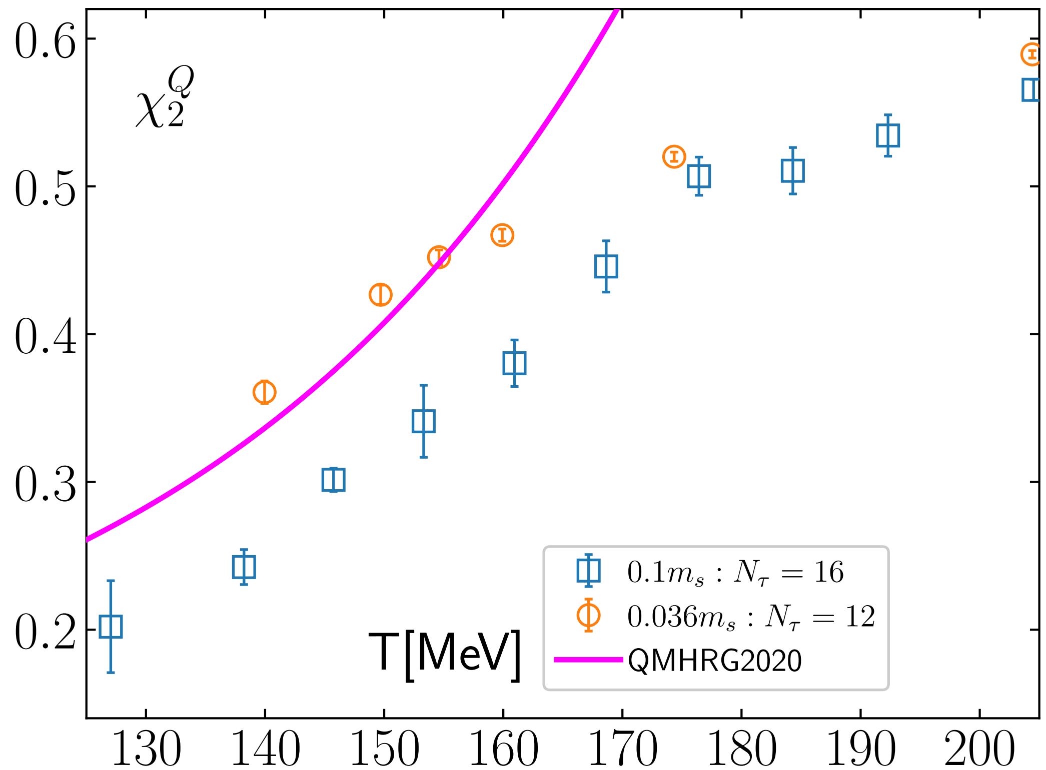


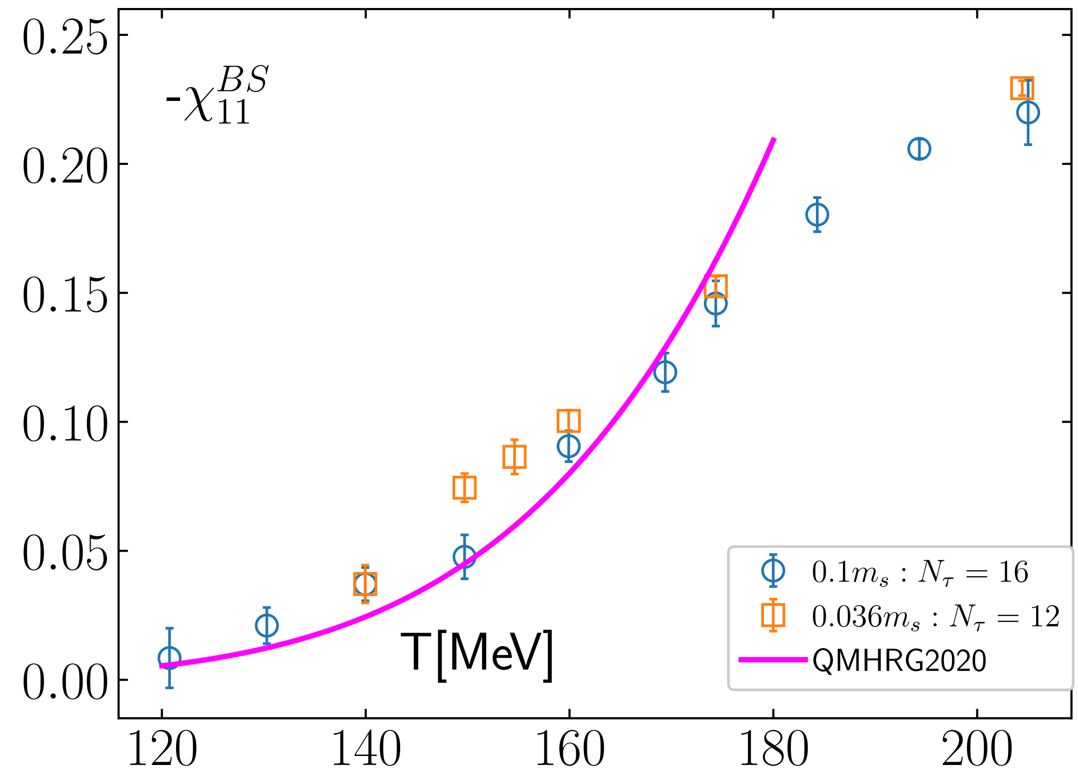


- We saw larger value in the χ_2^Q in the hadronic phase, compared to the HISQ and stout smeared staggered quarks calculations at finite lattice spacing.
- In a non-interacting HRG : χ_2^Q is dominated by pions and χ_2^S is dominated by kaons.

Refs: HISQ : D. Bollweg et al, arXiv:2107.10011 [hep-lat].
Stout : R. Bellwied et al, arXiv:1507.04627 [hep-lat]

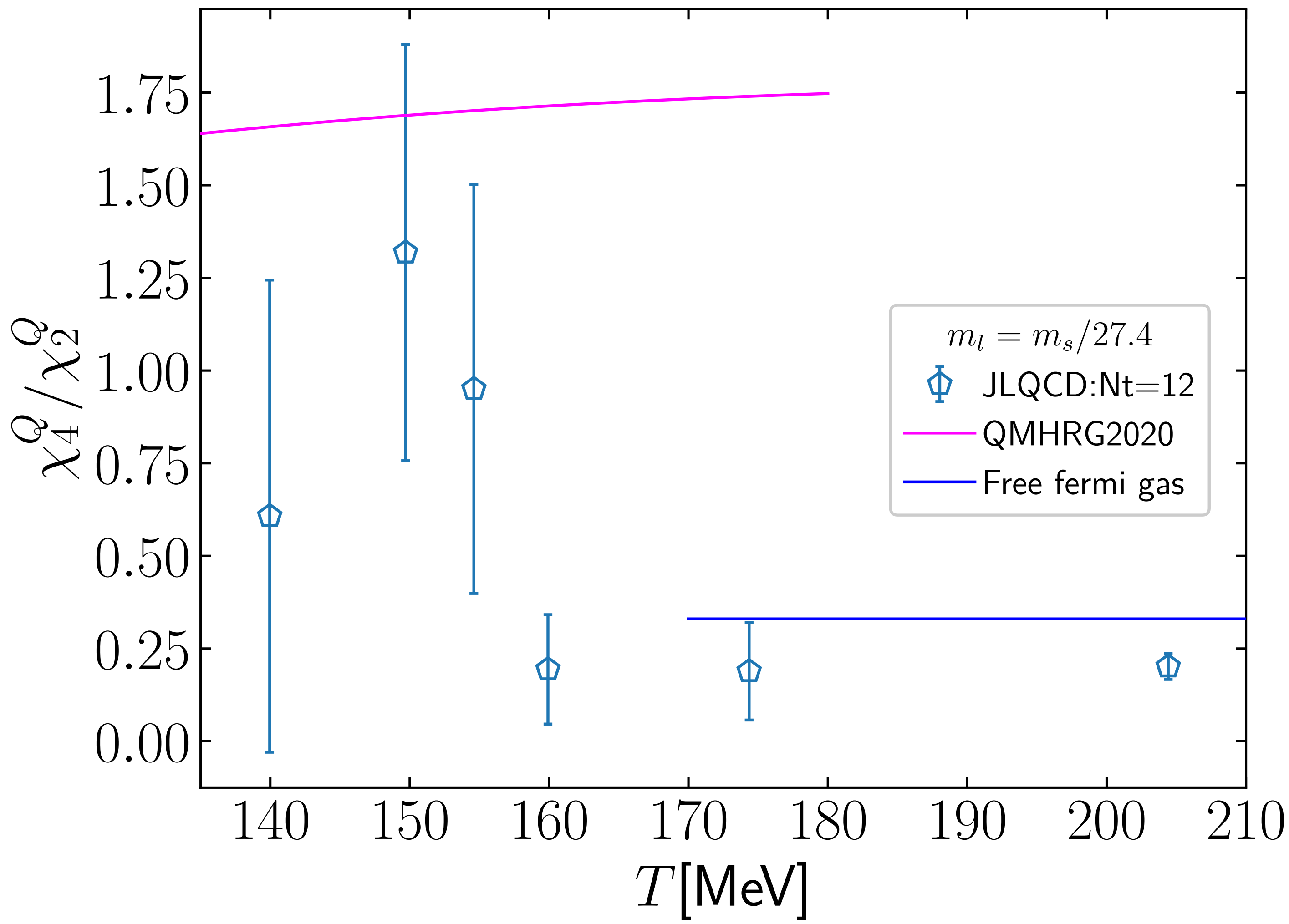
Sensitivity of χ_{11}^{BS}, χ_2^Q on different light quark masses





- In hadronic phase , $\chi_2^X \sim \exp(-m_H/T)$
- In a non-interacting HRG : χ_2^Q is dominated by pions and χ_{11}^{BS} is dominated by lambda baryons.
- We see that χ_2^Q is sensitive to the pion mass in the temperature, $T_{pc} \leq 160\ \mathrm{MeV}$ however χ_{11}^{BS} is not that sensitive to the hadron masses.

Leading order kurtosis of electric charge cumulants



$\vec{\mu} = \{\mu_B, \mu_Q, \mu_S\}$

$R_{42}^Q = \chi_4^Q/\chi_2^Q + O(\vec{\mu}^2)$

Leading order kurtosis value close to the Pseudo-critical temperature,

- $R_{42}^Q = 1.3(5)$, $T = 150$ MeV
- $R_{42}^Q = 0.9(5)$, $T = 155$ MeV

Summary and Conclusions

- We present results of conserved charge fluctuations using (2+1)-flavor QCD with a chiral fermion formalism, specifically Möbius Domain Wall Fermions.
- We compare our calculations of second order fluctuations with the staggered fermion formalism calculations at finite lattice spacing.
- We also present fourth order conserved charge fluctuations for the physical value of the quark masses.
- In future, we will extend our calculations to smaller lattice spacings to study the cut-off dependence of conserved charge fluctuations.

Thank you for your attention !!

Quark number susceptibility with Domain wall fermions

$$Z = \int DU \prod_{f=u,d,s} detM(m_f)\exp[-S_g], \qquad \det M(m_f, \hat{\mu}_f) = \left[\frac{\det D(m_f, \hat{\mu}_f)^{DWF}}{\det D(m_{PV}, \hat{\mu}_f)^{DWF}}\right]$$

$U_4(x) \to \exp(\hat{\mu}_f)U_4(x),\ U_4^\dagger(x) \to \exp(-\hat{\mu}_f)U_4^\dagger(x),$ J. Bloch and T. Wettig, Phys. Rev. Lett. 97, 012003 (2006)

$\hat{\mu}_f = \mu_f/T$, where , μ_f is the quark chemical potential for flavor f.

The diagonal and off-diagonal quark number susceptibilities can be written as,

$$\chi_2^f = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f^2}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\left[\left\langle \frac{\partial^2}{\partial \hat{\mu}_f^2}\ln\det M\right\rangle + \left\langle\left(\frac{\partial}{\partial \hat{\mu}_f}\ln\det M\right)^2\right\rangle\right]$$

$$= \frac{N_\tau}{N_\sigma^3}\langle D_2^f\rangle + \langle (D_1^f)^2\rangle,\ f = \{u, d, s\}$$

M. Cheng et al, Phys.Rev.D81:054510,2010 ; P. Hegde et al, PoS LATTICE2008:187,2008

$$\chi_{11}^{fg} = \frac{N_\tau}{N_\sigma^3}\frac{\partial^2 \ln Z}{\partial \hat{\mu}_f \partial \hat{\mu}_g}\bigg|_{\hat{\mu}_f=0} = \frac{N_\tau}{N_\sigma^3}\langle D_1^f D_1^g\rangle,\ f \neq g,\ f, g = \{u, d, s\}$$

$(D_1^f)^2$ and $D_1^f D_1^g$ are the most noisy part in our calculation

Quark number susceptibility with Domain wall fermions

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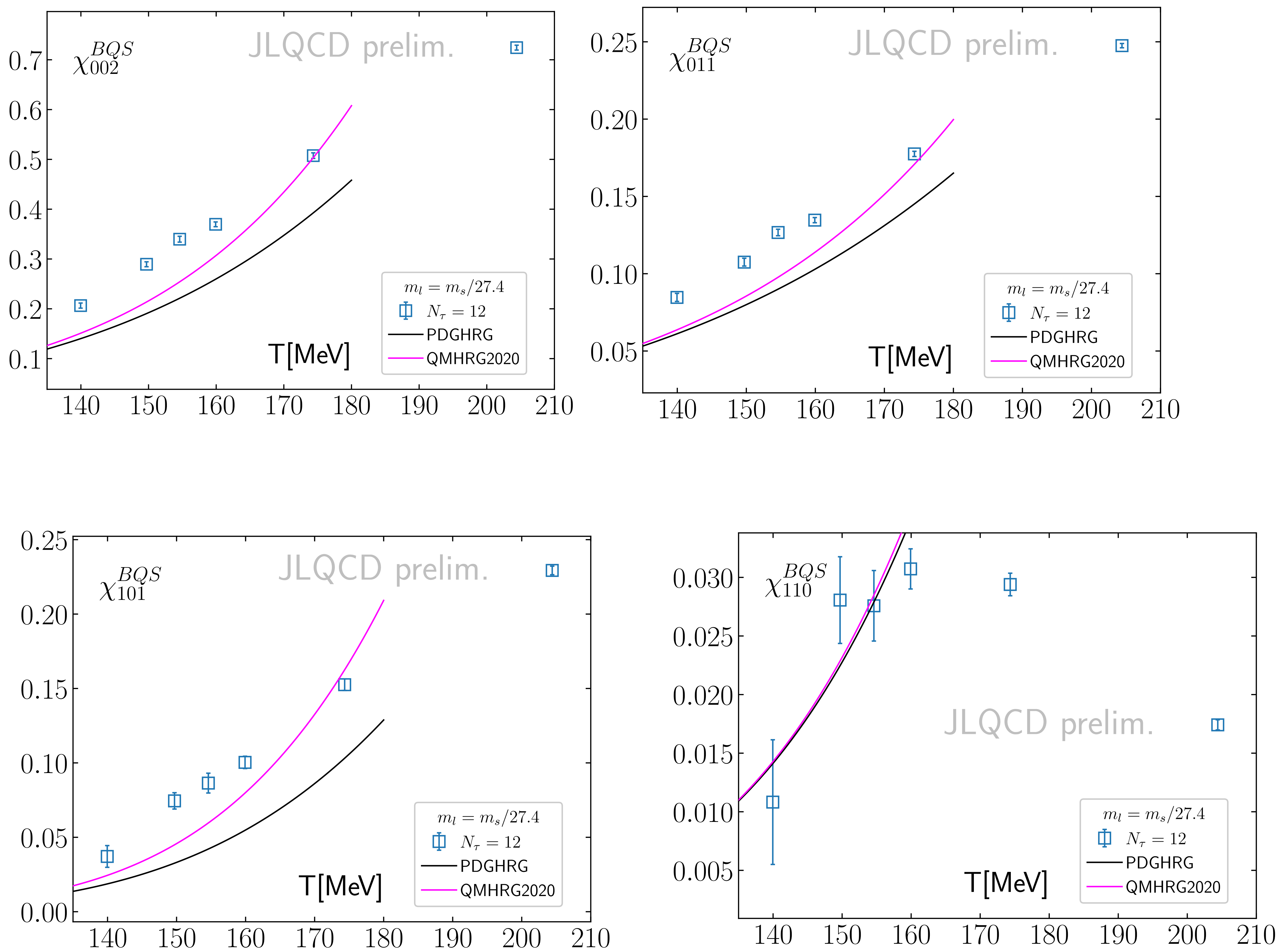
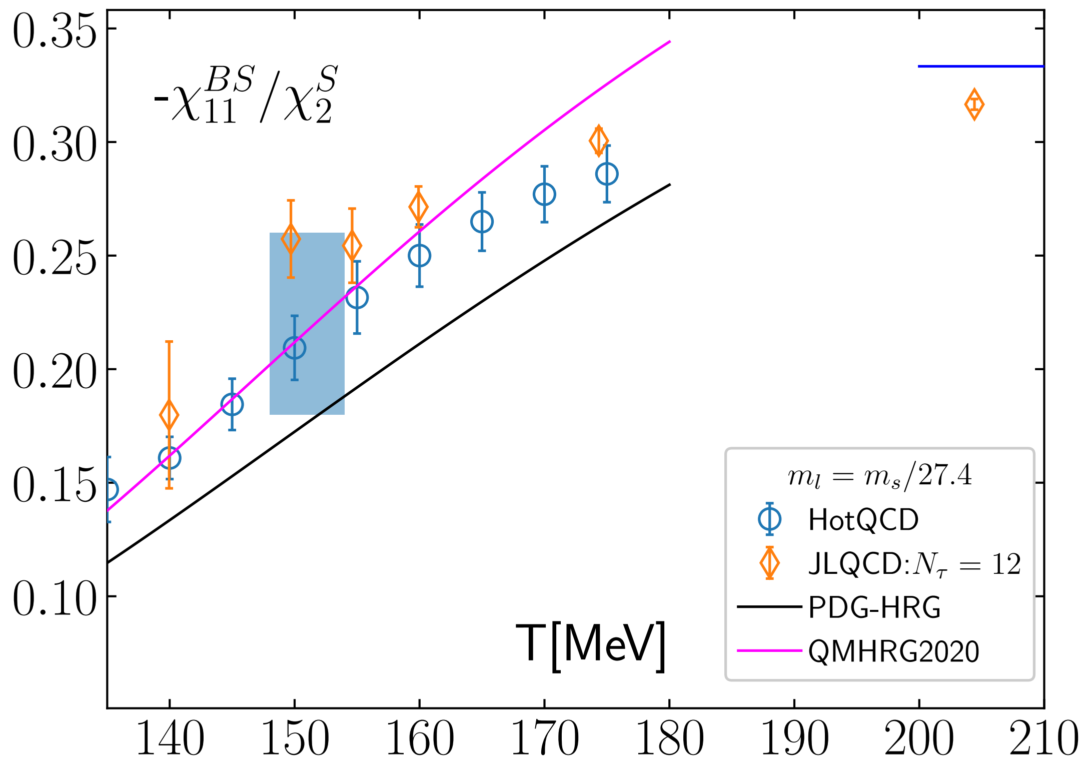

χ_{002}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{011}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{101}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]
χ_{110}^{BQS}
JLQCD prelim.
$m_l = m_s/27.4$
$N_\tau = 12$
PDGHRG
QMHRG2020
T[MeV]



-χ_{11}^{BS}/χ_2^S
0.35
0.30
0.25
0.20
0.15
0.10
140
150
160
170
180
190
200
210
T[MeV]
$m_l = m_s/27.4$
HotQCD
JLQCD:$N_\tau = 12$
PDG-HRG
QMHRG2020