

# **Finite temperature QCD phase transition with 3 flavors of Möbius domain wall fermions**

**Yu Zhang**

**Bielefeld University**

**In collaboration with**

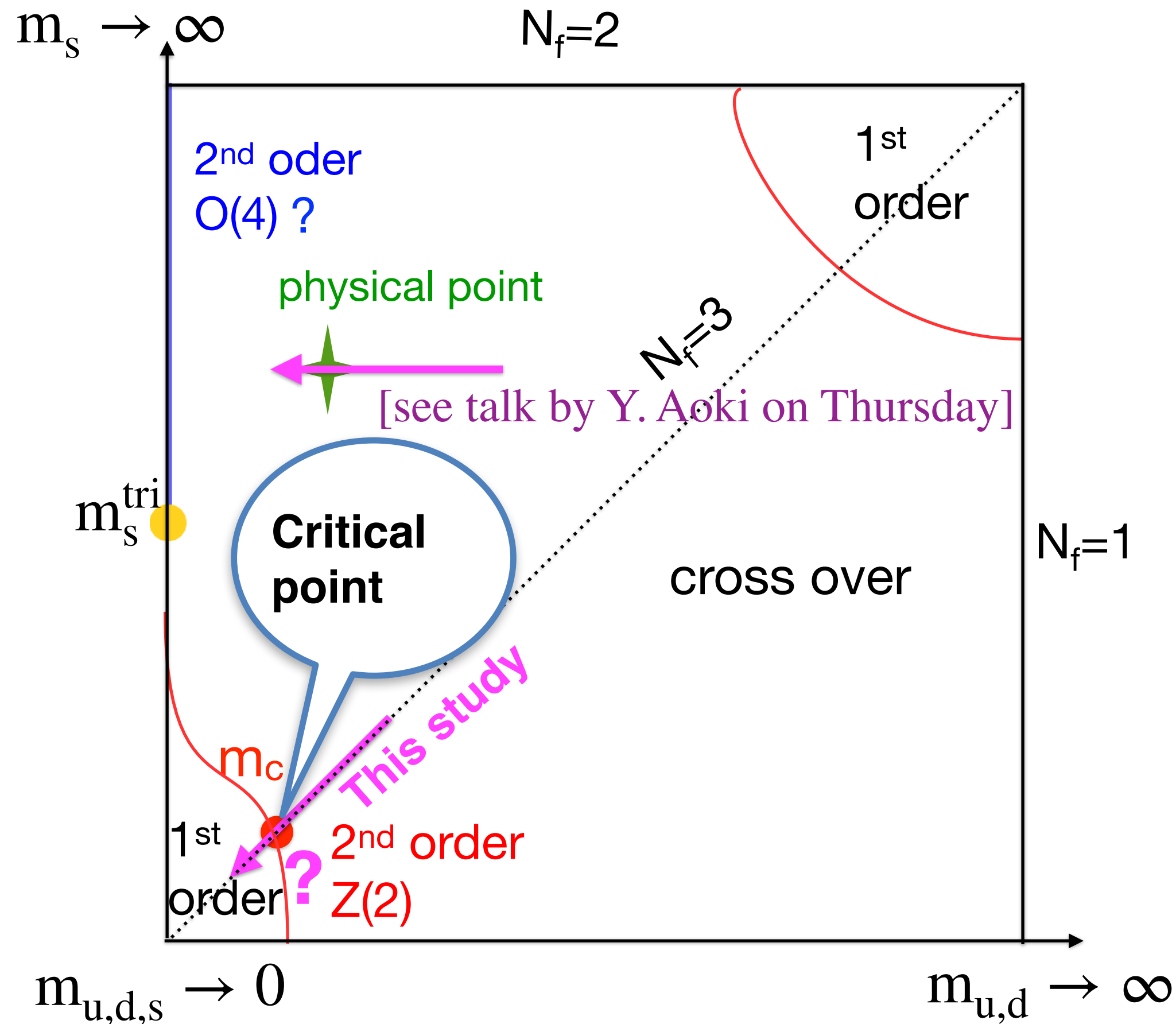
**Y. Aoki, S. Hashimoto, I. Kanamori, T. Kaneko, Y. Nakamura**

**German Japanese Workshop**

**Mainz, Sep 25-27, 2024**

# The nature of QCD phase transition at $\mu_B = 0$

## Columbia plot



- $\epsilon$  expansion: 1<sup>st</sup> order phase transition in the chiral limit for  $N_f = 3$   
[R. D. Pisarski, F. Wilczek PRD 84]
- Possible 2<sup>nd</sup> order phase transition in the  $N_f = 3$  chiral limit:  
[G. Fejos, PRD 22]  
[S. R. Kousvos, A. Stergiou SciPost 23]  
[J. Bernhardt, C. S. Fischer PRD 23]  
[R. D. Pisarski, F. Rennecke PRL 24]  
[G. Fejos, T. Hatsuda PRD 24]

Need to be checked by lattice QCD

### Critical point on the $N_f = 3$ chiral region:

- Location? (existence?)
- Universality class?

# Previous Nf=3 lattice QCD studies

Action	$N_t$	$m_\pi^{Z_2}$ [MeV]	Ref.
Staggered, standard	4	290	Karsch et al. (2001)
Staggered, standard	6	150	de Forcrand et al. (2007)
Staggered, HISQ	6	$\lesssim 50$	Bazavov et al. (2017)
Staggered, stout	4-6	0?	Varnhost (2014)
Wilson, standard	4	$\lesssim 670$	Iwasaki et al. (1996)
Wilson-Clover	6-10	$\lesssim 170$	Jin et al. (2017)
Wilson-Clover	6-12	$\lesssim 110$	Kuramashi et al. (2020)

➔ Strong cutoff and discretization effects

Evidence for chiral limit to feature 2nd order PT with staggered and HISQ fermion [F. Cuteri et al. JHEP 2021; S. Sharma et al. PRD 2022]

[see talk by O. Philipsen on Thursday]

**We propose to use chiral fermion (Mobius domain wall fermion)**

- Exact chiral symmetry at finite  $a$  for infinite Ls
- Reduced  $\chi_{SB}$  parameterized by residual mass when Ls is finite

# Lattice Setup

•  $N_f=3$  Mobius Domain Wall Fermion

• Tree-level Symanzik improved gauge action and stout smearing

★  $T=0$ :

$$\beta = 4.0, 24^3 \times 48 \times 16 : \quad 0.02 \leq am_q \leq 0.045, am_{\text{res}}(\text{estimated}) \approx 0.006$$

$$\beta = 4.1, 24^3 \times 48 \times 16 : \quad 0.015 \leq am_q \leq 0.040$$

$$\beta = 4.17, 32^3 \times 64 \times 16 : \quad 0.012 \leq am_q \leq 0.026:$$

★  $T=121(2)$  MeV ( $\beta = 4.0$ , ( $a = 0.1361(20)$  fm, determined from Wilson flow  $t_0$ ))

$$N_s^3 \times 12 \times 16 : \quad N_s = 48, -0.004 \leq am_q \leq -0.003$$

$$N_s = 36, -0.005 \leq am_q \leq 0.001$$

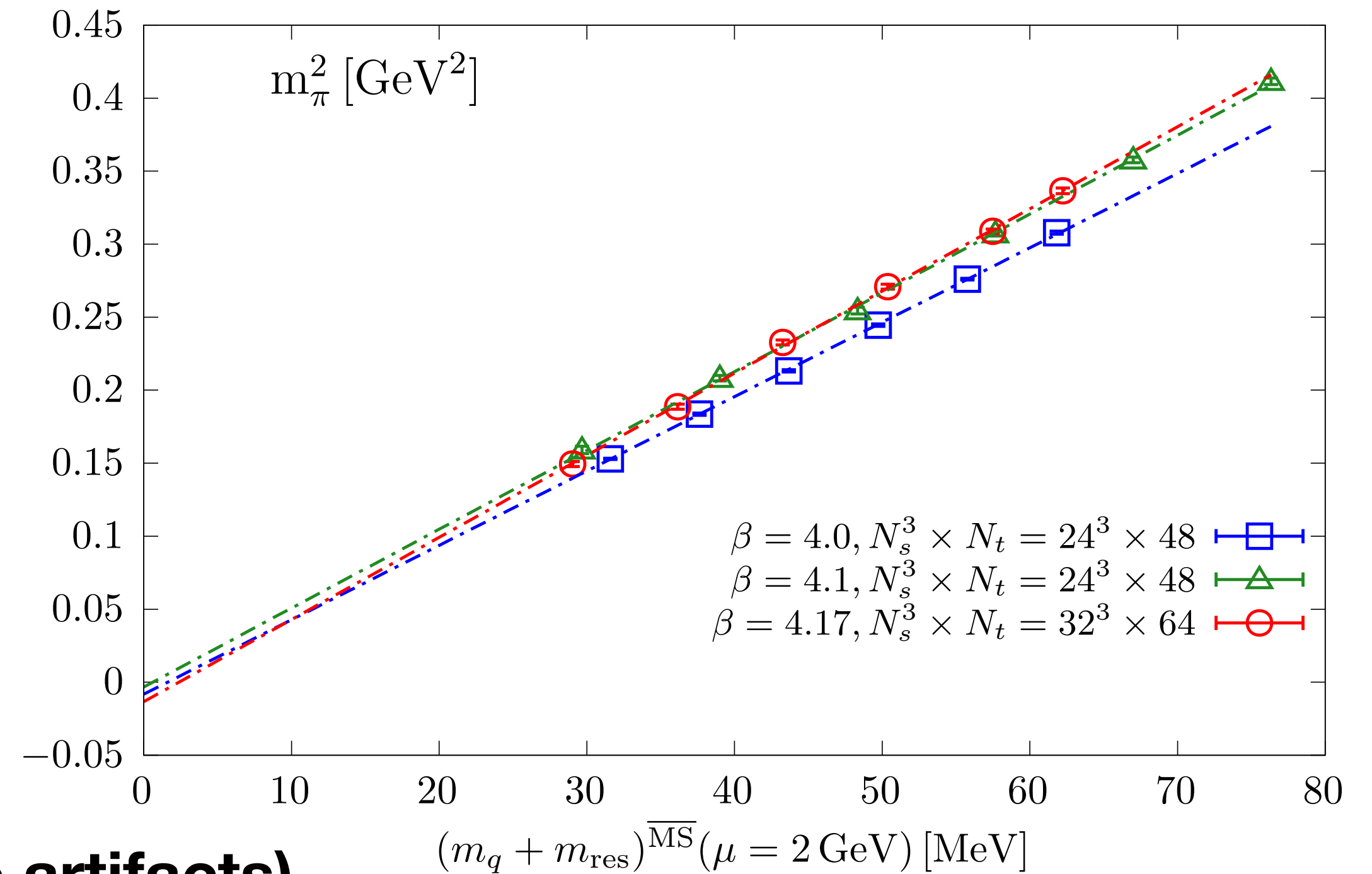
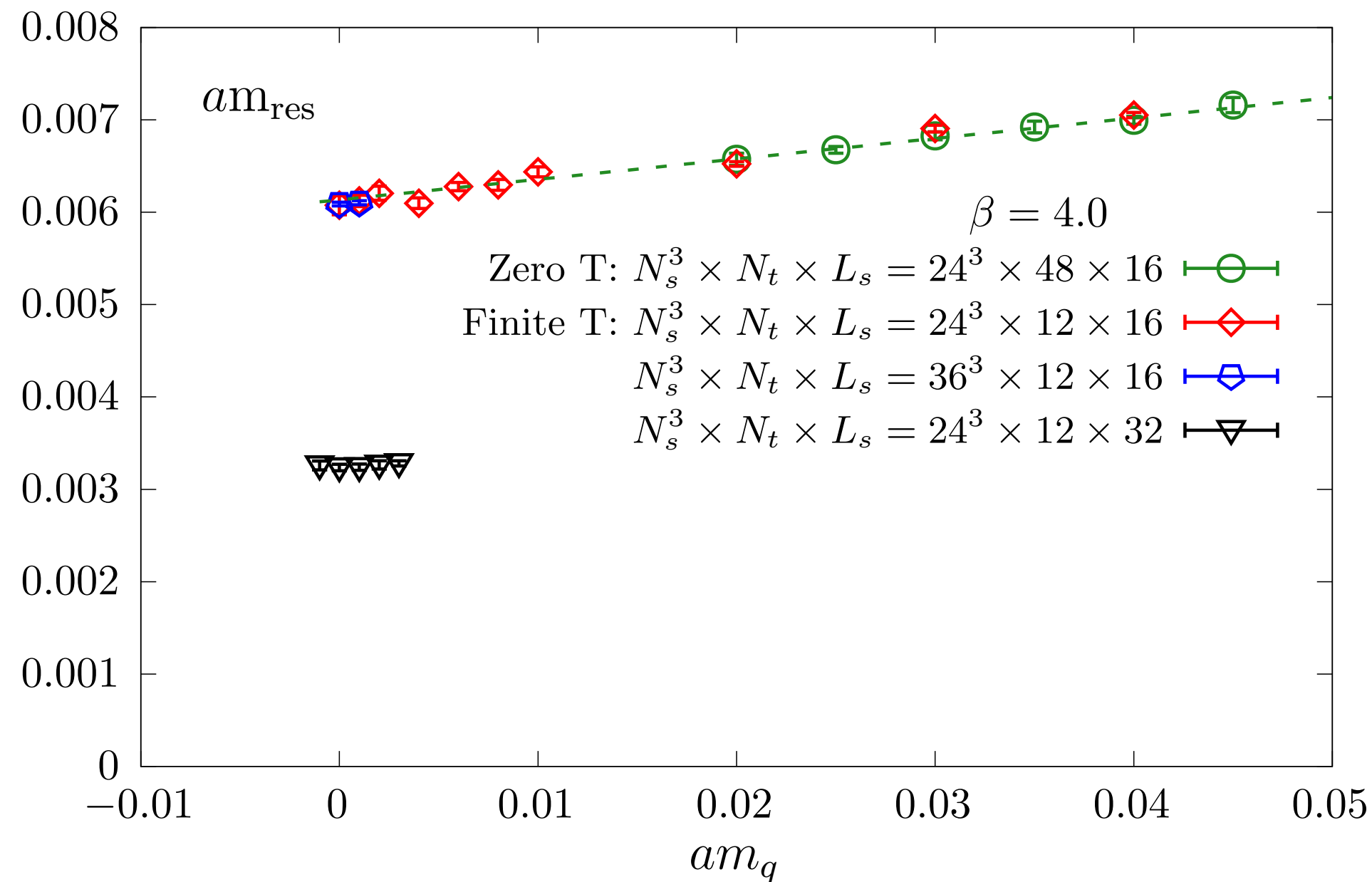
$$N_s = 24, -0.006 \leq am_q \leq 0.1$$

$$N_s^3 \times 12 \times 32 : \quad N_s = 24, -0.001 \leq am_q \leq 0.003$$

# Residual chiral symmetry breaking

$$m_{\text{res}} = \frac{\left\langle \sum_{\vec{x}} J_{5q}^a(\vec{x}, t) P^a(\vec{0}, 0) \right\rangle}{\left\langle \sum_{\vec{x}} P^a(\vec{x}, t) P^a(\vec{0}, 0) \right\rangle} \Big|_{t \geq t_{\text{min}}}$$

- For finite  $L_s$  chiral symmetry is broken, leading to an additive renormalization of the mass:  $m_q \rightarrow m_q + m_{\text{res}}$



- $m_{\text{res}}$  has a linear dependence on  $m_q$  (lattice artifacts)
  - ▶ At  $am_q = 0$ :  $am_{\text{res}} = 0.00613(9)$  for  $L_s = 16$ ,  $am_{\text{res}} = 0.00324(3)$  for  $L_s = 32$
- At strong coupling,  $m_{\text{res}}$  dominated by gauge field dislocations, suppressed by  $1/L_s$
- $m_\pi$  almost vanishes at chiral limit

# Chiral condensate

$$\langle \bar{\psi}\psi \rangle |_{DWF} \sim \langle \bar{\psi}\psi \rangle |_{\text{cont.}} + C^D \frac{m_q + xm_{res}}{a^2} + \dots$$

[S. Sharpe, arXiv: 0706.0218]

•  $x = \mathcal{O}(1)$  but  $x \neq 1$

• Additive divergence remains by  $m = m_q + m_{res} \rightarrow 0$  :

[see talk by Y. Aoki on Thursday]

$$\lim_{m \rightarrow 0} \lim_{L \rightarrow 0} \langle \bar{\psi}\psi \rangle |_{DWF} \sim \langle \bar{\psi}\psi \rangle |_{\text{cont.}} + C^D \frac{(x-1)m_{res}}{a^2}$$

## Two ways of subtracting divergences:

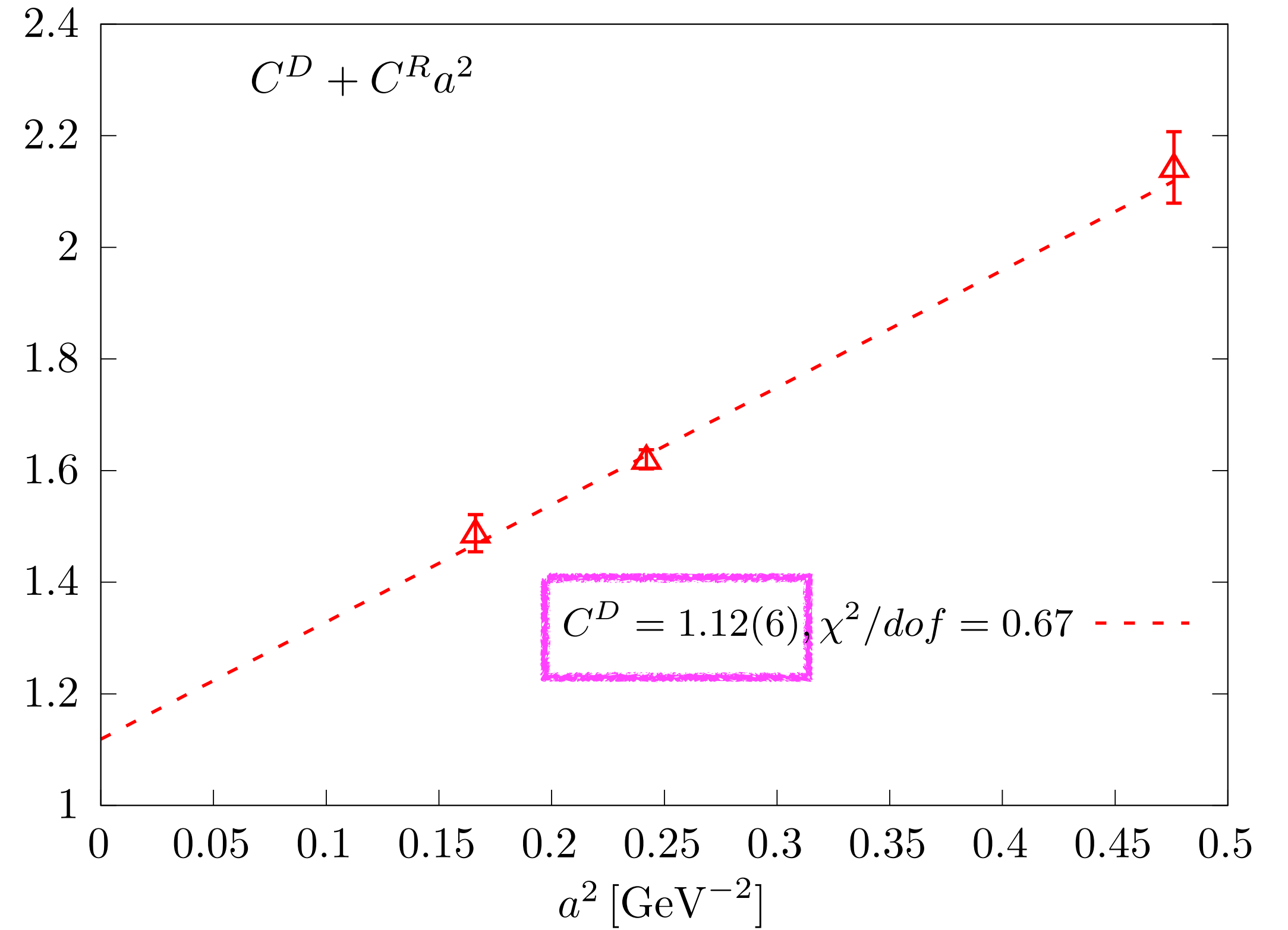
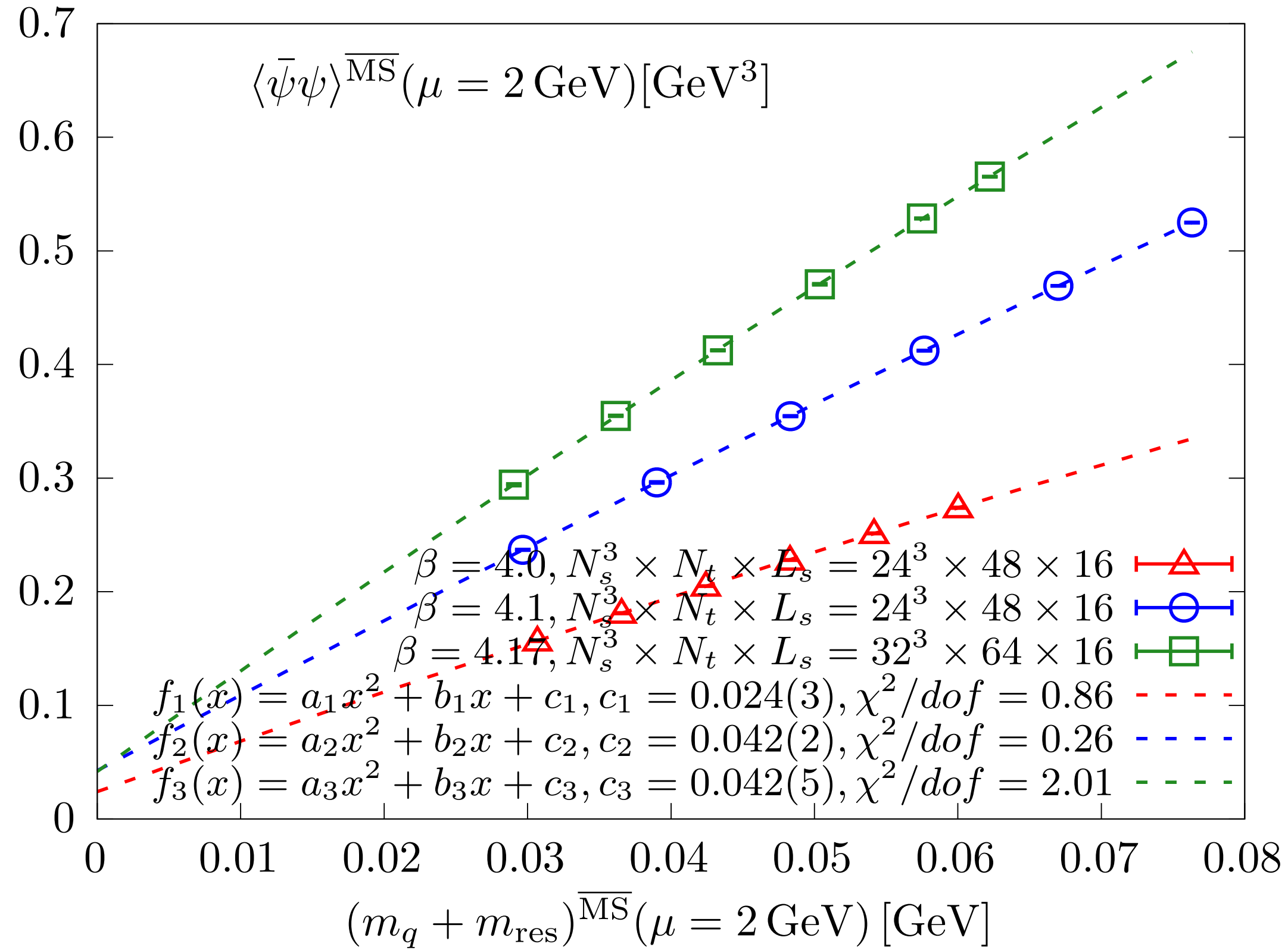
$$\bullet \langle \bar{\psi}\psi \rangle^{\text{ren}} = Z_m^{-1} \left[ \langle \bar{\psi}\psi \rangle^{T>0} - \langle \bar{\psi}\psi \rangle^{T=0} \right]$$

$$\bullet \langle \bar{\psi}\psi \rangle^{\text{ren}} = Z_m^{-1} \left[ \langle \bar{\psi}\psi \rangle - C^D \frac{m_q + xm_{res}}{a^2} \right], \text{ If we know } C^D \text{ and } x$$



# Zero T results

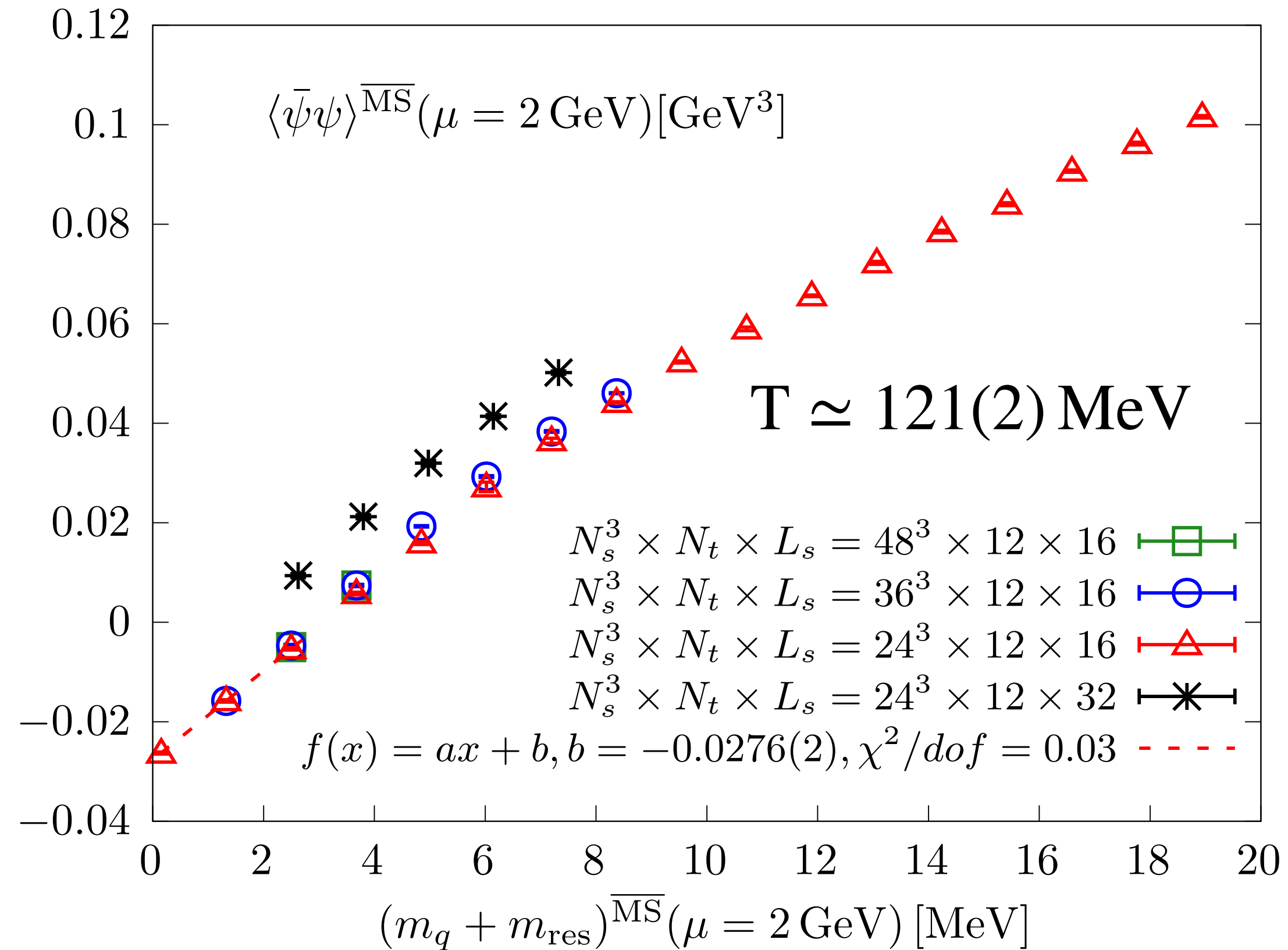
Calculate  $C^D$  for subtracting UV divergence term  $C^D \frac{m_q + xm_{res}}{a^2}$



$$\begin{aligned}
 \langle \bar{\psi}\psi \rangle(m_q + m_{res}) &= \langle \bar{\psi}\psi \rangle(0) + C^D \frac{m_q + xm_{res}}{a^2} + C^R(m_q + m_{res}) + A(m_q + m_{res})^2 \\
 &= \langle \bar{\psi}\psi \rangle(0) + (C^D + C^R a^2) \frac{m_q + m_{res}}{a^2} + C^D \frac{(x-1)m_{res}}{a^2} + A(m_q + m_{res})^2
 \end{aligned}$$

# Finite T results

Calculate  $x$  for subtracting UV divergence term  $C^D \frac{m_q + xm_{res}}{a^2}$



$$\lim_{(m_q + m_{res}) \rightarrow 0} \langle \bar{\psi}\psi \rangle |_{DWF} \sim \langle \bar{\psi}\psi \rangle |_{cont.} + C^D \frac{(x-1)m_{res}}{a^2}$$

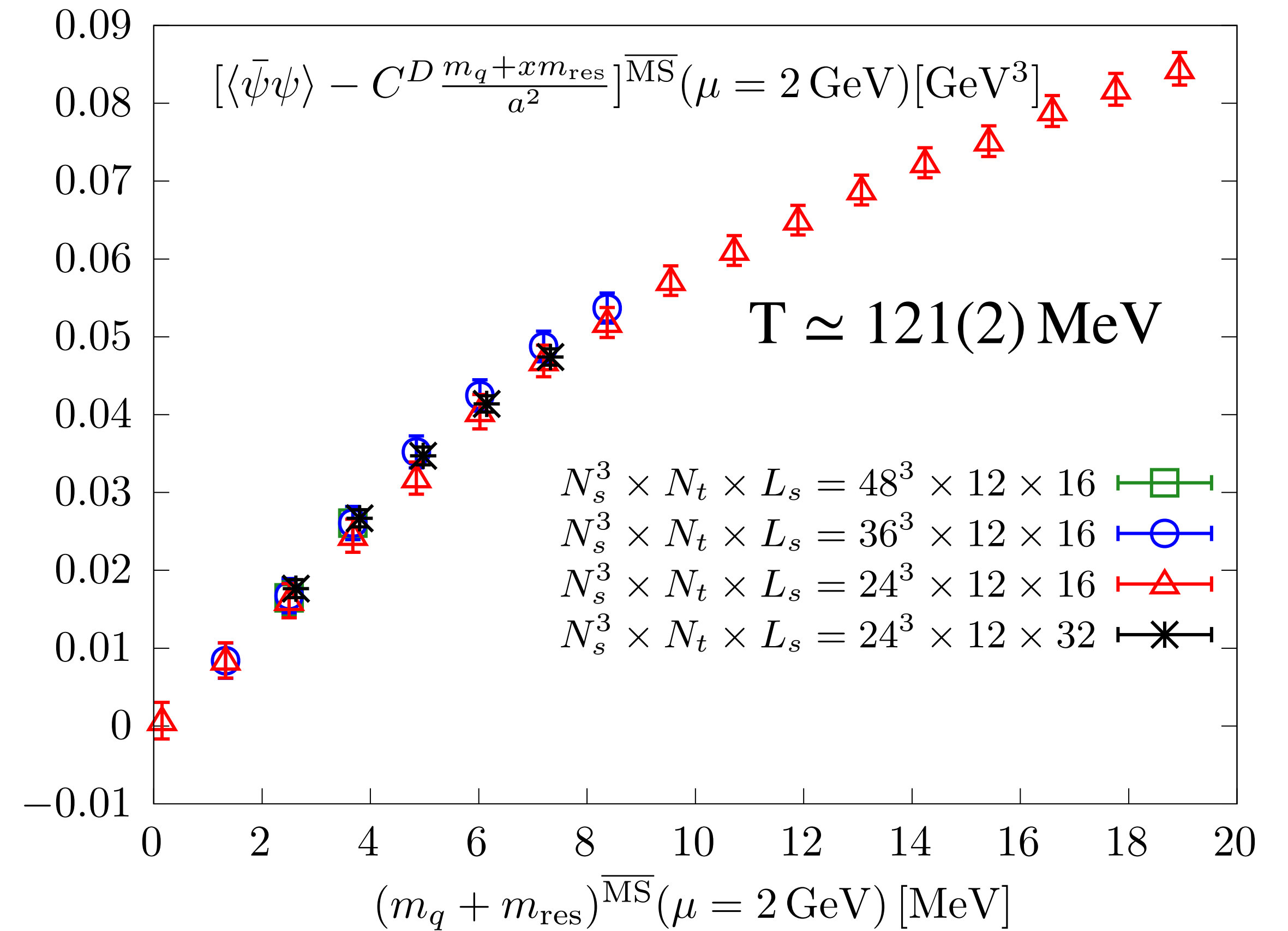
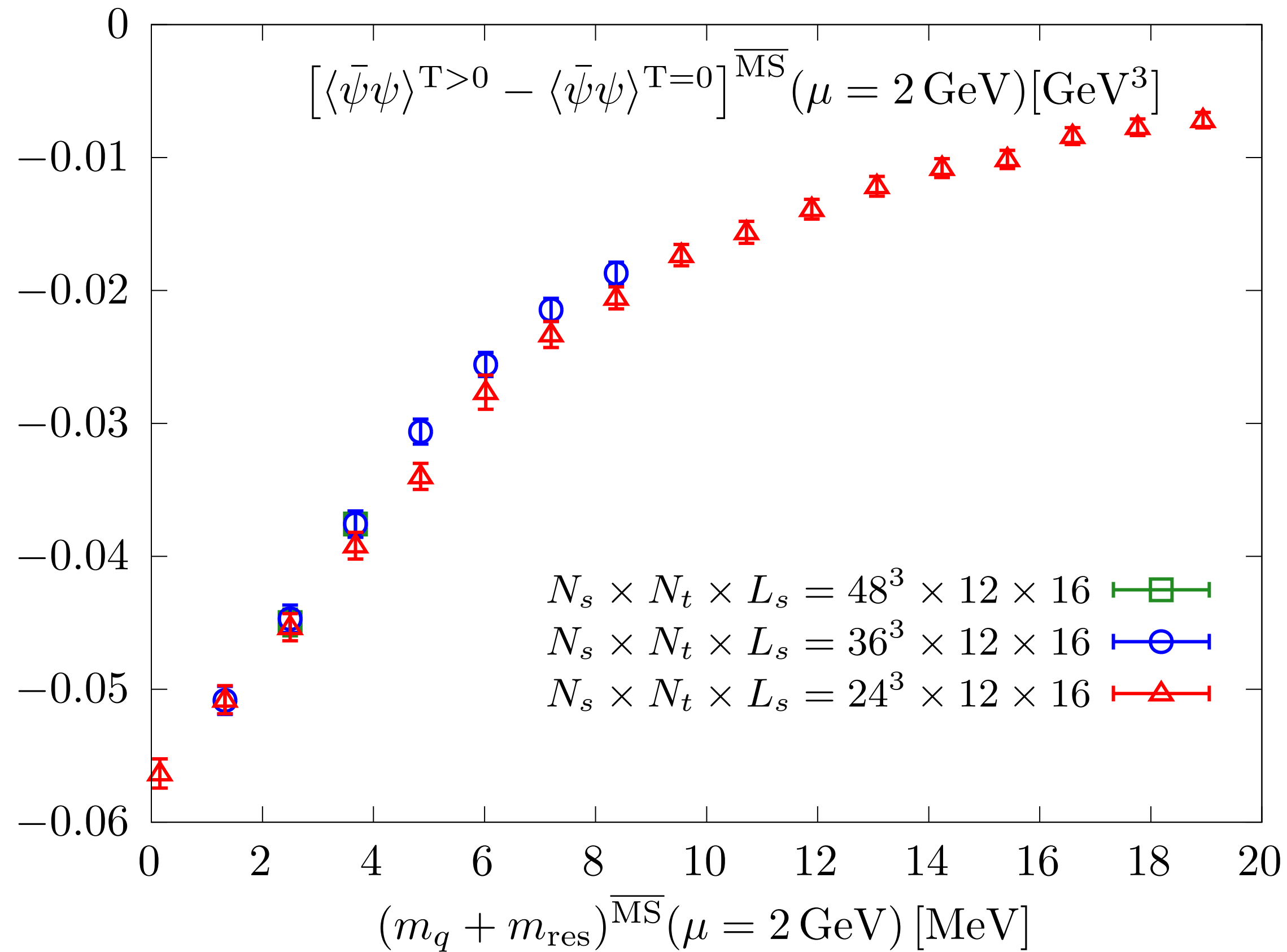
$x = -0.6(1) \text{ for } T > T_c$

$$T_c = 98_{-6}^{+3} \text{ MeV}$$

[S. Sharma et al. PRD 2022]

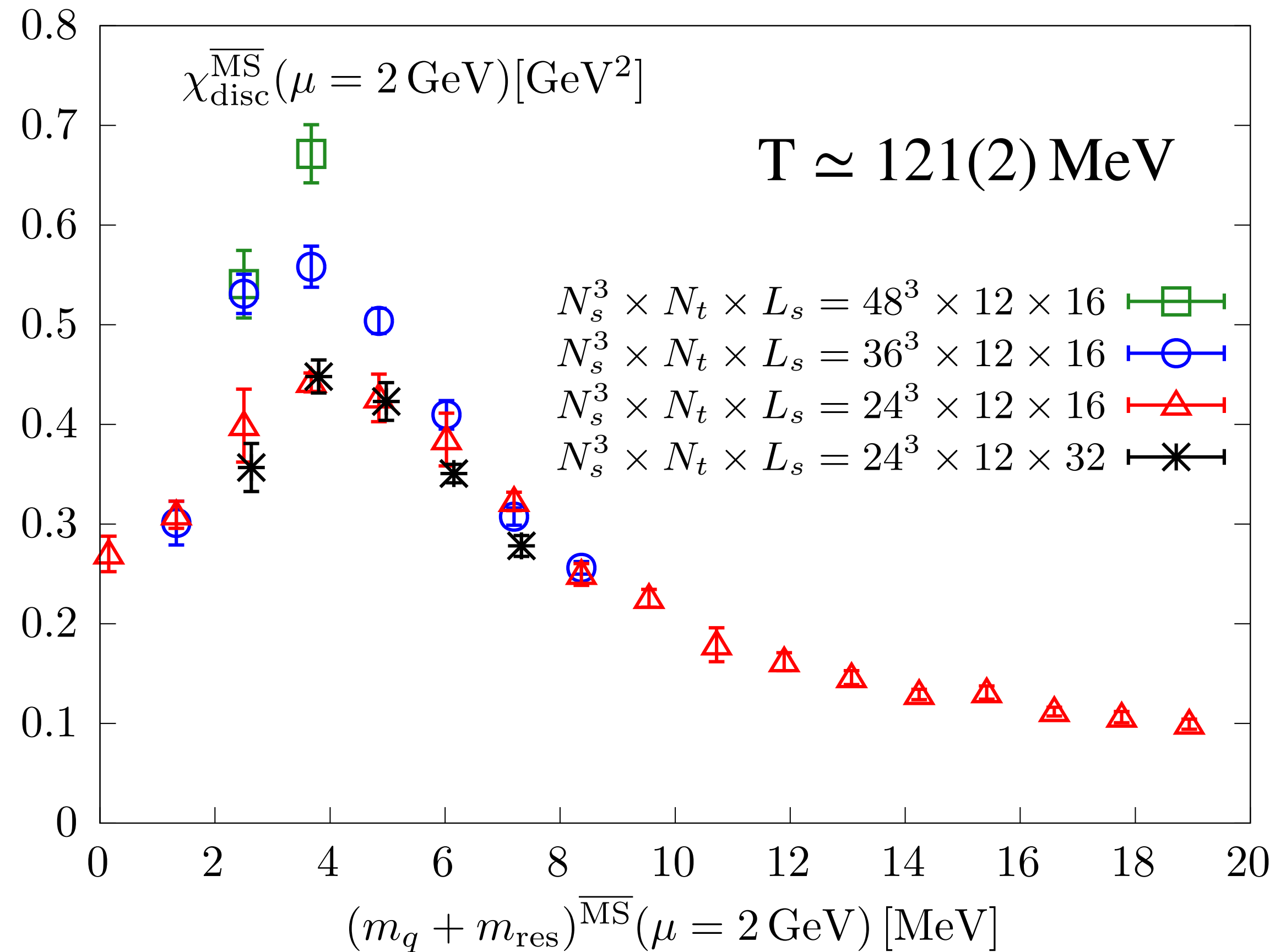


# Renormalized chiral condensate



- **Subtracted chiral condensate vanishes in the chiral limit**
- **Finite volume effect is visible at low T**
- **Subtracted chiral condensate remains the same with same total quark mass for varying Ls**

# Disconnected chiral susceptibility



- **Large finite volume effect near the transition point, but the change in peak height is not as large as anticipated from a real phase transition**
- ➔ **Consistent with the crossover transition**

- **The transition mass point is around 3.6 MeV**

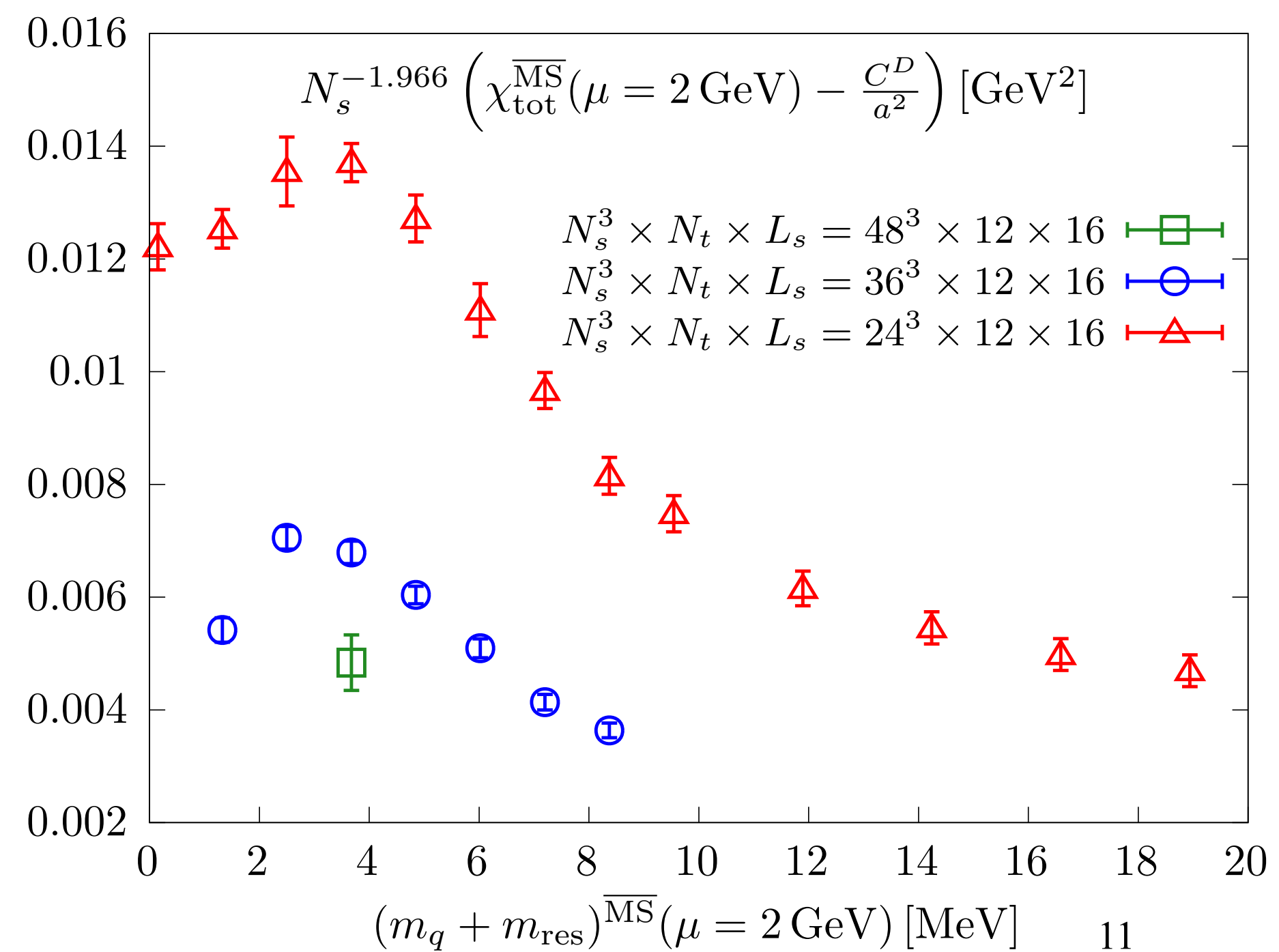
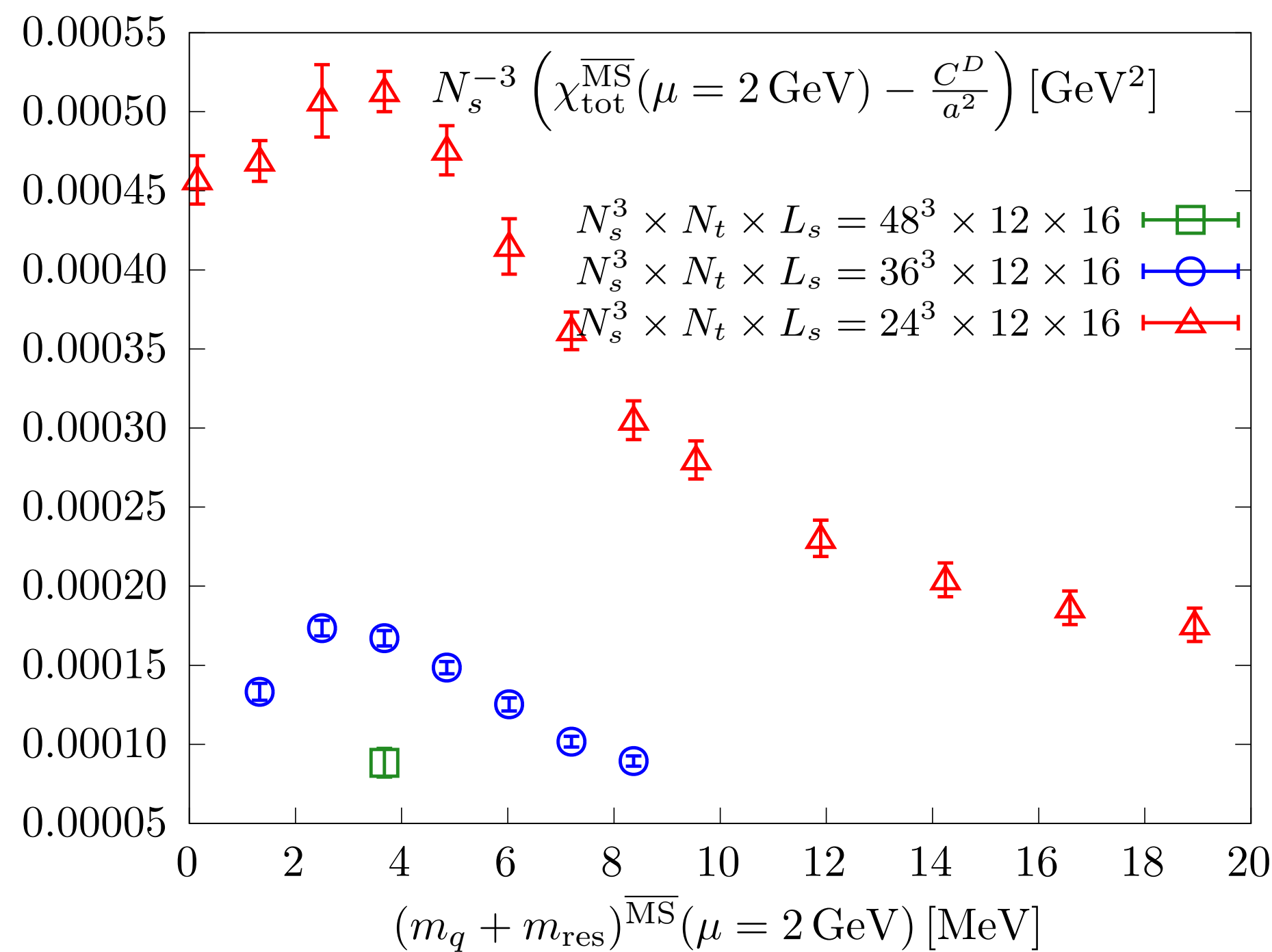
(FLAG Review '21:  $m_{ud}^{\overline{MS}}(2 \text{ GeV}) = 3.381(40) \text{ MeV}$ )

- $\chi_{disc}$  **seems to be function of total quark mass**

# Finite size scaling: susceptibility

- Crossover:  $\chi_{tot}^{\max}(N_s, N_t)$  independent of  $V$
- 1st order PT:  $\chi_{tot}^{\max}(N_s, N_t) \propto V$
- Z(2) 2nd order PT: a singular behavior should be observed in  $\chi_{tot}^{\max}(N_s, N_t)$  with  $V$   
 $((N_s^3 \times N_t)^\alpha, \alpha = 1.966$  is the critical exponent)

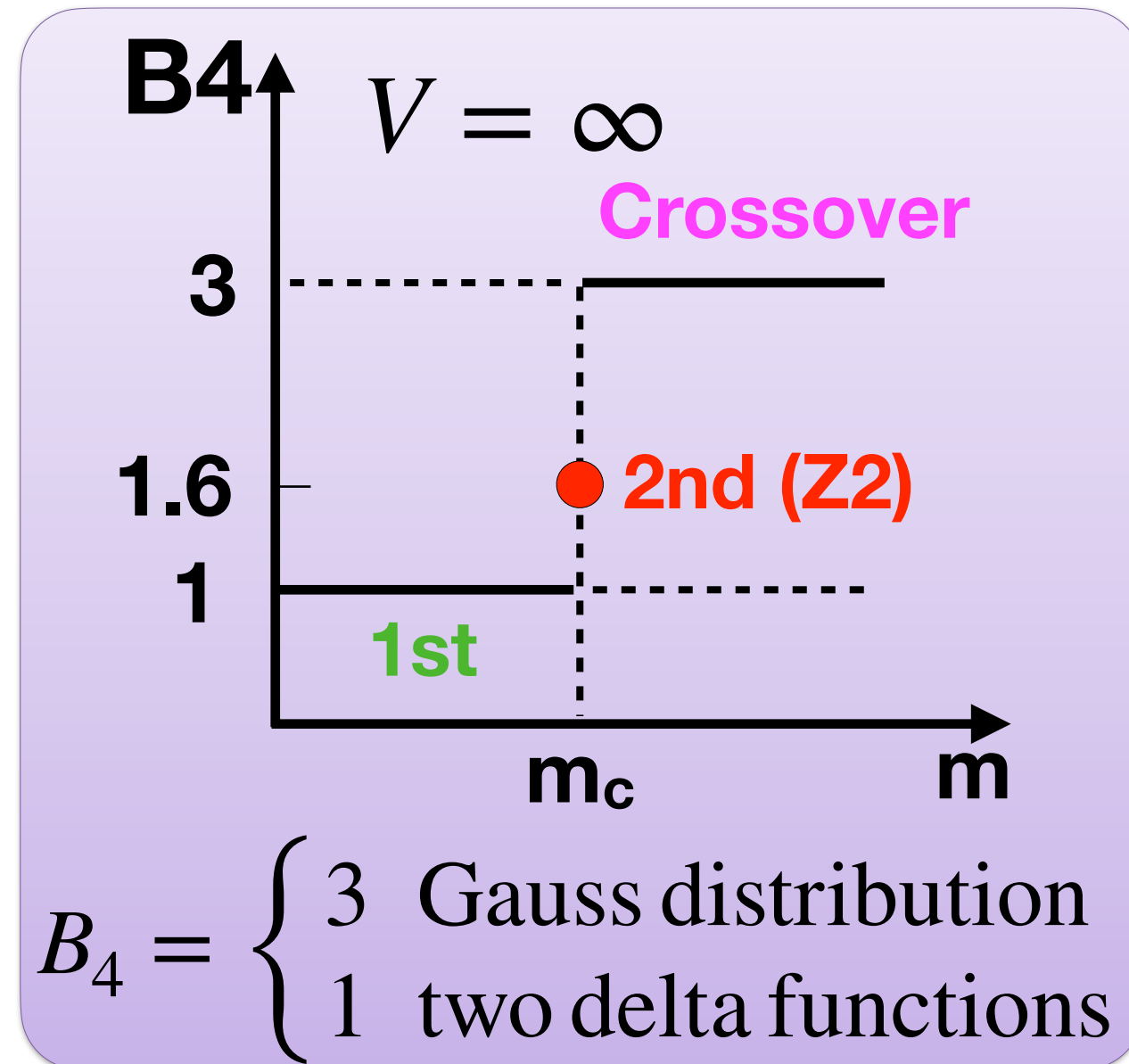
**The peak height of  $\chi_{tot}$  does not scale like a 1st or Z(2) second order PT**  
**finite-size scaled renormalized chiral susceptibility**



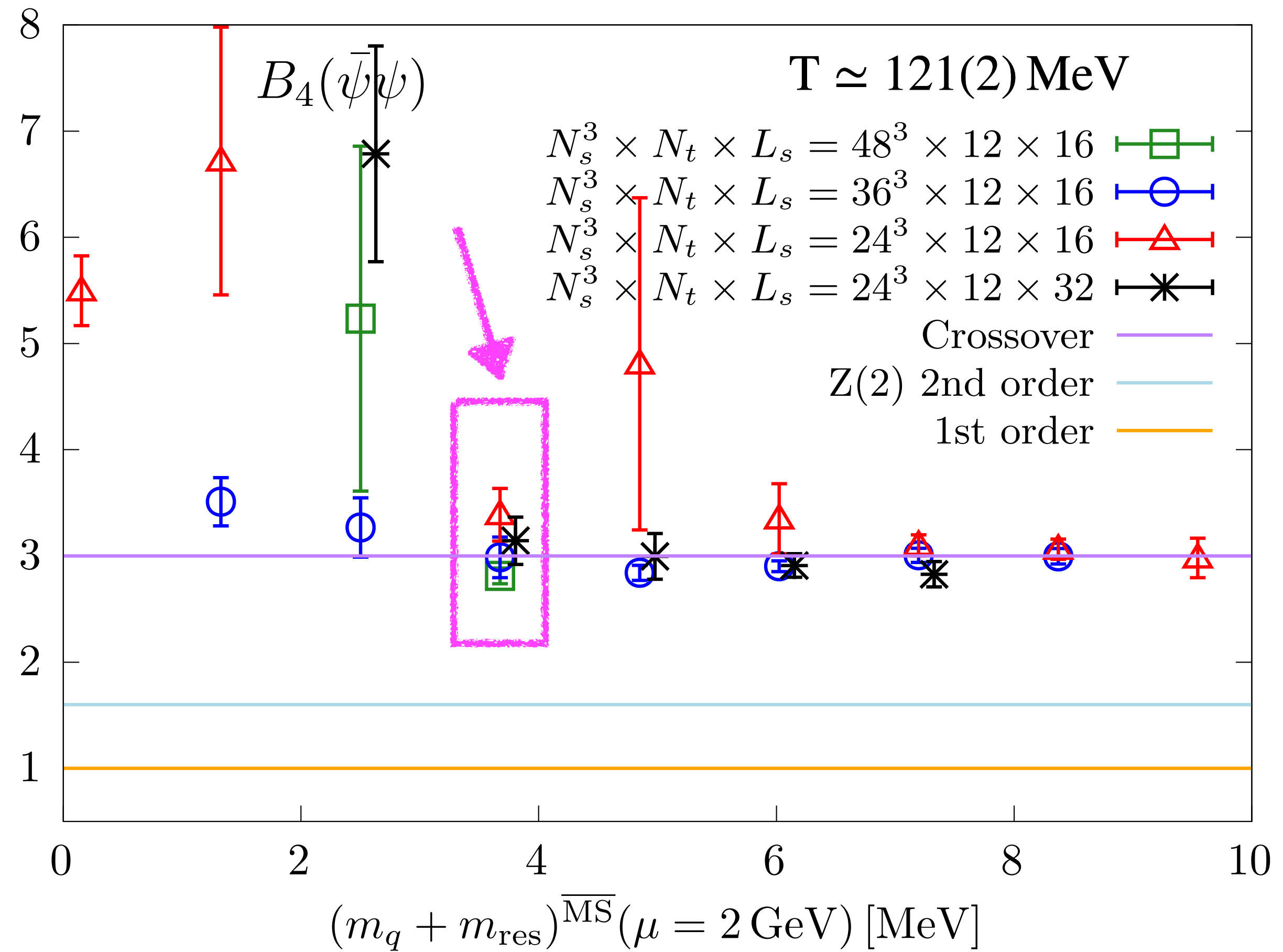
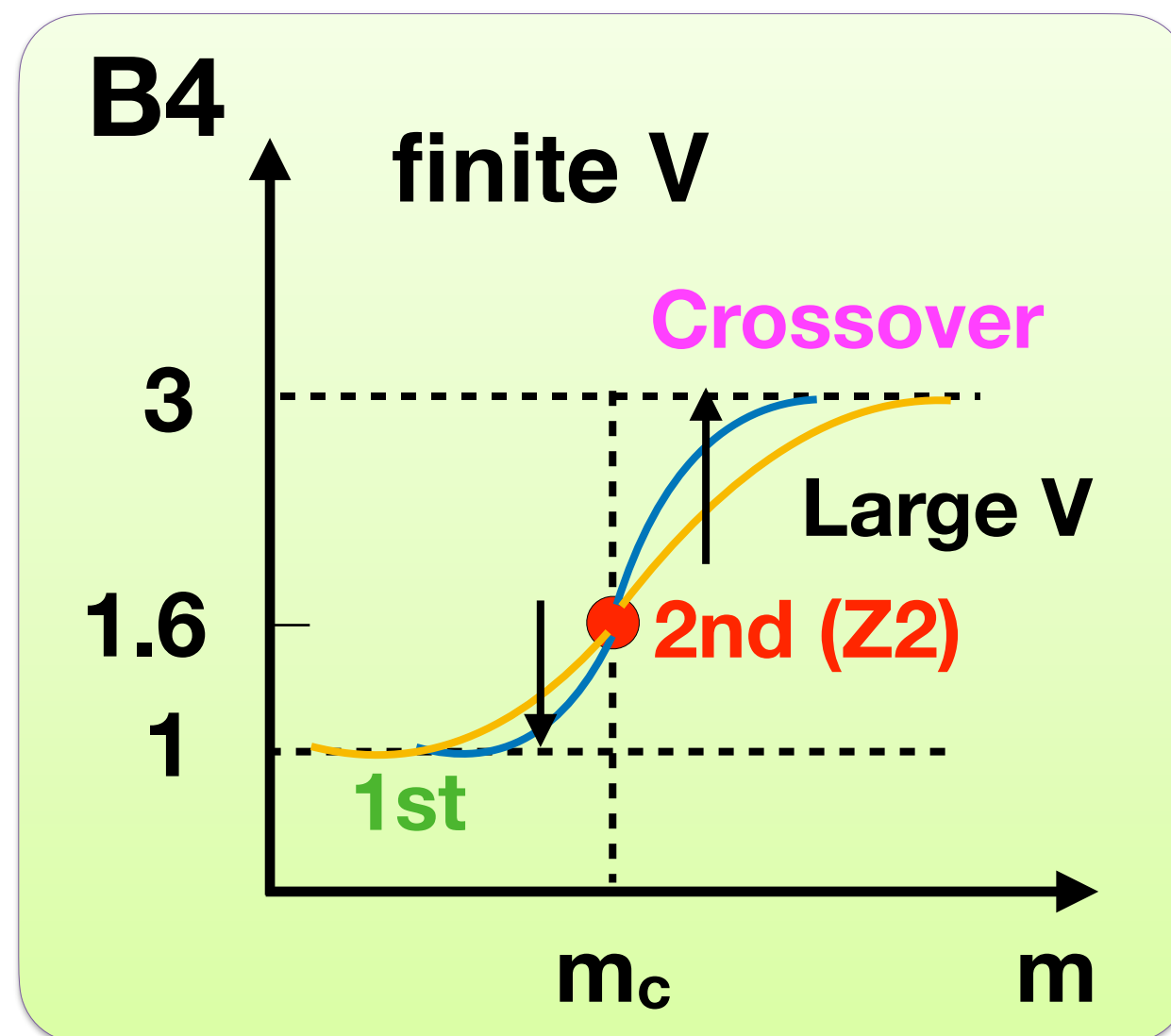
# Binder Cumulant

$$B_4(\bar{\psi}\psi) = \frac{\langle (\delta\bar{\psi}\psi)^4 \rangle}{\langle (\delta\bar{\psi}\psi)^2 \rangle^2}, \quad \delta\bar{\psi}\psi = \bar{\psi}\psi - \langle \bar{\psi}\psi \rangle$$

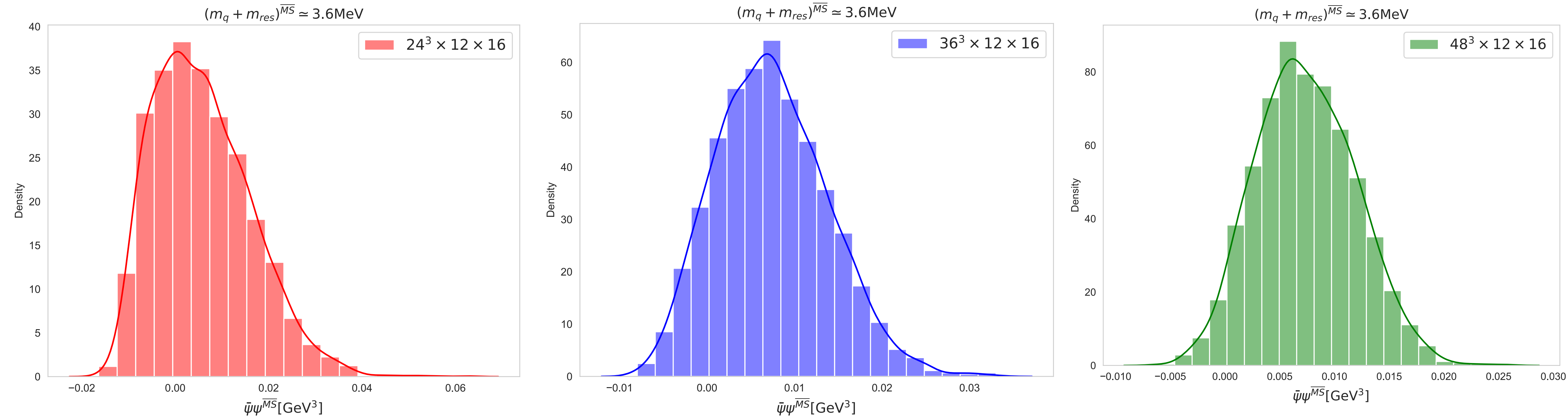
Suggests a crossover transition at  $(m_q + m_{\text{res}})^{\overline{\text{MS}}}(\mu = 2 \text{ GeV}) \simeq 3.6 \text{ MeV}$



A.Kiyohara et al., PRD 104, 114509 (2021)



# Histogram of chiral condensate near transition point

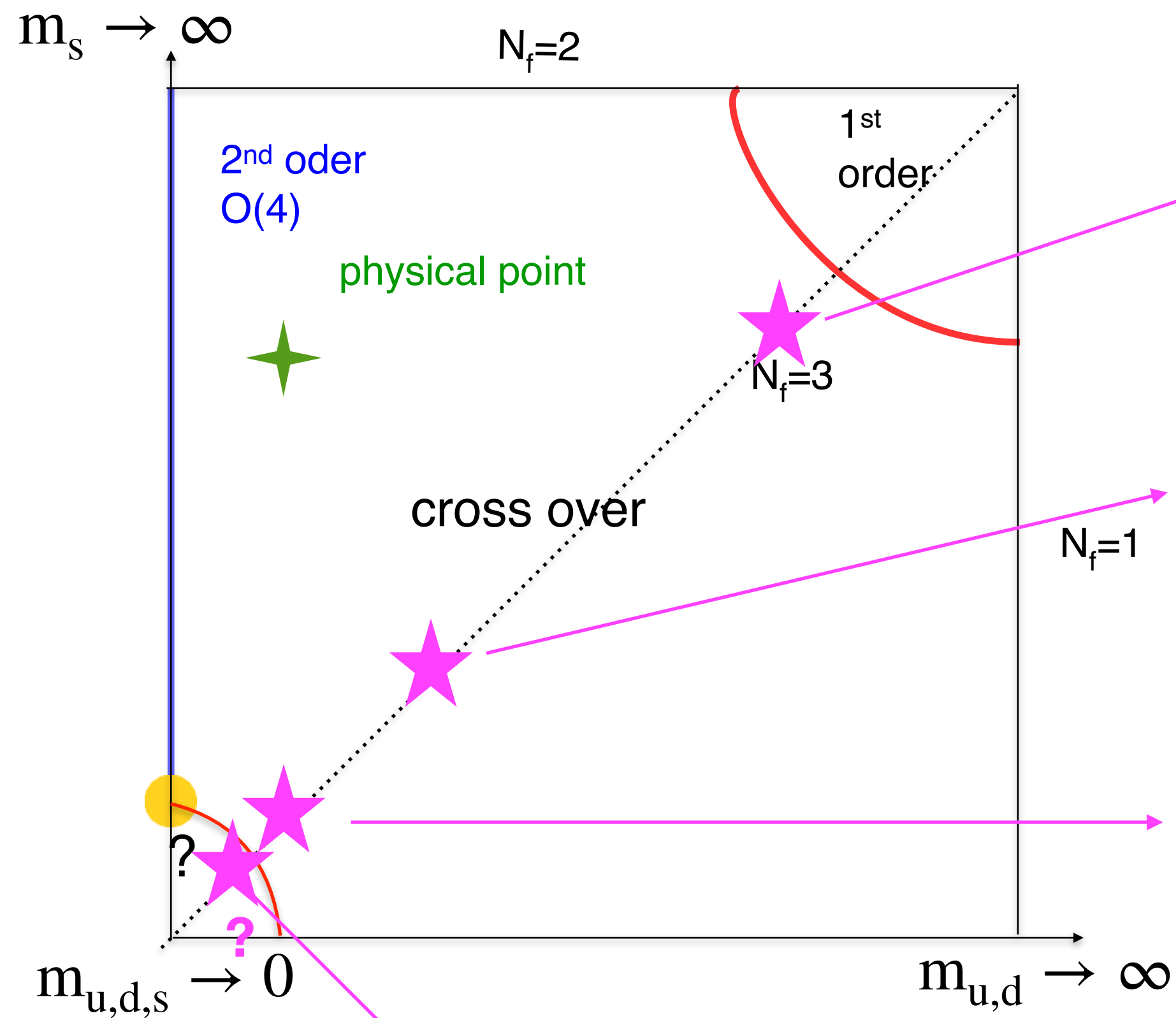


**Behaves like a Gaussian distribution, no evidence of a double peak structure would appear as  $V \uparrow$**



# Summary and outlook

**No evidence of a 1st order PT in our explored quark mass range**  
**If a 1st order region exist, the critical mass should be less than 3.6 MeV**



- $T \sim 242 \text{ MeV} (N_t = 6), (m_q + m_{\text{res}})_{pc}^{\overline{\text{MS}}} \sim 183 \text{ MeV}$   
 $\Leftrightarrow m_{\pi}^{pc} \sim 997 \text{ MeV}$ , crossover transition

Y. Nakamura, Y. Zhang et al., *PoS LATTICE2021*

- $T \sim 181 \text{ MeV} (N_t = 8), (m_q + m_{\text{res}})_{pc}^{\overline{\text{MS}}} \sim 42 \text{ MeV}$   
 $\Leftrightarrow m_{\pi}^{pc} \sim 476 \text{ MeV}$ , crossover transition

Y. Zhang et al., *PoS LATTICE2022*

- $T \sim 121 \text{ MeV} (N_t = 12), (m_q + m_{\text{res}})_{pc}^{\overline{\text{MS}}} \sim 3.6 \text{ MeV}$   
 $\Leftrightarrow m_{\pi}^{pc} \sim 141 \text{ MeV}$ , crossover transition

Y. Zhang et al., *PoS LATTICE2023*

- $T \sim 104 \text{ MeV} (N_t = 14)$ , lighter quark mass simulation is underway



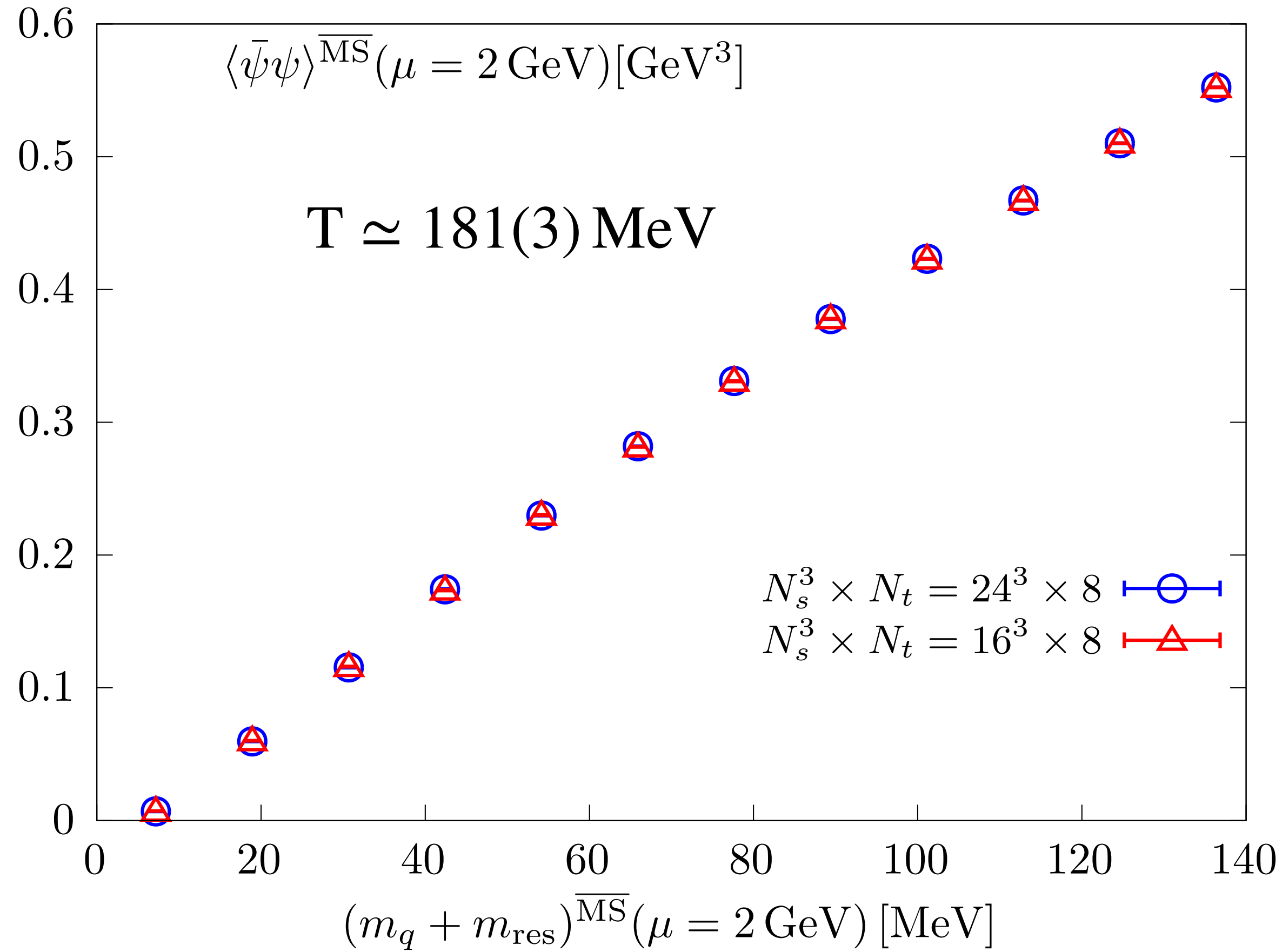
# Acknowledgements

- Codes
  - HMC
    - Grid (implementation for A64FX: thanks to the Regensburg group)
  - Measurements
    - Bridge++
    - Hadrons / Grid
- Computers
  - Supercomputer Fugaku provided by the RIKEN Center for Computational Science through HPCI project #hp210032 and Usability Research ra000001.
    - Wisteria/BDEC-01 Oddysey at Univ. Tokyo/JCAHPC through HPCI project #hp220108
  - Ito supercomputer at Kyushu University through HPCI project #hp190124 and hp200050
    - Hokusai BigWaterfall at RIKEN
- Grants
  - JSPS Kakenhi (20H01907)

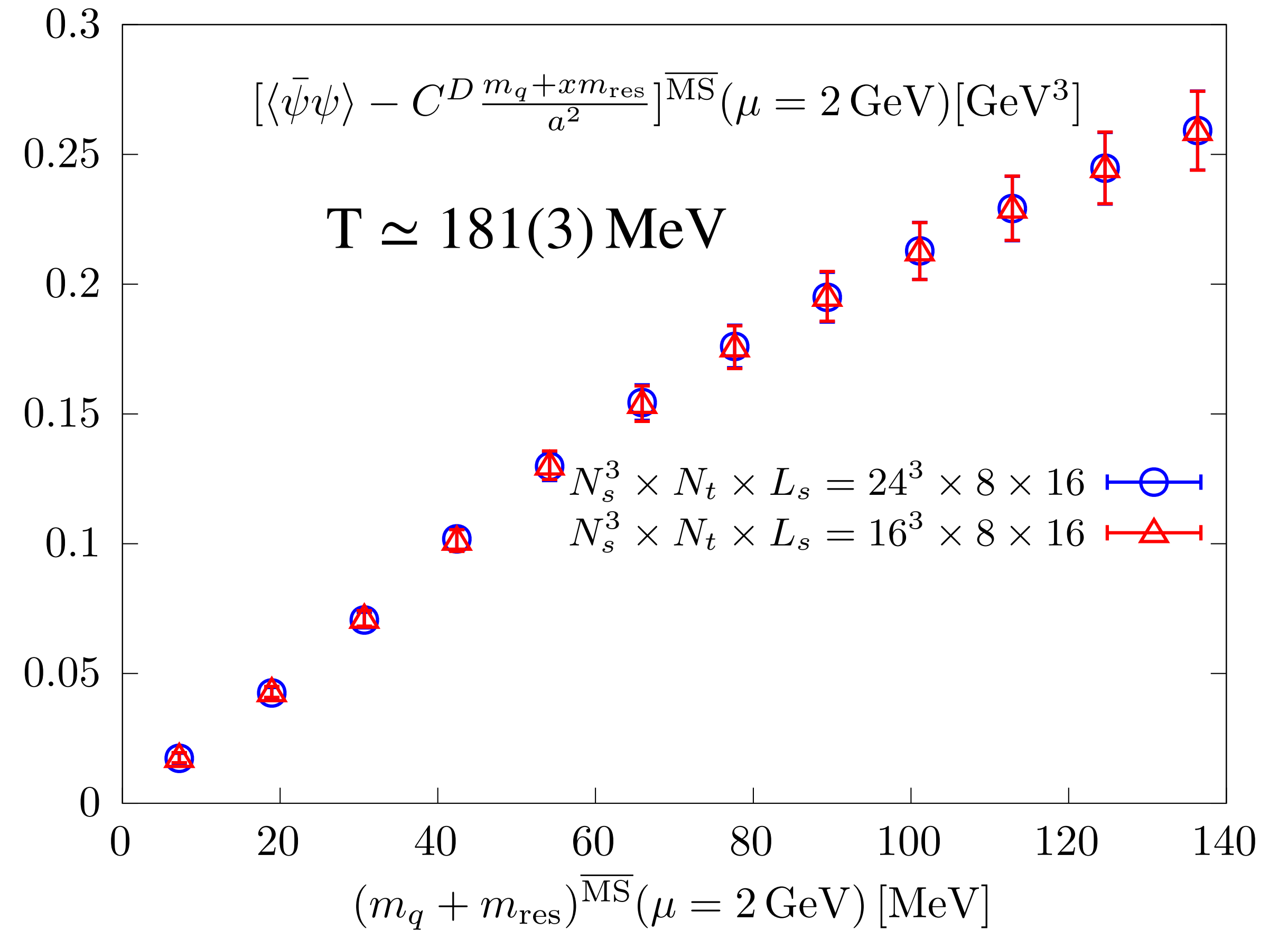
**Backup slide**

# Renormalized chiral condensate

## Multiplicatively renormalized chiral condensate

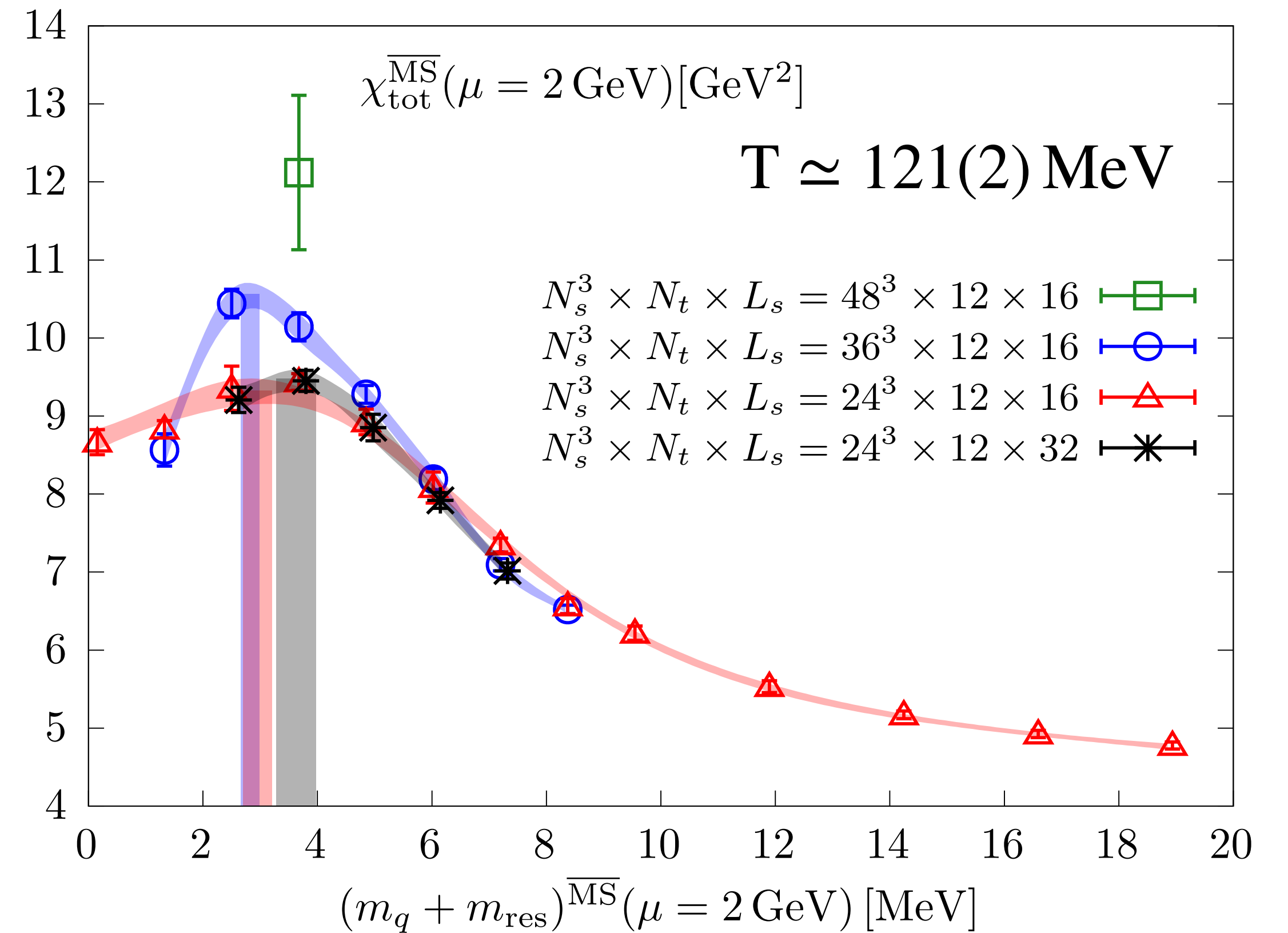
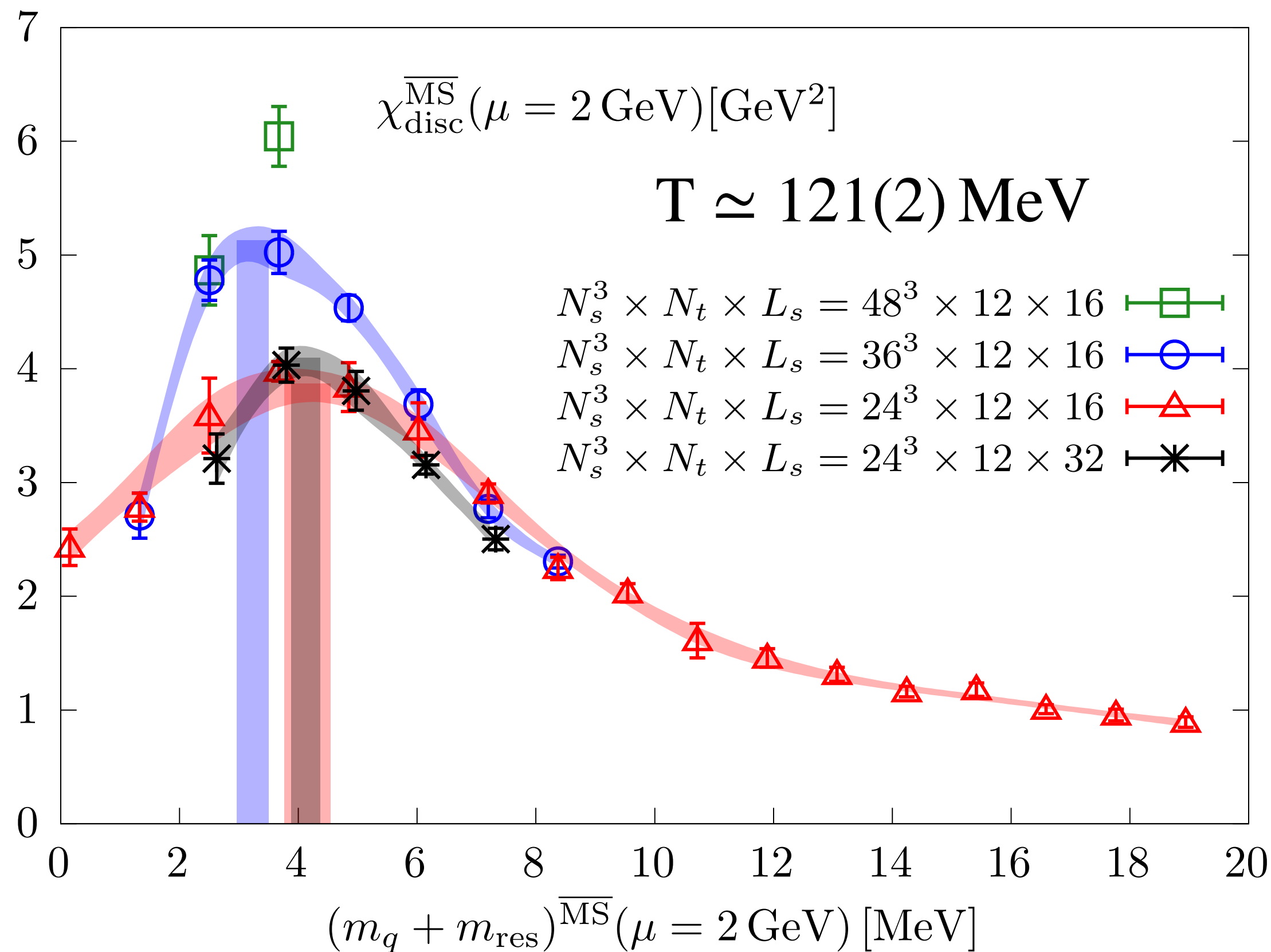


## Additive and multiplicatively renormalized chiral condensate

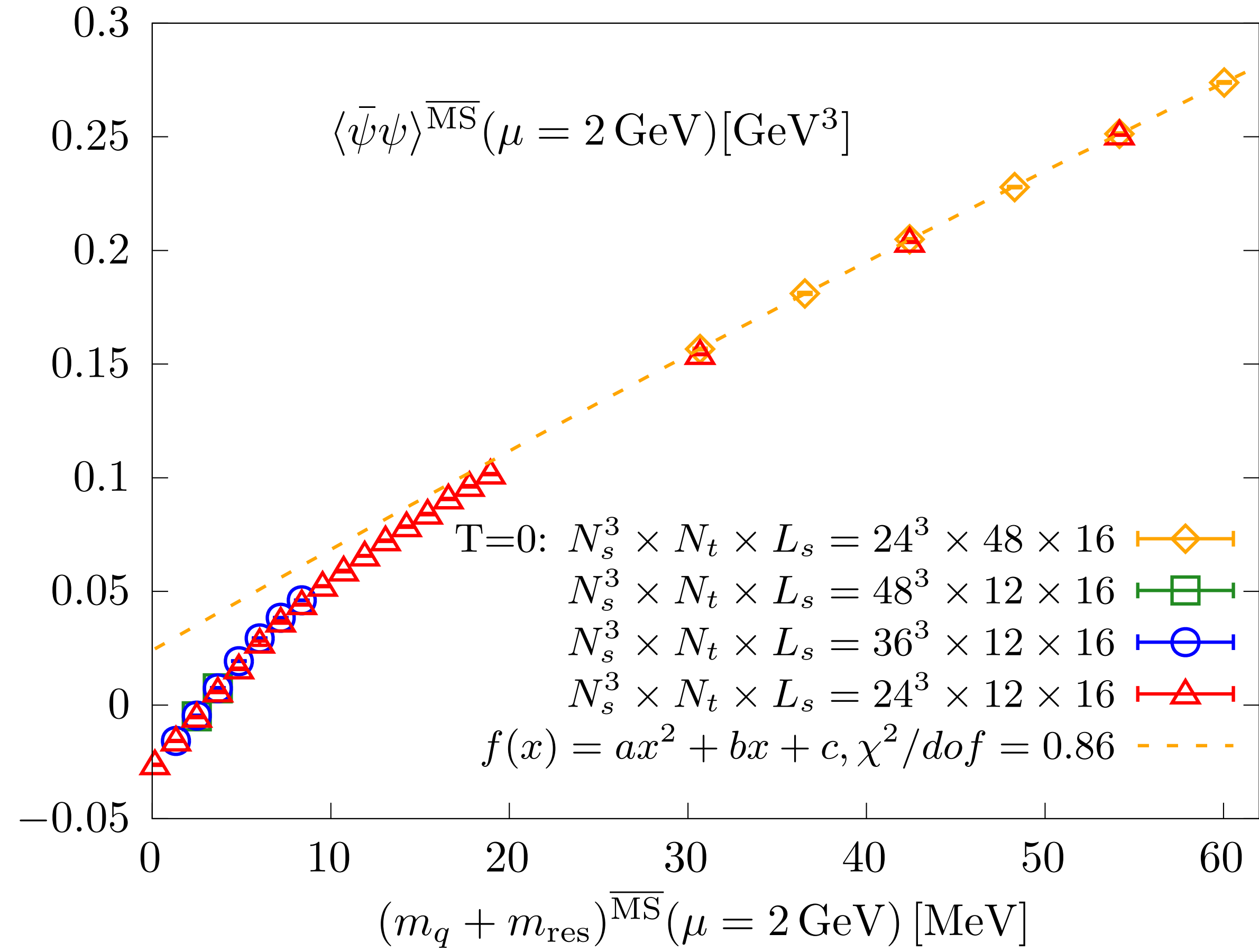


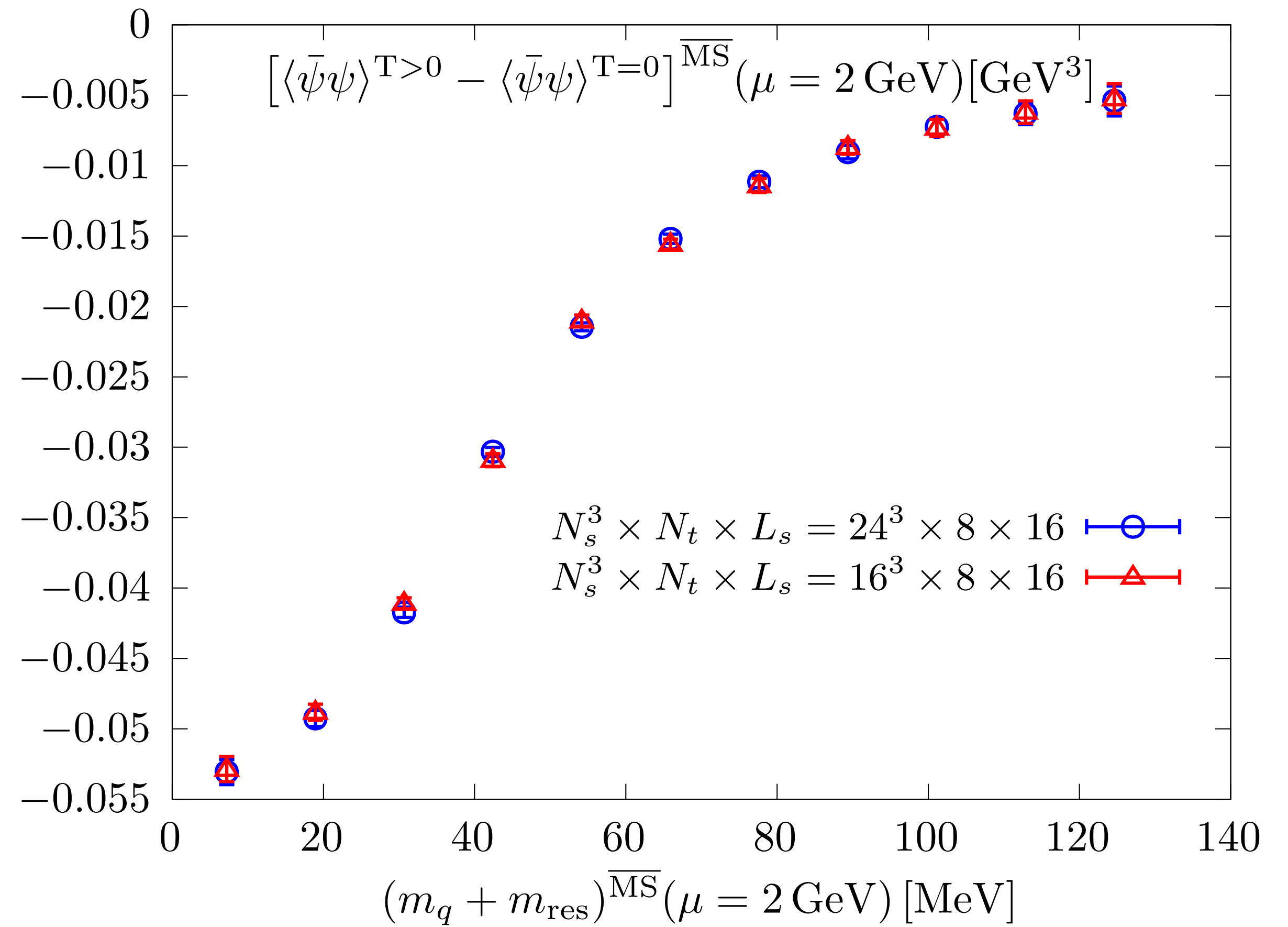
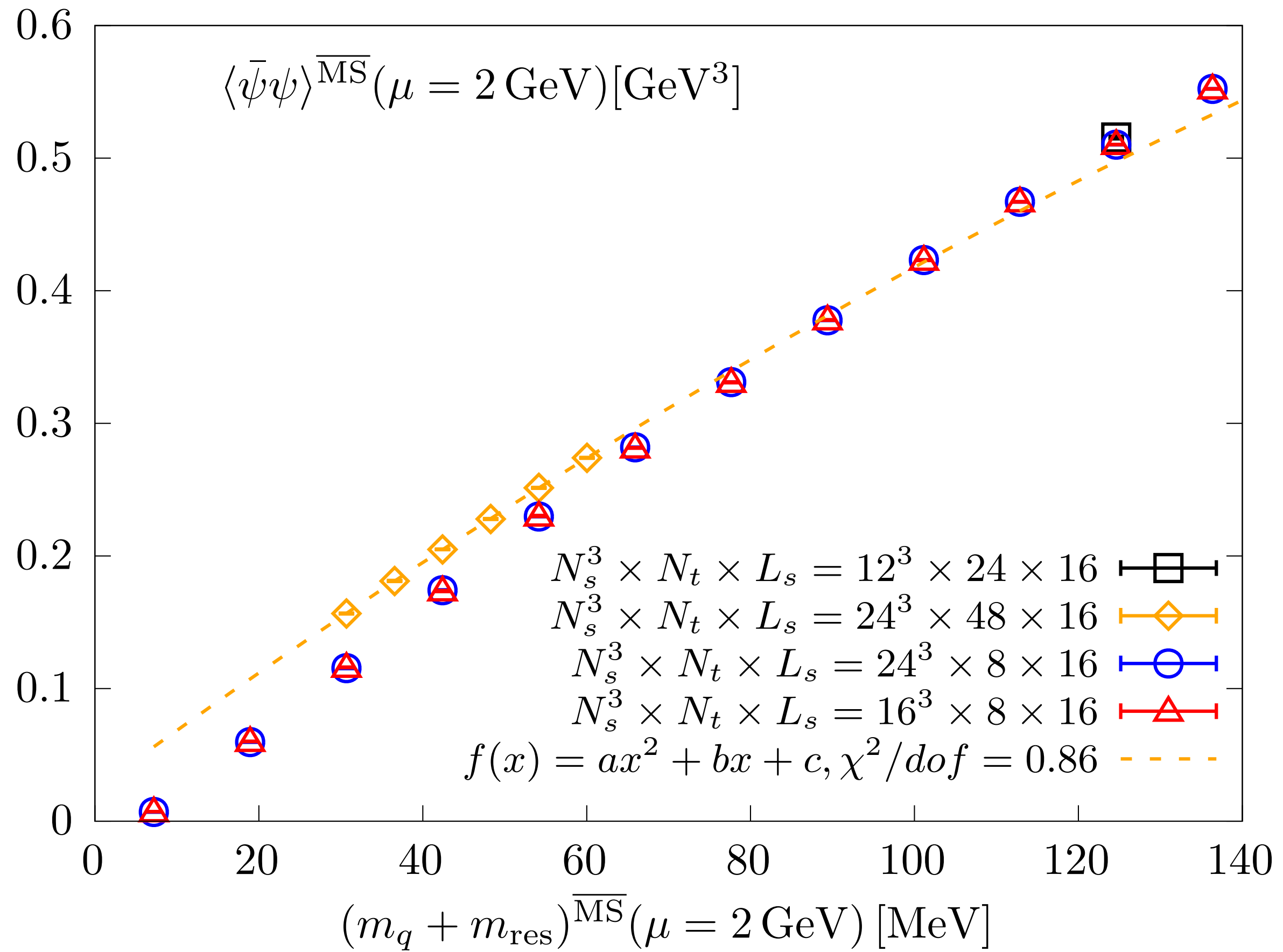
**Subtracted chiral condensate vanishes in the chiral limit as expected since  $T > T_c$**

# Chiral susceptibility

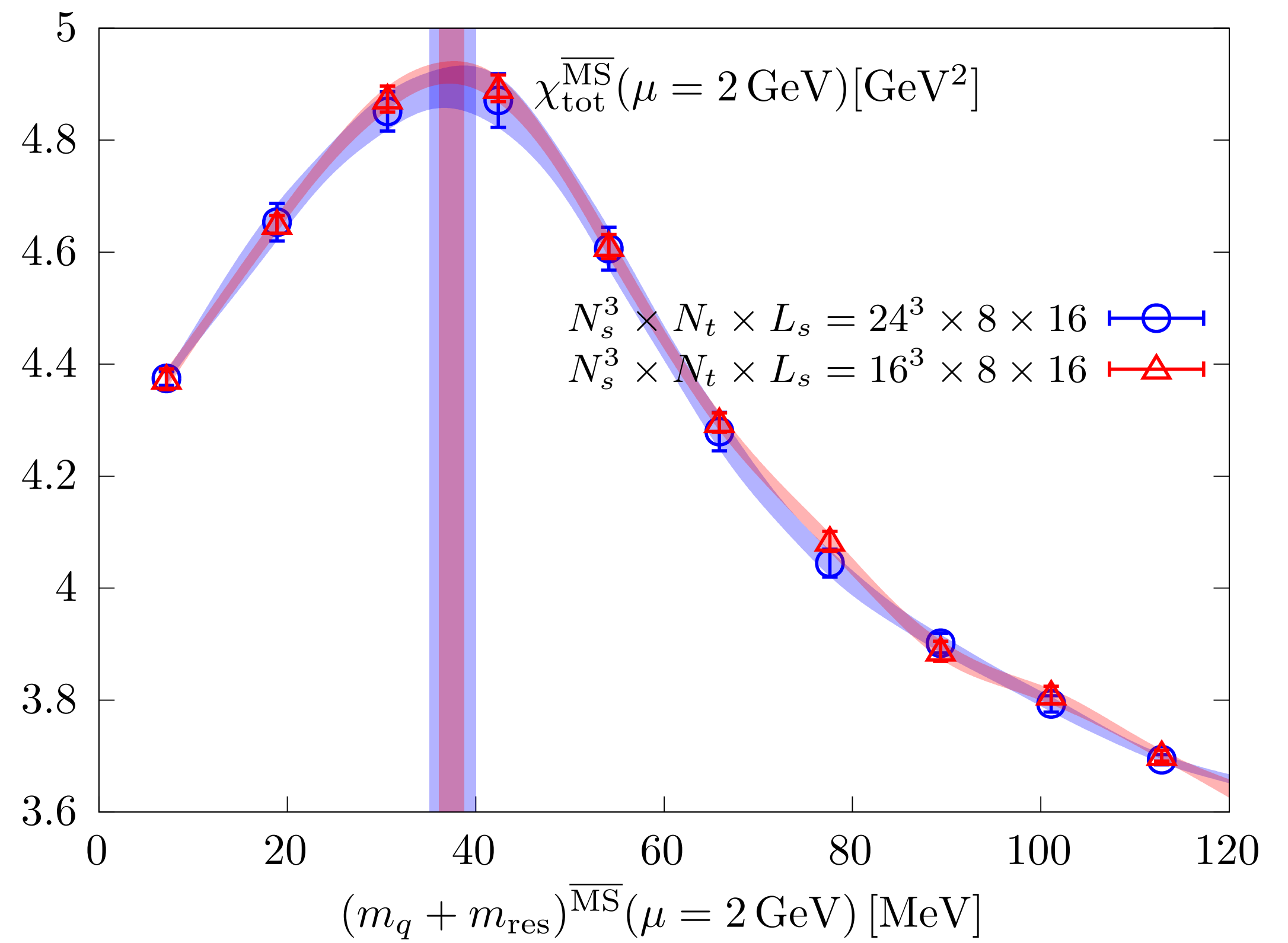
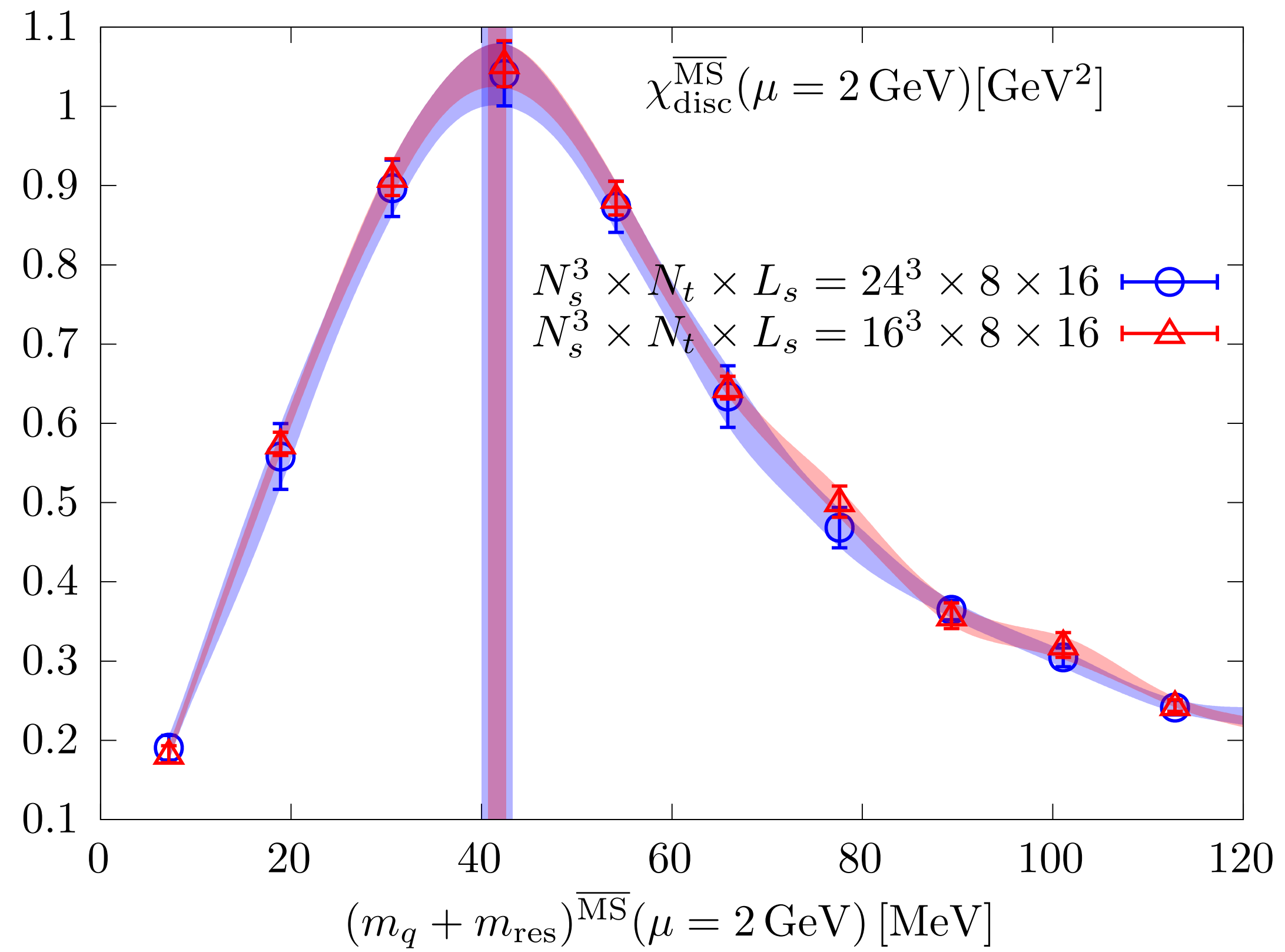


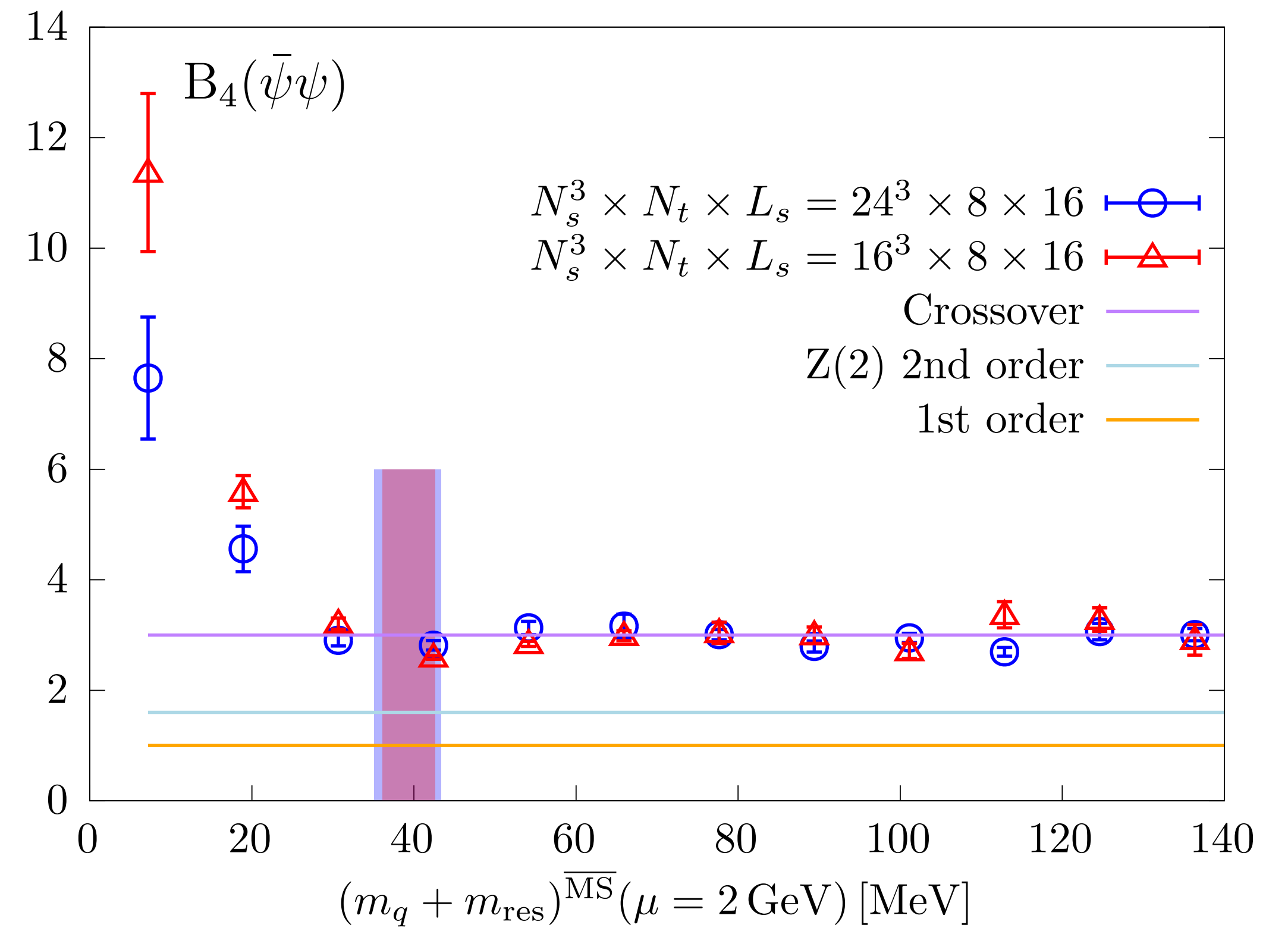
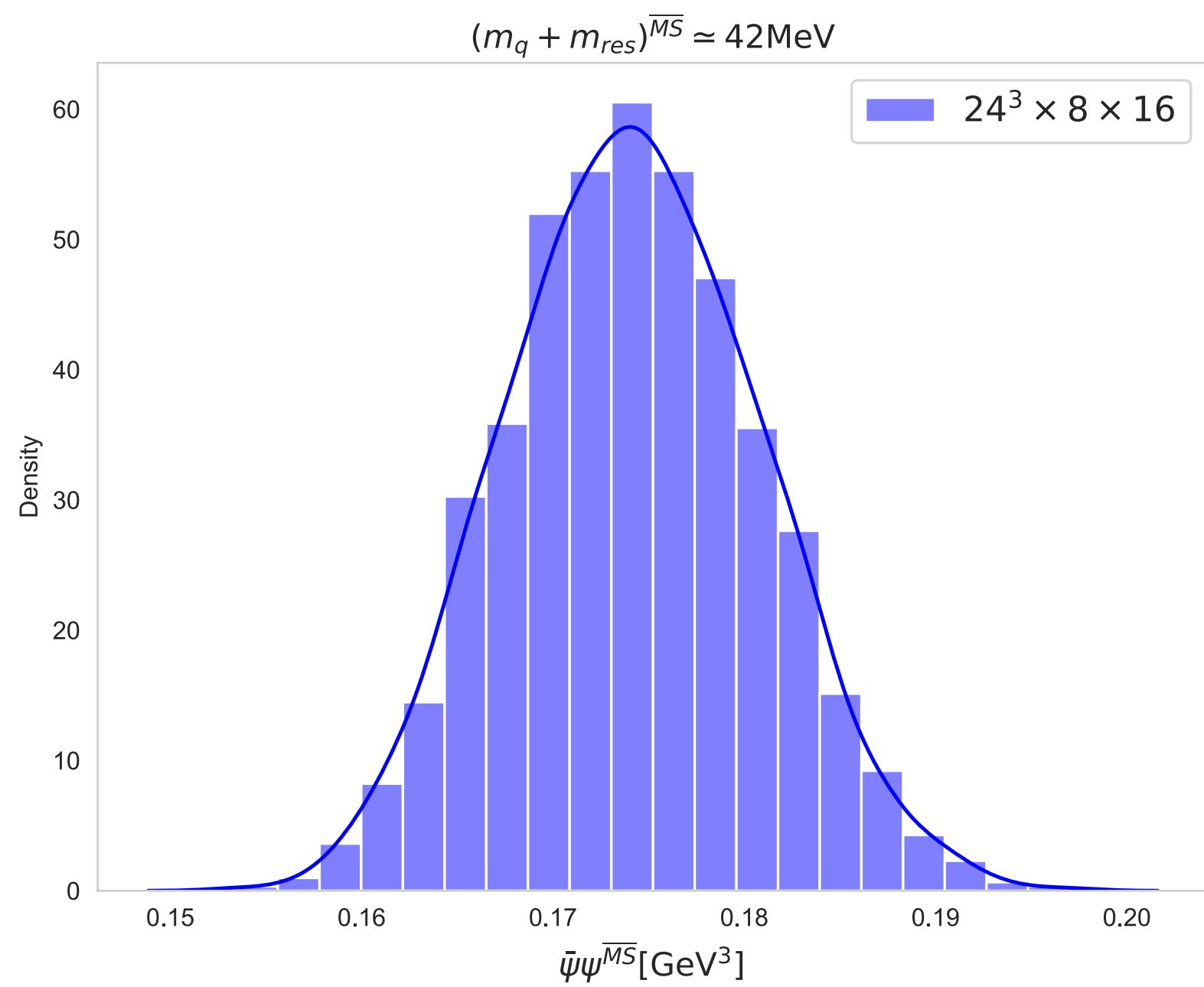
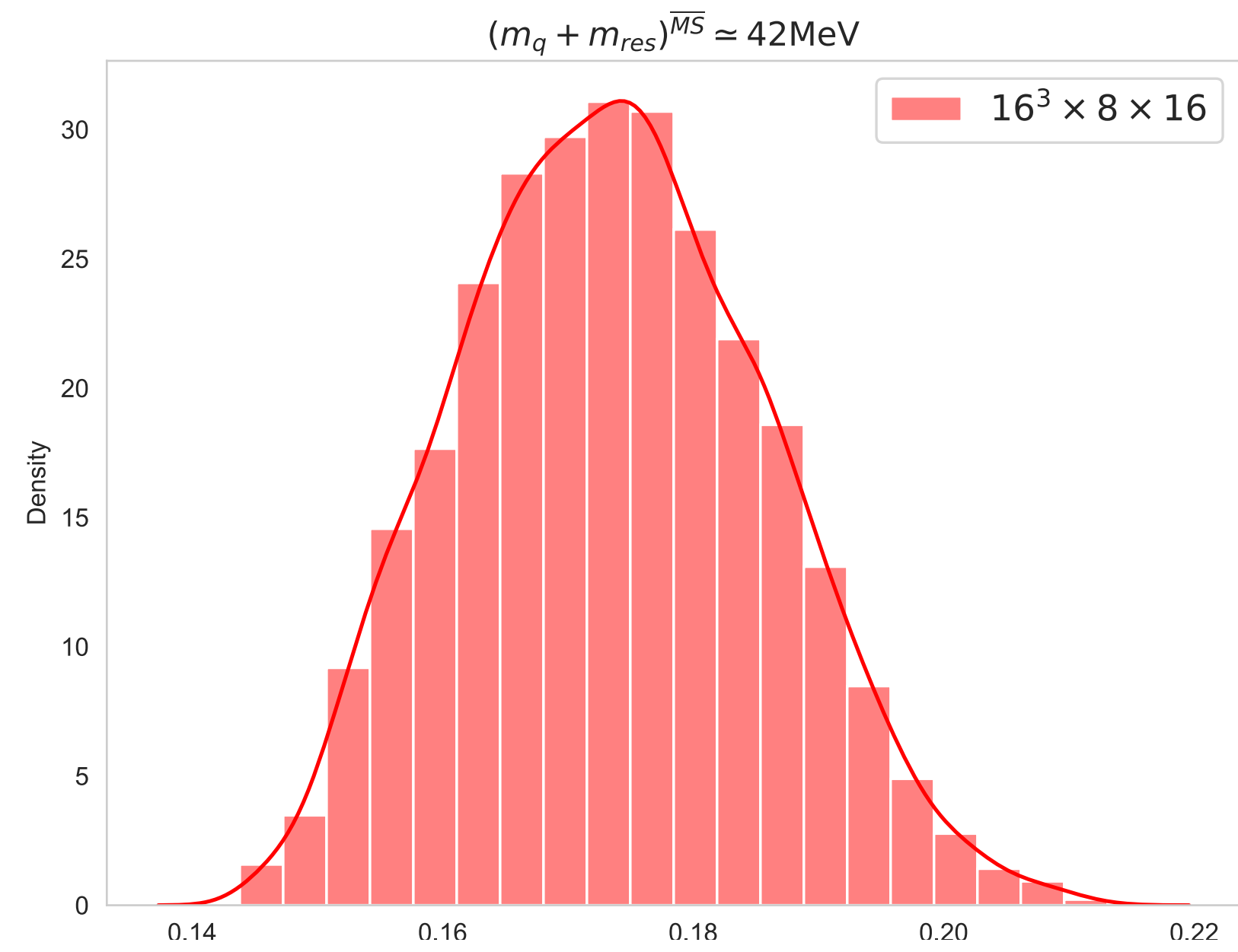
- Large finite volume effect near the transition point, but the change in peak height is not as large as anticipated from a real phase transition → **Consistent with the crossover transition**
- The transition mass point is around **2.7-4.5 MeV** (FLAG Review '21:  $m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) = 3.381(40) \text{ MeV}$ )
- Chiral susceptibilities seem to be function of total quark mass

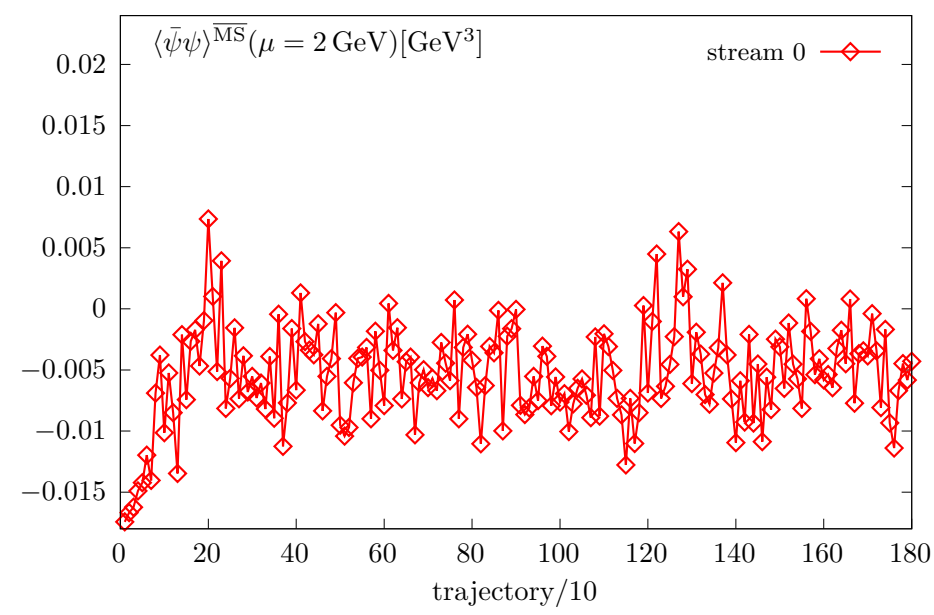
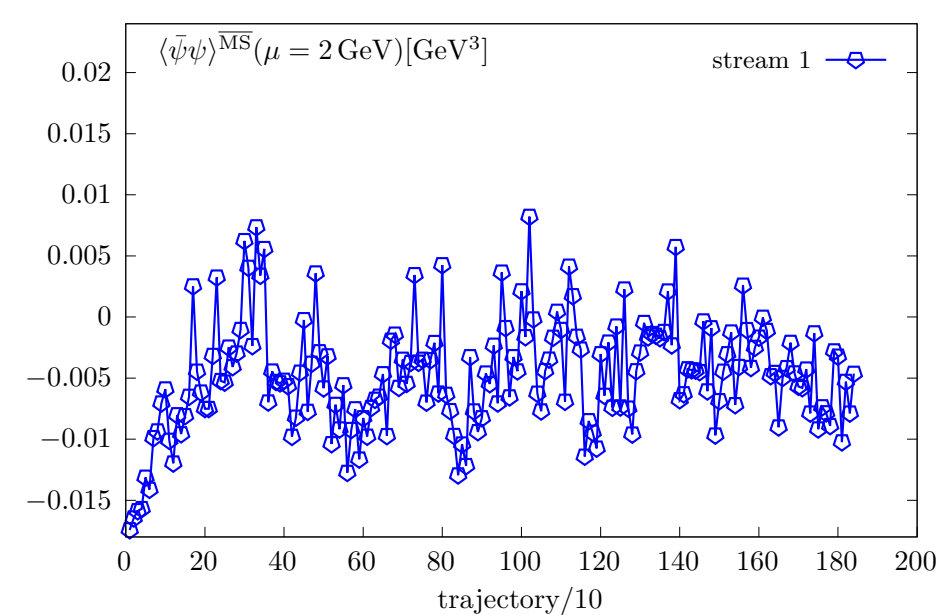
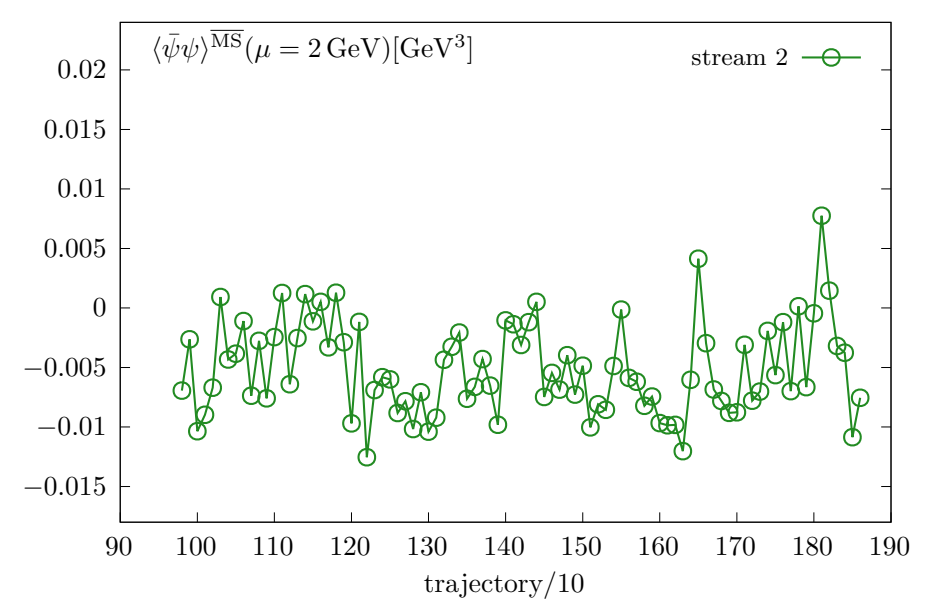
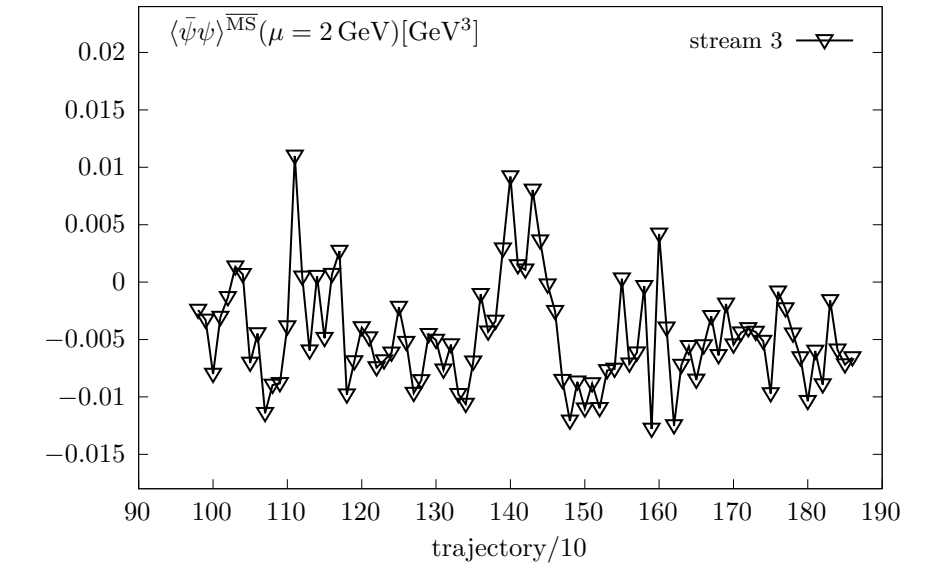
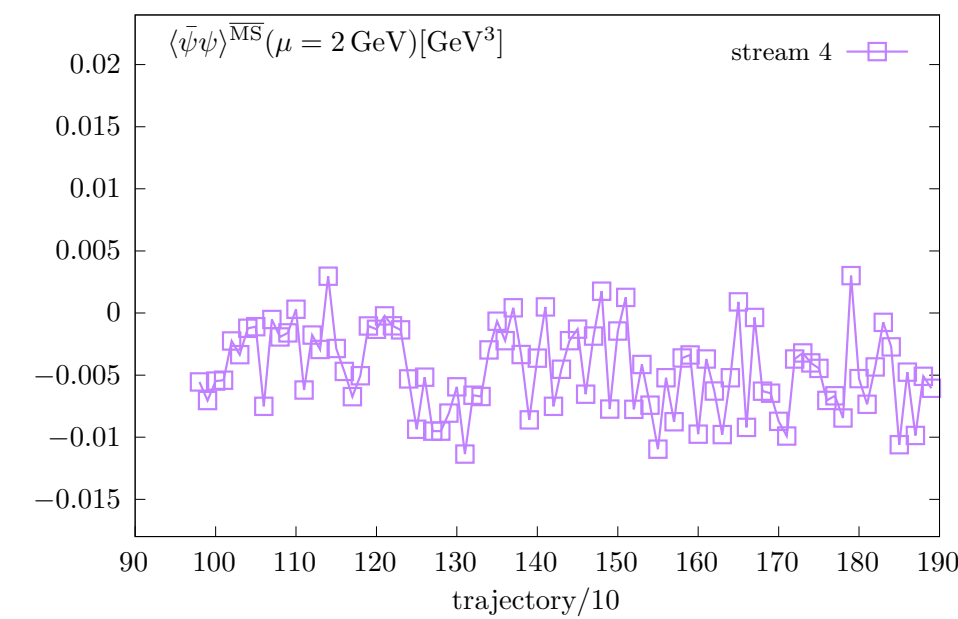
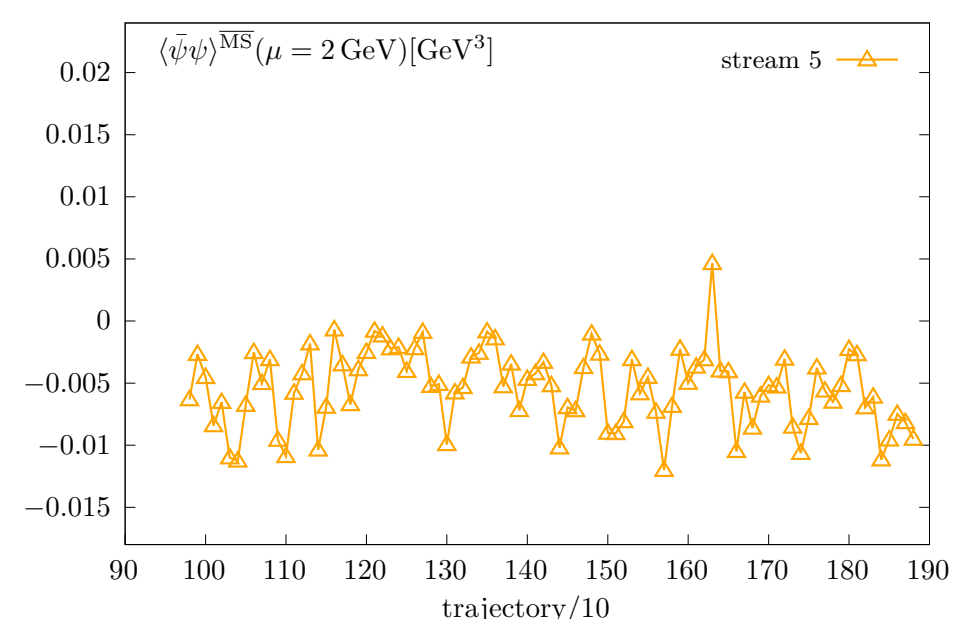
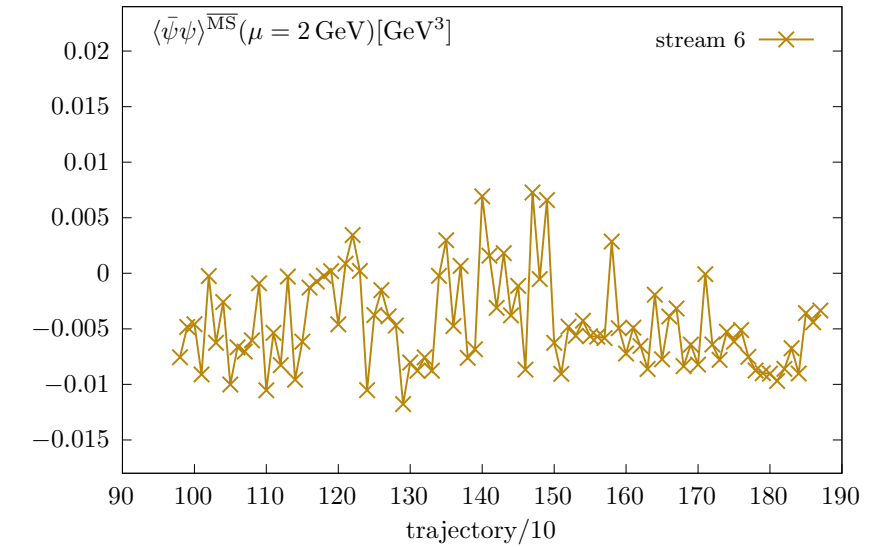
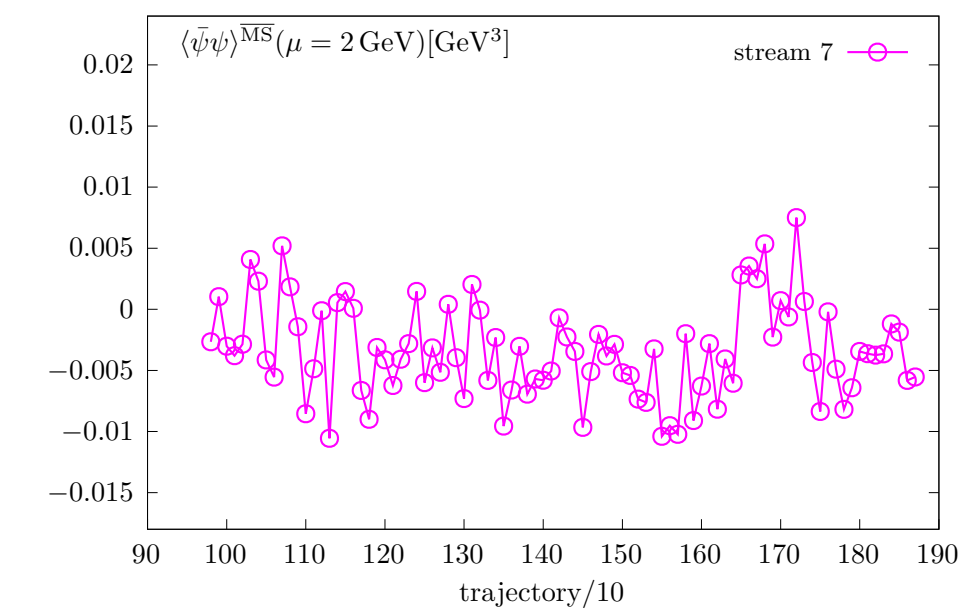
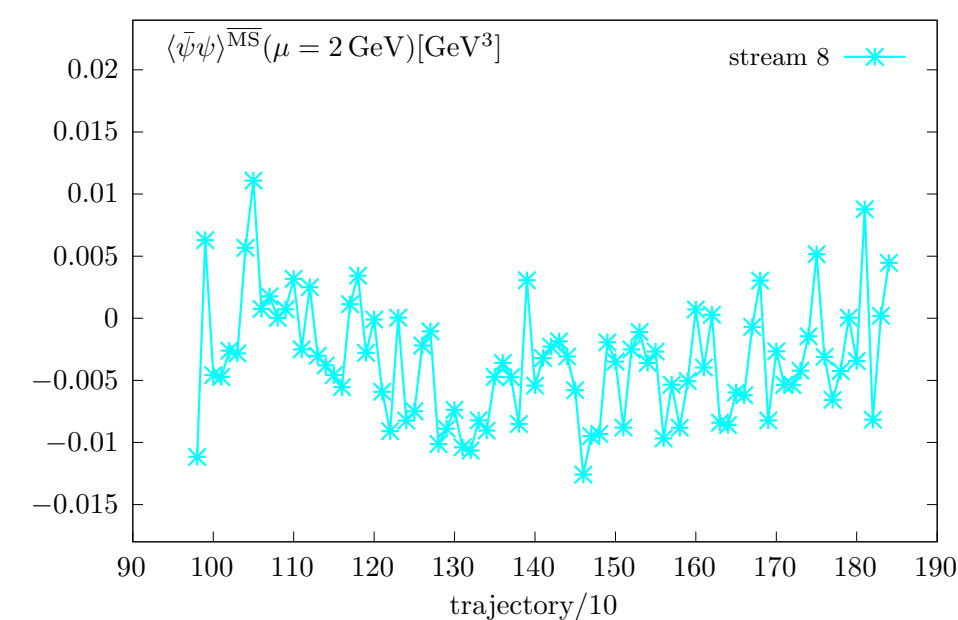
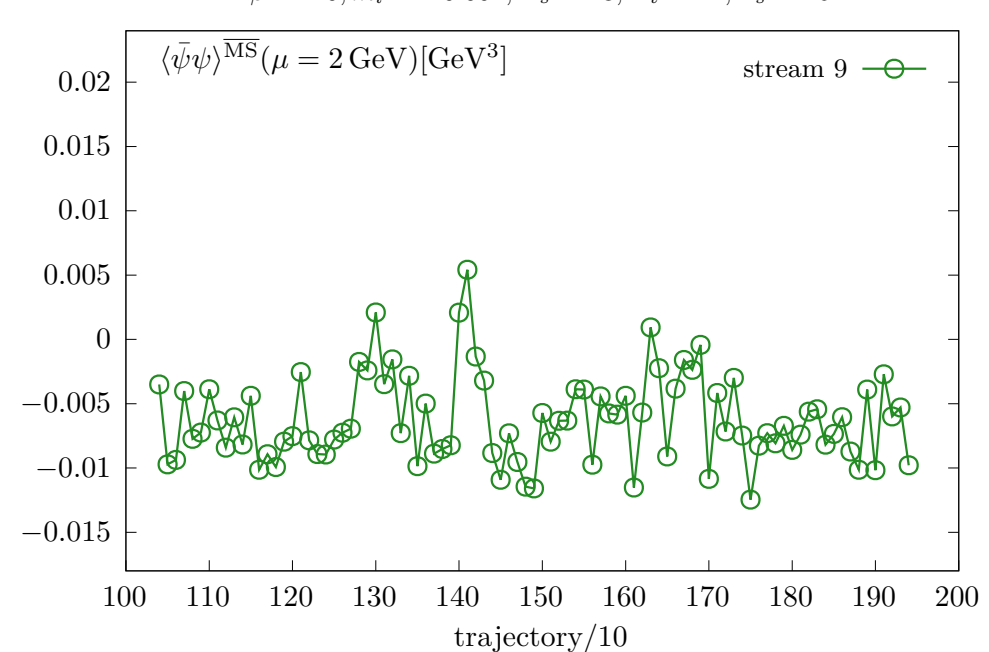
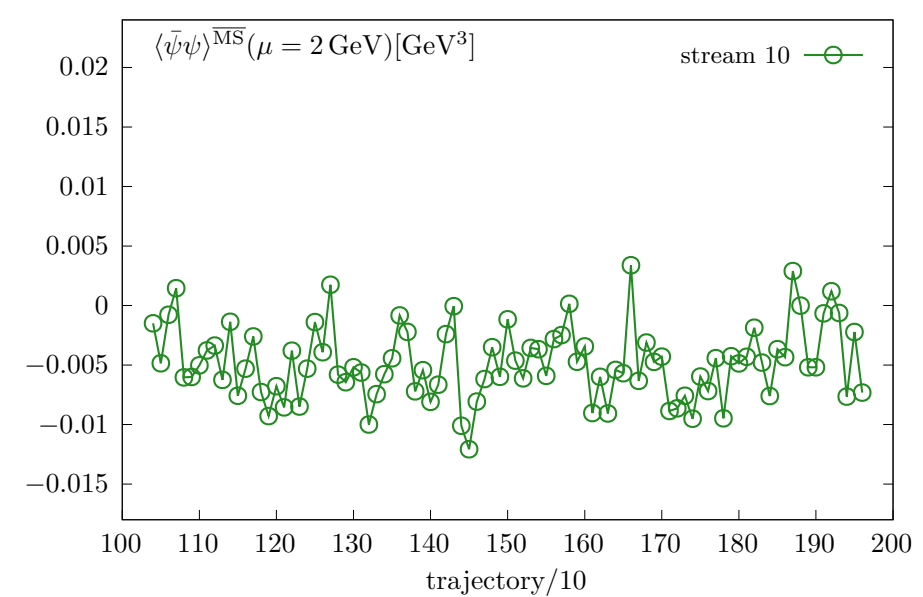
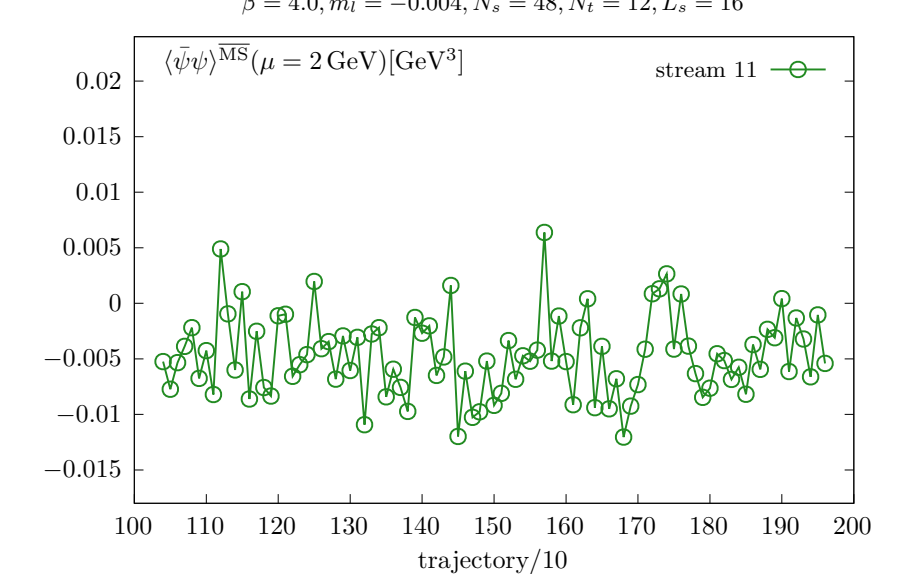
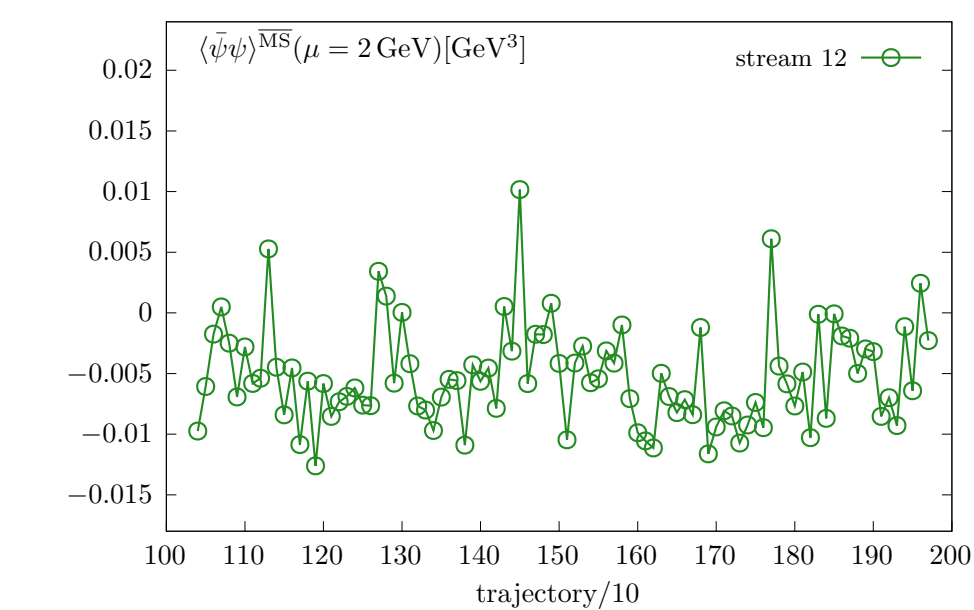
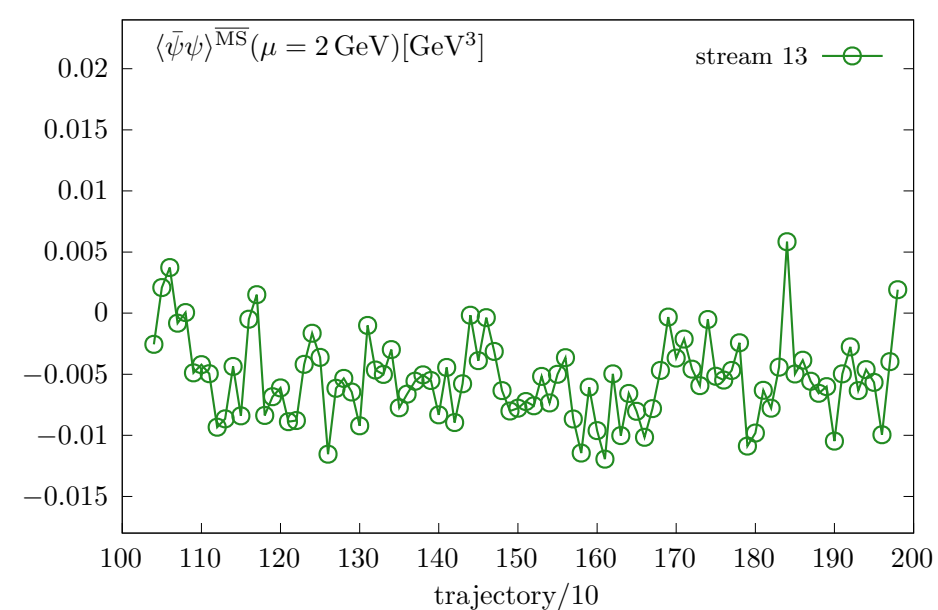
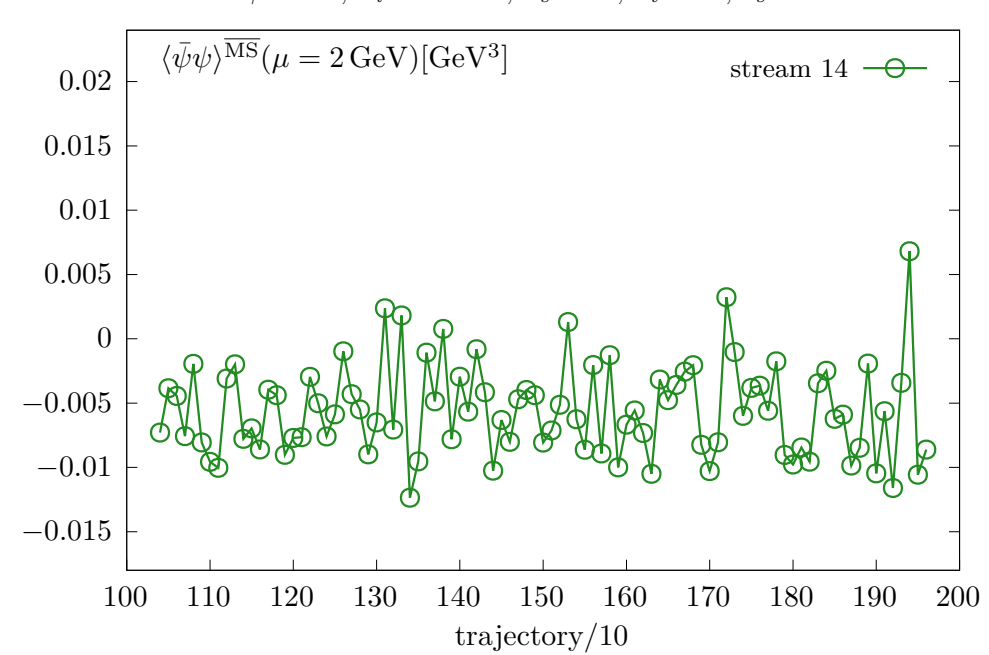
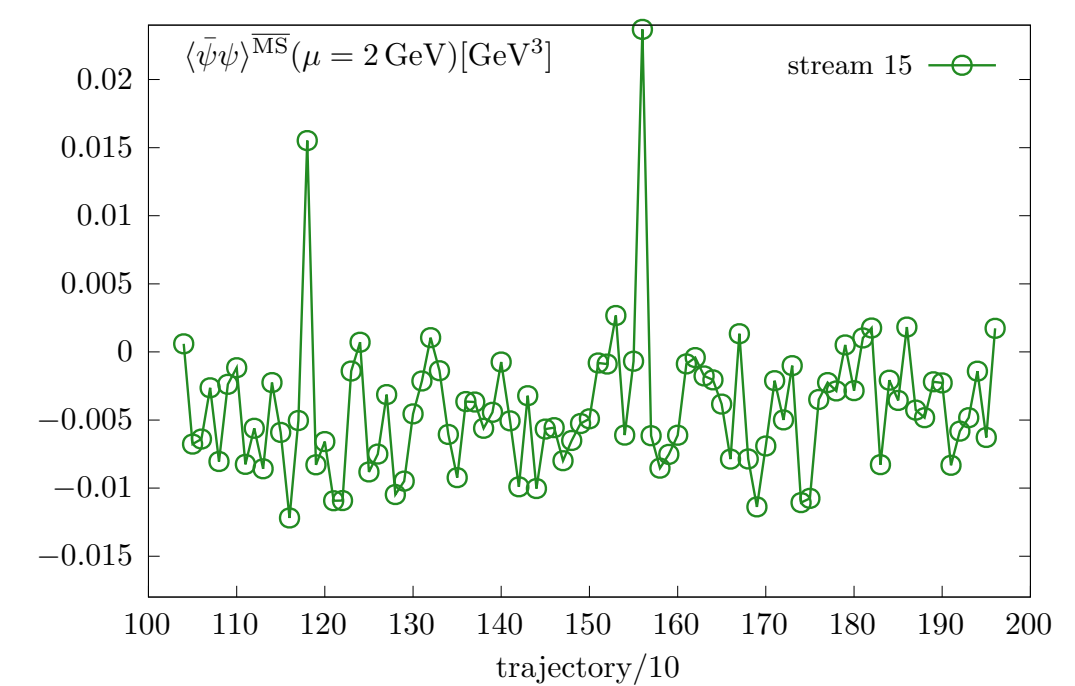










$\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$  $\beta = 4.0, m_l = -0.004, N_s = 48, N_t = 12, L_s = 16$ 

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