Calculation of $K_{\ell 3}$ form factor at the physical point on large volume

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Large-scale lattice QCD simulation and development of AI technology

Collaborators

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for PACS Collaboration

Refs: PRD101,9,094504(2020), PRD106,9,094501(2022), PoS(LATTICE2023)276(2024)

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Outline

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2. Simulation parameters
3. Calculation method
4. Results
   - $K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$
   - Continuum extrapolation of $f_+(q^2), f_0(q^2)$
   - $f_+(0)$ and $|V_{us}|$
5. Summary

Introduction

Indirect search for physics beyond standard model through $|V_{us}|$

$|V_{us}|$ determinations

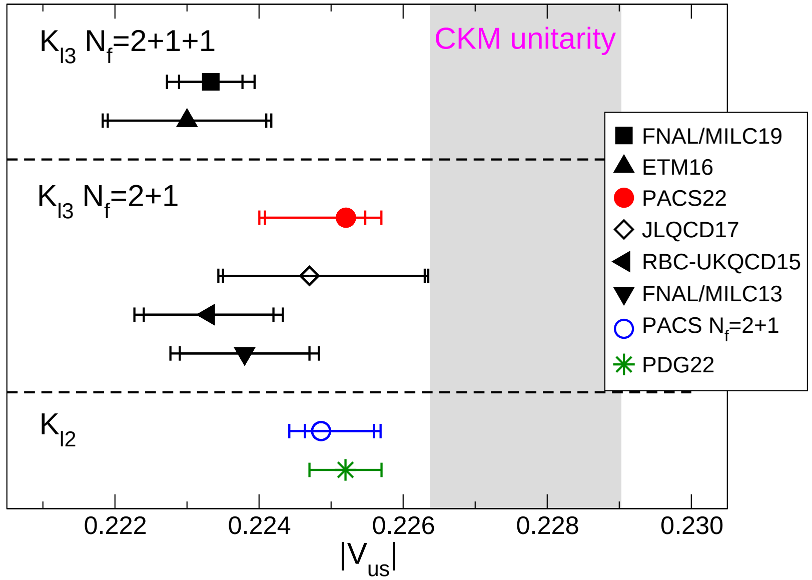
CKM unitarity

$|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$ w/ $|V_{ud}|$ ['20 Hardy, Towner]

$K_{\ell 2}$ decay [PDG22]

$K_{\ell 3}$ decay Most accurate ['19 FNAL/MILC]

Tensions among three $|V_{us}|$



Independent and precise calculations are important.

$K_{\ell 3}$ form factors with two PACS10 configurations ['20 PACS, '22 PACS]

$L \gtrsim 10$[fm] at very close to physical point

Negligible finite L effect, tiny q^2 region, tiny chiral extrapolation

Largest uncertainty form finite a effect

This talk:

Preliminary result with 3rd PACS10 configuration

Simulation parameters

PACS10 configurations: $L \gtrsim 10$[fm] at very close to physical point

$N_f = 2+1$ six-stout-smeared non-perturbative $\mathcal{O}(a)$-improved Wilson action

$+$ Iwasaki gauge action

	$L^3 \cdot T$	L[fm]	a[fm]	a^{-1}[GeV]	M_π[MeV]	M_K[MeV]	$N_{\rm conf}$
PACS10/L256	256^4	10.5	0.041	4.792	142	514	20
PACS10/L160	160^4	10.2	0.063	3.111	137	501	20
PACS10/L128	128^4	10.9	0.085	2.316	135	497	20

Physical observables

– Hadron spectrum

– Hadron vacuum polarization

– Proton decay matrix element

– Nucleon form factor — Ryutaro Tsuji

– Meson charge radius — Kohei Sato

– $K_{\ell 3}$ form factor — TY

Simulation parameters

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	$L^3 \cdot T$	L[fm]	a[fm]	a^{-1}[GeV]	M_π[MeV]	M_K[MeV]	$N_{\rm conf}$	$t_{\rm sep}$[fm]
PACS10/L256	256^4	10.5	0.041	4.792	142	514	20	3.1–4.0
PACS10/L160	160^4	10.2	0.063	3.111	137	501	20	2.3–4.1
PACS10/L128	128^4	10.9	0.085	2.316	135	497	20	3.4–3.9

(1000–2500 measurements in each $t_{\rm sep}$)

All the results on PACS10/L256 are preliminary.

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21635(39)$ ['22 Seng *et al.*]

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$ from 3-point function

w/ $Z(2)\otimes Z(2)$ random source spread in L^3, color, spin ['08 RBC-UKQCD]

random smeared source also adopted in $L = 160, 256$ calculations

$$C_{V_\mu}(t,p) = \langle 0|O_K(t_{\rm sep},\mathbf{0})V_\mu(t,\mathbf{p})O_\pi^\dagger(0,\mathbf{p})|0\rangle$$

V_μ : { Local vector current with Z_V; Conserved vector current }

$$\langle\pi(p)|V_\mu|K(0)\rangle = (p_K+p_\pi)_\mu f_+(q^2) + (p_K-p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K,\vec{0}), p_\pi = (E_\pi,\vec{p})$

$q^2 = -(M_K - E_\pi)^2 + p^2$

Resources: Fugaku in HPCI System Research Project

(hp200062, hp200167, hp210112, hp220079, hp230199, hp240207)

Calculation method w/ local vector current

Details in PACS:PRD101,9,094504(2020)

2-point function* $X = \pi, K$

$$C_X(t,p) = \langle 0|O_X(t,\mathbf{p})O_X^\dagger(0,\mathbf{p})|0\rangle = \frac{Z_X^2}{2E_X}\left(e^{-E_X t} + e^{-E_X(2T-t)}\right) + \cdots$$

$p \equiv |\mathbf{p}| = \left|\frac{2\pi}{L}\mathbf{n}\right|$, $|\mathbf{n}| = 0$–6 with periodic boundary

3-point function*

$$C_{V_\mu}(t,t_{\rm sep},p) = \langle 0|O_K(t_{\rm sep},\mathbf{0})V_\mu(t,\mathbf{p})O_\pi^\dagger(0,\mathbf{p})|0\rangle = \frac{Z_\pi Z_K}{Z_V}\frac{M_\mu(p)}{4E_\pi M_K}e^{-E_\pi t}e^{-M_K(t_{\rm sep}-t)} + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = 1/\sqrt{F_\pi^{\rm bare}(0)F_K^{\rm bare}(0)}$ determined w/ electromagnetic form factor $F_{\pi,K}(0) = 1$

Ratio $(0 \ll t \ll t_{\rm sep})$

$$\frac{Z_\pi Z_K Z_V C_{V_\mu}(t,t_{\rm sep},p)}{C_\pi(t,p)C_K(t_{\rm sep}-t,0)} = M_\mu(p) + \underbrace{A(p)e^{-\Delta_\pi(p)t} + B(p)e^{-\Delta_K(t_{\rm sep}-t)}}_{\text{1st excited state, } \pi', K', \text{ contributions}}$$

$\Delta_\pi(p) = E_{\pi'}(p) - E_\pi(p), \Delta_K = M_{K'} - M_K$

Extract $M_\mu(p) = \langle\pi(p)|V_\mu|K(0)\rangle$ w/ fit including 1st excited states

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$

Form factors at each q^2 except for $p = 0$ (only $f_0(q^2)$)

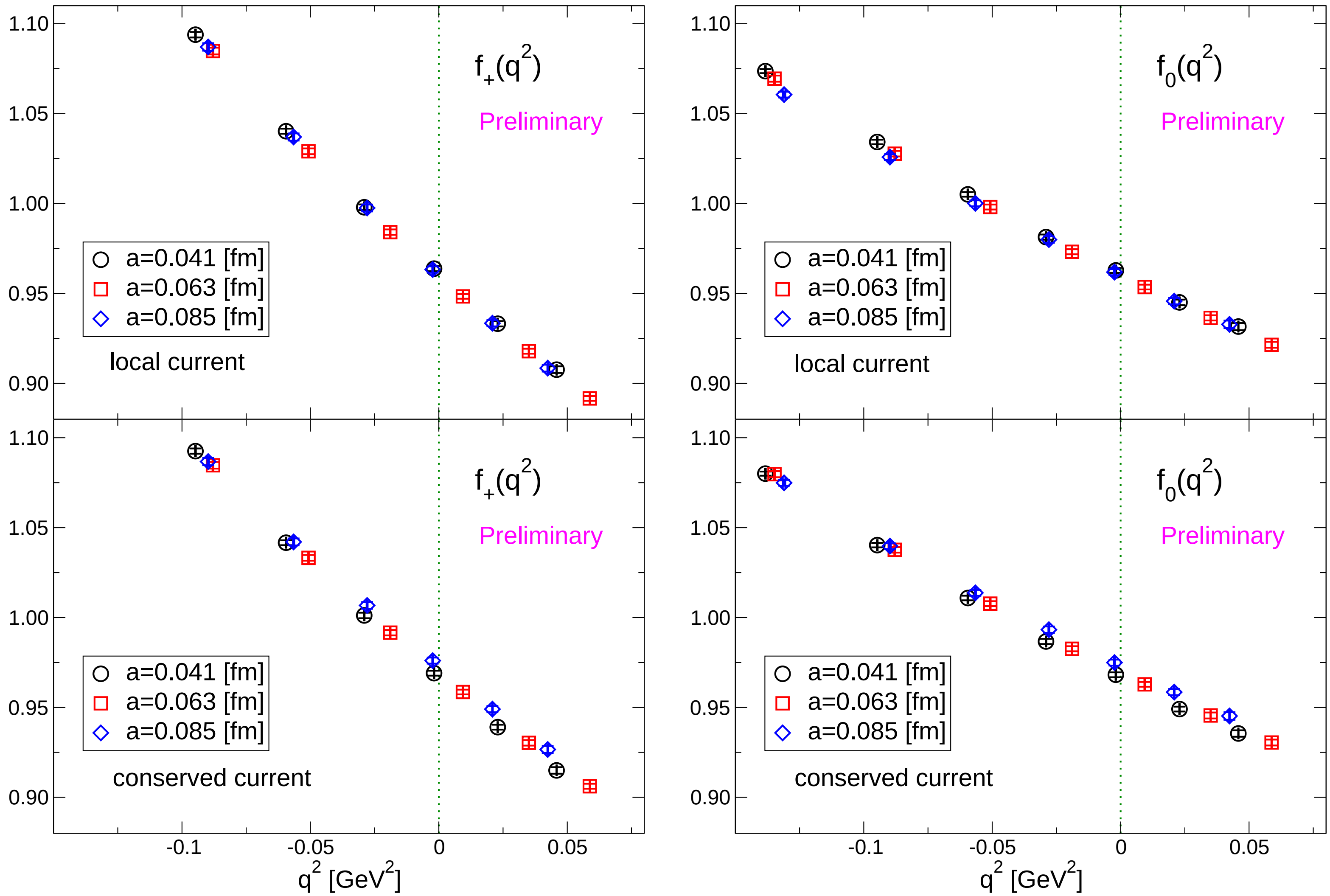
$$M_\mu(p) = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2}f_-(q^2)$$

$p_K = (M_K, \vec{0}), p_\pi = (E_\pi, \vec{p})$

$q^2 = -(M_K - E_\pi)^2 + p^2$

$f_+(q^2)$ and $f_0(q^2)$ at three lattice spacings



Clear signal at three lattice spacings

Several data in tiny q^2 region thanks to huge volume

Update from Lat23: Removing $\mathcal{O}(a^2)$ factor in conserved $f_+(q^2), f_0(q^2)$ due to point-splitting current

q^2 interpolation + $a \to 0$ extrapolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ c.f.) PACS:PRD106,9,094501(2022)

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_M \Delta M + c_2^+ q^4 + g_+^{\mathrm{cur}}(a, q^2)$$

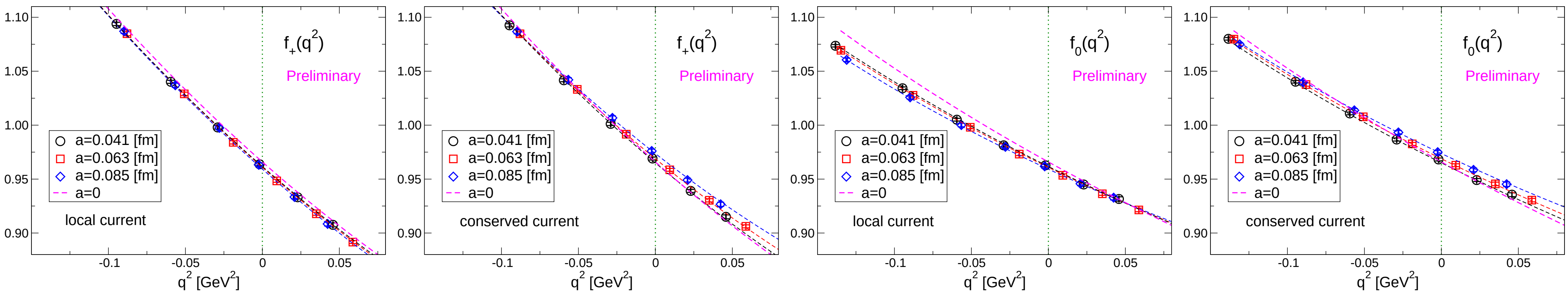
$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_M \Delta M + c_2^0 q^4 + g_0^{\mathrm{cur}}(a, q^2)$$

K_+, K_0: known functions ['85 Gasser, Leutwyler], $\Delta M = (M_K^2 - M_\pi^2)^2$

$g_{+,0}^{\mathrm{cur}} = \sum_{n,m} e_{+,0}^{\mathrm{cur},nm} a^n q^{2m}$, cur = local, conserved: a^2, a extrapolation investigated

free parameters: $L_5(\mu), L_9(\mu), c_0, c_2^+, c_2^0 + e_{+,0}^{\mathrm{cur},nm}$

fixed parameters: $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV F_0 estimated from FLAG $F^{\mathrm{SU(2)}}/F_0$ w/ $F^{\mathrm{SU(2)}} = 0.129$ GeV



magenta dashed-line : $a = 0$ @ physical point

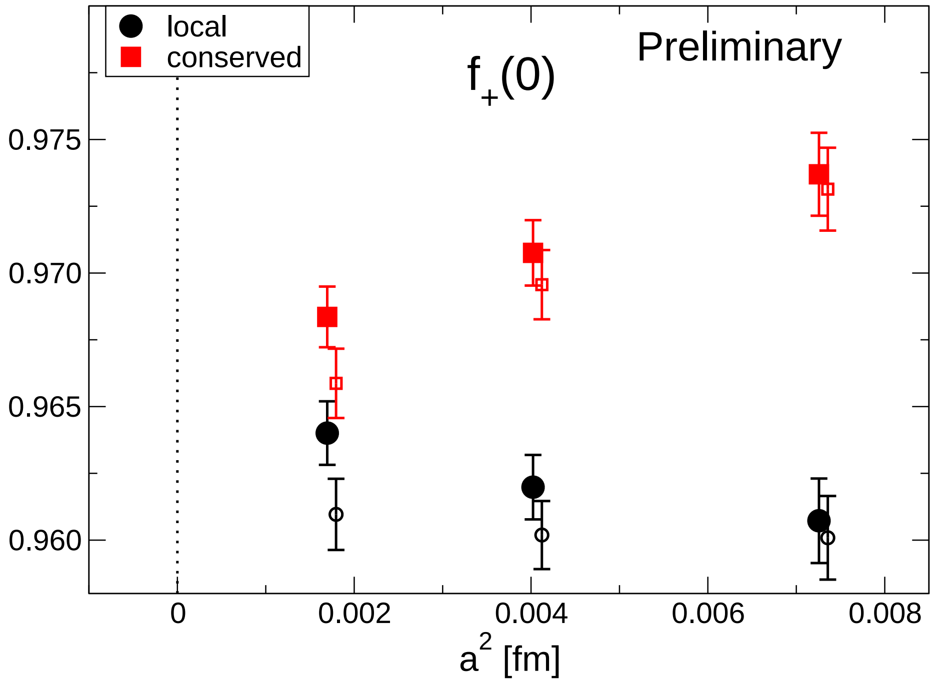
Simultaneous fit for (f_+, f_0) with (local,conserved)

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

a[fm]	M_π[MeV]	M_K[MeV]
0.041	142	514
0.063	137	501
0.085	135	497

$m_{\pi^-} = 139.57061$ MeV, $m_{K^0} = 497.611$ MeV

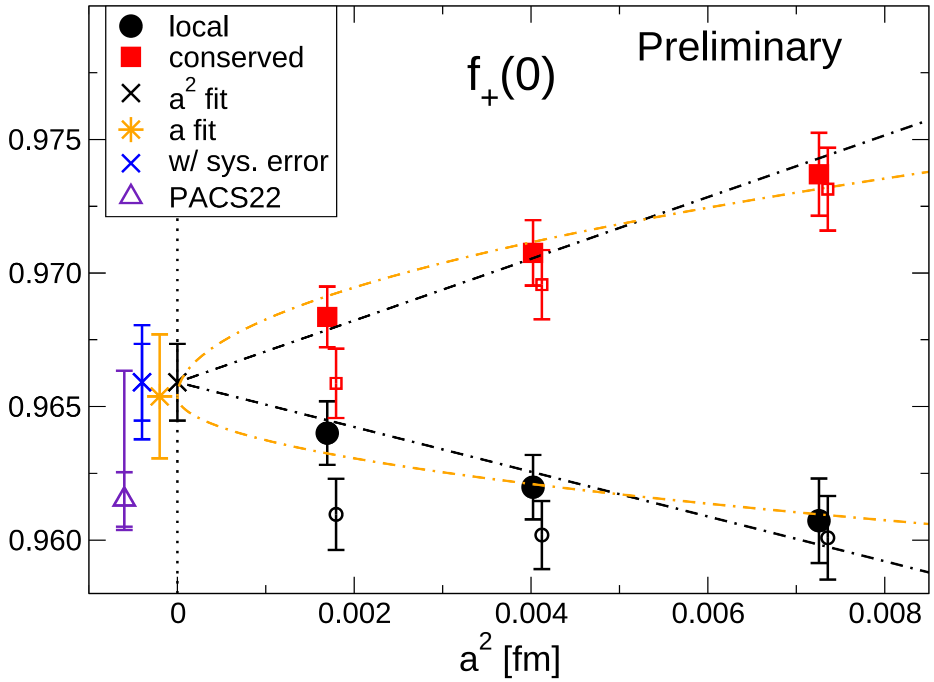
Continuum extrapolation of $f_+(0)$

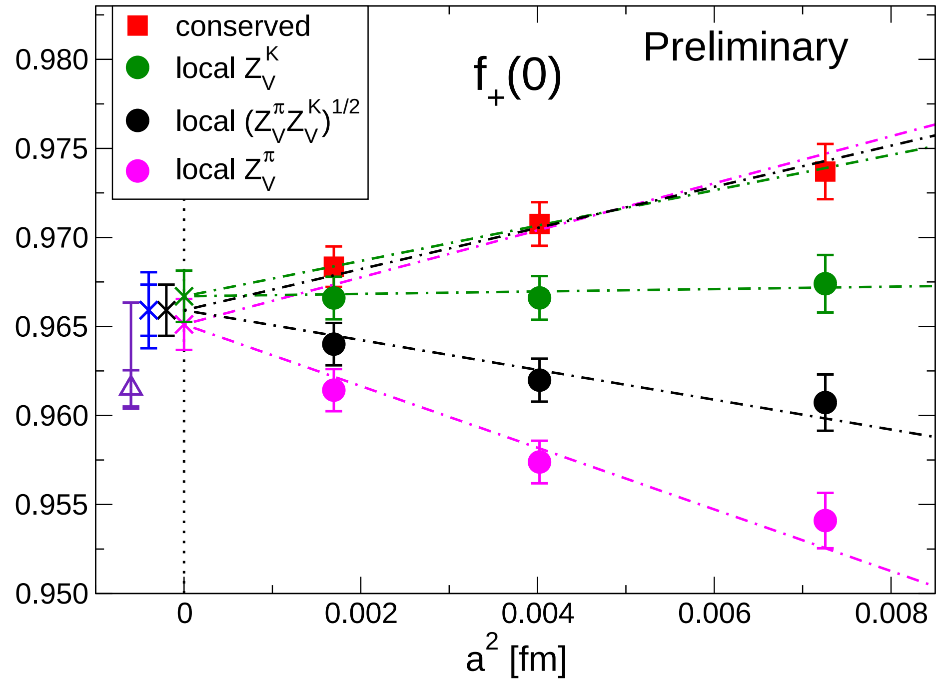


closed (open) symbol: physical (simulation) point data

Chiral extrapolation: tiny upward shift especially at $a = 0.041$ fm

Continuum extrapolation of $f_+(0)$ (simultaneous fit result at $q^2 = 0$)





closed (open) symbol: physical (simulation) point data

$Z_V^K = 1/F_K^{\text{bare}}(0), Z_V^\pi = 1/F_\pi^{\text{bare}}(0)$

Chiral extrapolation: tiny upward shift especially at $a = 0.041$ fm

Systematic error estimated various different analyses

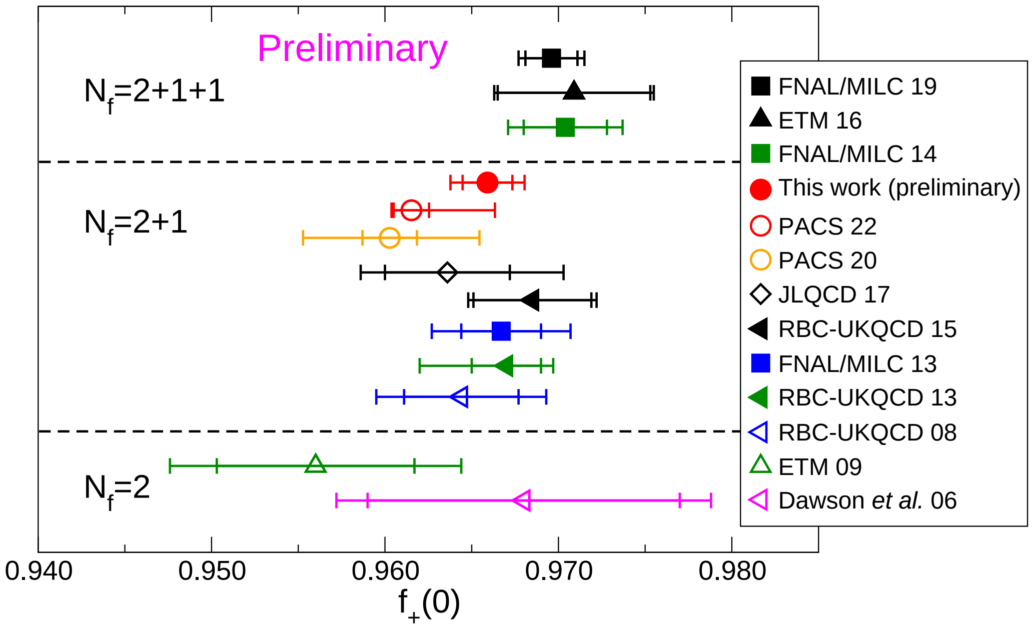
e.g., isospin breaking effect w/ SU(3) ChPT and different Z_V in local data

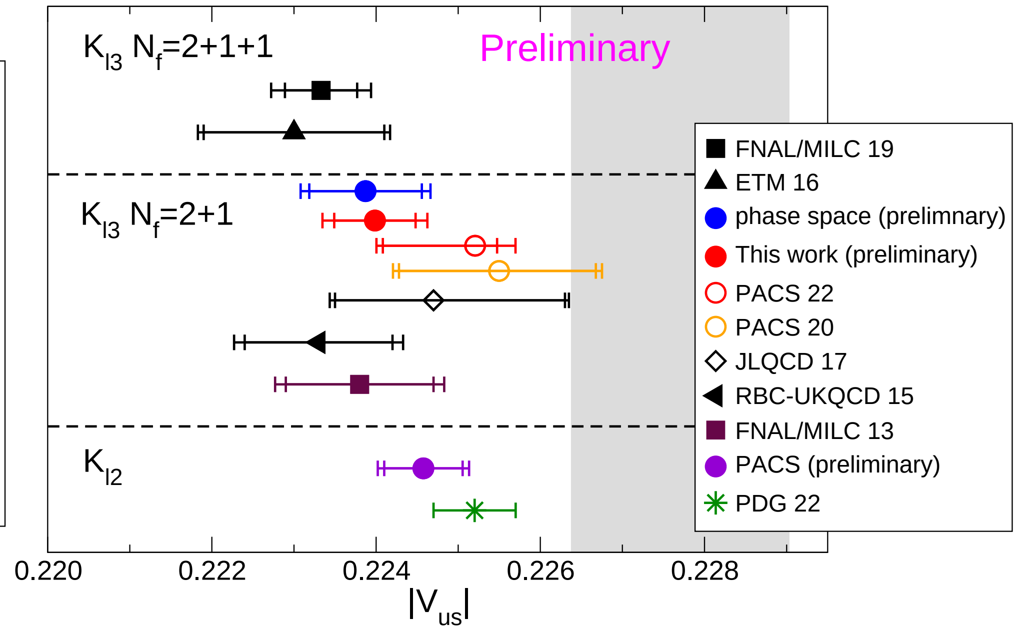
a^2 fit result agrees with a fit result and PACS22 result

$\mathcal{O}(a)$-improved current $V_\mu^{\text{imp}} = V_\mu + c_V \partial_\nu T_{\nu\mu}$ does not affect $f_+(0)$

$f_+^{\text{imp}}(q^2) = f_+(q^2) - c_V q^2 f_T(q^2)$, $f_0^{\text{imp}}(q^2) = f_0(q^2)$ through $\langle \pi | T_{\nu\mu} | K \rangle = (q_\nu^+ q_\mu - q_\mu^+ q_\nu) f_T(q^2)$

$f_+(0)$ and $|V_{us}|$





inner, outer = statistical, total(stat.+sys.)   inner, outer = lattice, total(lat.+exp.)

grey band: Standard model prediction using $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$ ['20 Hardy, Towner]

$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 2\sigma$

$|V_{us}|$: from $|V_{us}|f_+(0) = 0.21635(39)$ ['22 Seng *et al.*]

agree with $|V_{us}|$ from K_{l2} using f_K/f_π $\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(35)$ ['19 Di Carlo *et al.*]

2σ difference from CKM unitarity (grey band)

consistent with $|V_{us}|$ from phase space integral

Phase space integral I_K^ℓ

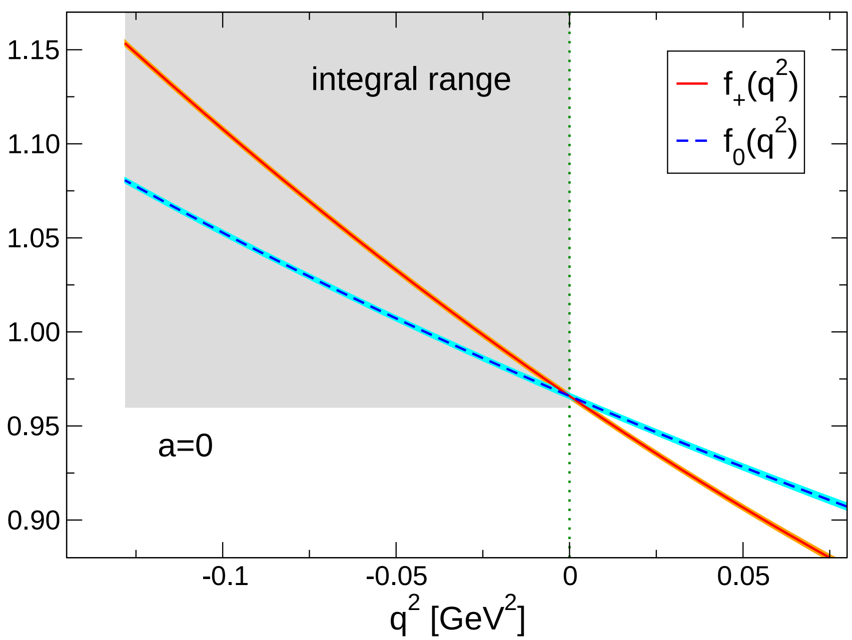
$\Gamma_{K_{\ell 3}} = C_{K_{\ell 3}} (|V_{us}| f_+(0))^2 I_K^\ell$ $\quad$ $\Gamma_{K_{\ell 3}}$: decay width, $C_{K_{\ell 3}}$: known factor, $\ell = e, \mu$

$|V_{us}| f_+(0) = 0.21654(41)$ ['17 Moulson]

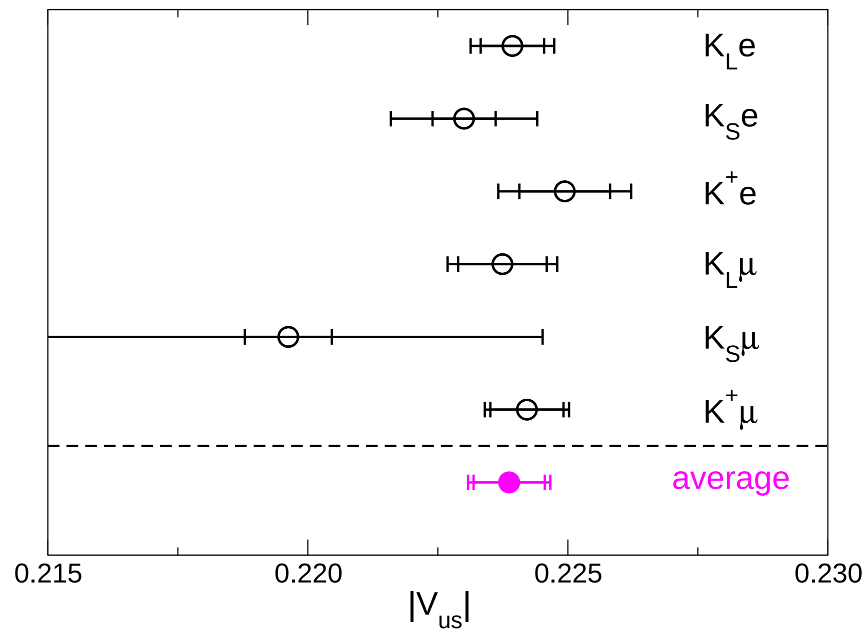
$\leftarrow$ I_K^ℓ from dispersive representation of experimental $\overline{F}_{+,0}(t)$

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t) \overline{F}_+^2(t) + J_0(t) \overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$J_{+,0}(t)$: known function ['84 Leutwyler, Roos]

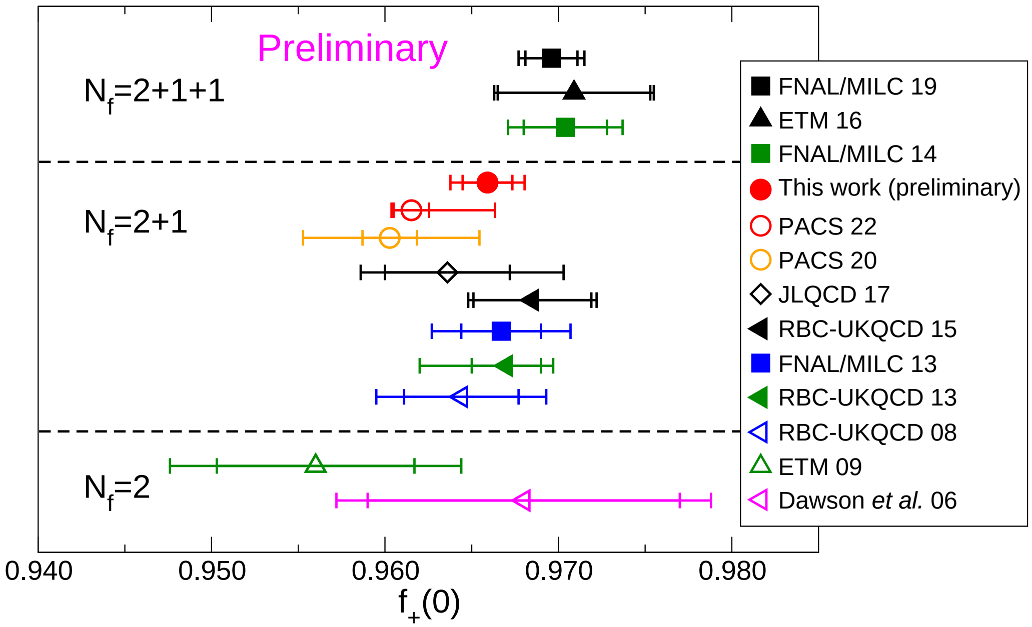


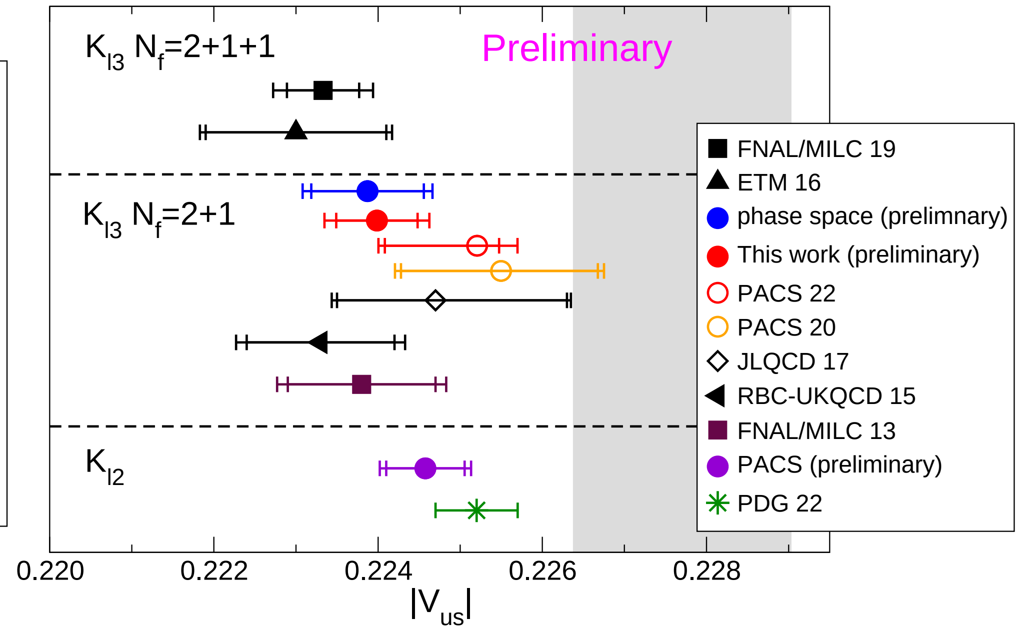
$K \to \pi \ell \nu$ decay channels



inner(outer) error = lat.(lat. + exp.) error

$f_+(0)$ and $|V_{us}|$





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2σ difference from CKM unitarity (grey band)

consistent with $|V_{us}|$ from phase space integral

Summary

Preliminary result of $K_{\ell 3}$ form factors with 3rd PACS10 configuration

$a = 0.041$ fm, $L \gtrsim 10$[fm] very close to physical point

- Clear signal for $f_+(q^2), f_0(q^2)$ in tiny q^2 region
- Continuum extrapolation of $f_+(0)$ with local and conserved data

  Small, clear effect from chiral extrapolation

  Reasonably consistent with PACS22 and previous results
- $|V_{us}|$ from our preliminary result

  Reasonably consistent with previous $K_{\ell 3}$ determinations

  Consistent with $K_{\ell 2}$ determinations

  Different from CKM unitarity by 2 σ

Future works

Isosping breaking effect from lattice calculation

dynamical charm quark effect

$N_f = 2+1+1$ simulation

Program for Promoting Researches on the Supercomputer Fugaku

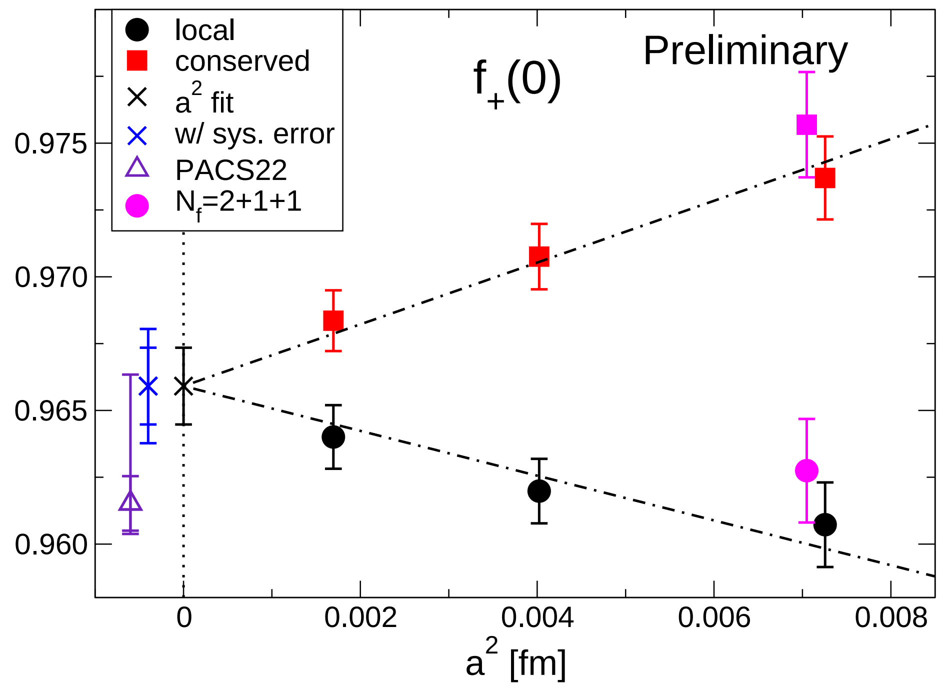
Large-scale lattice QCD simulation and development of AI technology

PACS10$_c$ configuration generation

the same actions, but $N_f = 2+1+1$ quark

(very close to) physical point on more than (10 fm)^4 volume

at three lattice spacings

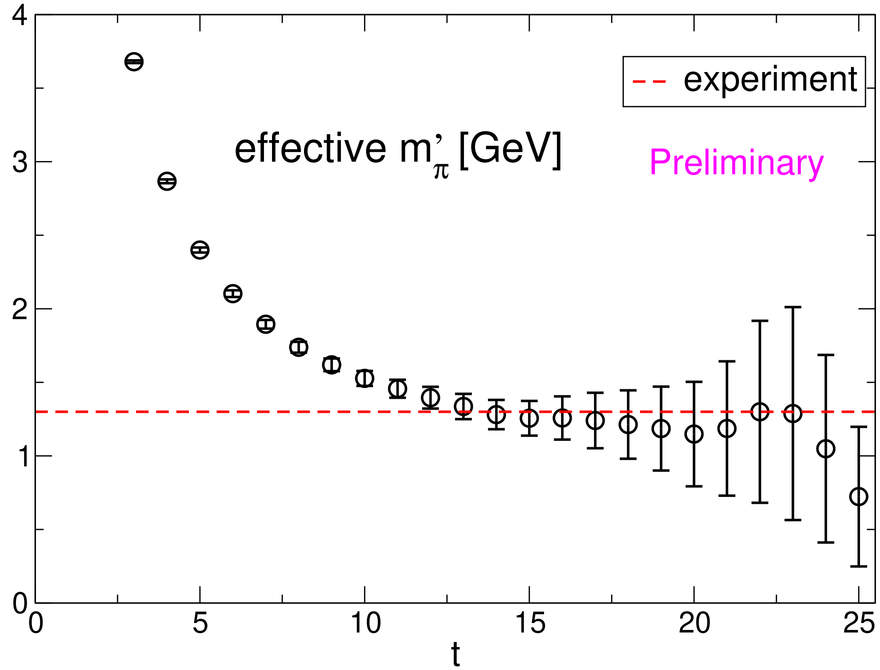


Preliminary PACS10$_c$/L128 result agrees PACS10/L128 result

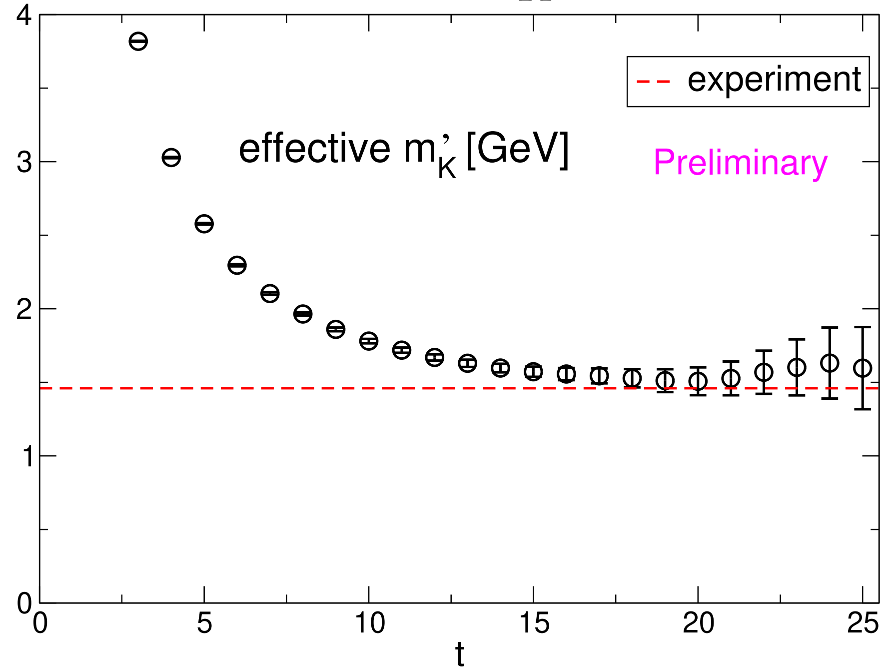
Back up

Effective mass for 1st excited states π', K' on 256^4

Eff. M'_π



Eff. M'_K



Effective mass of 1st excited state (local source)

$$M'(t) = \log\left(\frac{C'(t)}{C'(t+1)}\right), \; C'(t) = C(t) - \underline{A_0 e^{-M_0 t}}$$

ground state contribution from fit in $t \gg 1$

Slope and curvature for $f_+(q^2)$ and $f_0(q^2)$