Finite temperature QCD explored with chiral fermions

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Acknowledgements

Codes used:

- Grid (HMC)
- BQCD (Measurements)
- Bridge++ (Measurements)
- Hadrons (Measurements)

Grants:

- KAKANHI – (FY2020-2024) - QCD phase diagram explored by chiral fermions – 20H01907
- MEXT Program for Promoting Researches on the Supercomputer **Fugaku** (PPR-Fugaku)
  - (FY2020-2022) - Simulation for basic science: from fundamental laws of particles to creation of nuclei - JPMXP1020200105
  - (FY2023-2025) - Simulation for basic science: approaching the new quantum era - JPMXP1020230411

Computers:

- RIKEN Hokusai BW
- Ito at Kyushu University (hp190124, hp200050)
- Polaire and Grand Chariot at Hokkaido University (hp200130)
- supercomputer **Fugaku** at R-CCS (ra000001; hp210032,hp220108,hp220233; hp200130, hp230207)

Projects

Nf=2:
- DWF → Overlap; high T:
  - chiral symmetry, fate of U(1)A, topology
- DWF
  - spectrum (see Lattice 2024 talk by David Ward)

common set-up for :
- JLQCD type domain wall fermion (DWF)
  - Gauge: tree-level Symanzik
  - Fermions: Möbius DWF (scale factor=2 Shamir) with stout smeared links
- good knowledge of T=0 fine lattices for flavor physics
  - calibration for finite temperature needs only small effort (computational)

Nf=2+1:
- DWF → Overlap for high T (led by Hidenori Fukaya)
- **DWF: LCP analysis near and on the physical point**
  - **transition / crossover; topology**
  - charge fluctuation (see talk by Jishnu Goswami)

Nf=3:
- DWF: phase hunting near three-flavor degenerate chiral limit (see talk by **Yu Zhang**)

Members involved in the main topics of this talk






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Y. Nakamura[(1)], Y. Zhang[(6)],,,,

(1): RIKEN Center for Computational Science
(2): Osaka University
(3): KEK
(4): SOKENDAI
(5): Kobayashi-Maskawa Institute, Nagoya Univ.
(6): Bielefeld University

QCD phase transition near and on the physical point

- N_f=2+1, 2 fine lattice DWF simulation and reweighting to overlap [PRD(2021), PTEP(2022)]
  - Profound relation among: chiral symmetry, axial anomaly and topological susceptibility
- R & D for the N_f=2+1 thermodynamics with Line of Constant Physics (LCP)
  - Codes: Grid, Hadrons, Bridge++
  - LCP / Reweighting
  - Chiral order parameter and renormalization
  - Quark number susceptibility

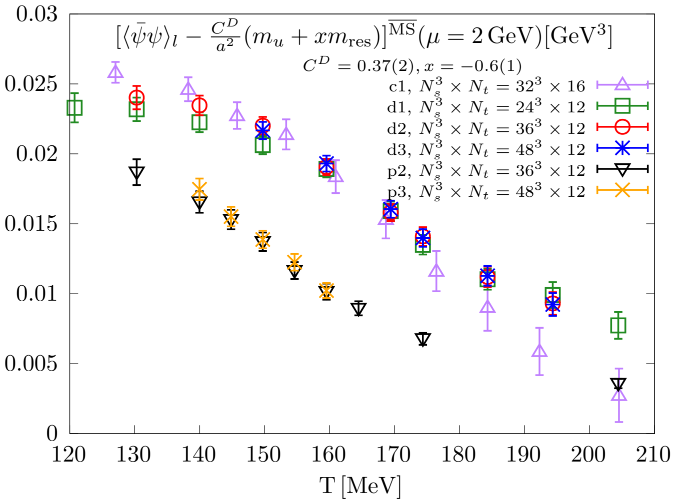


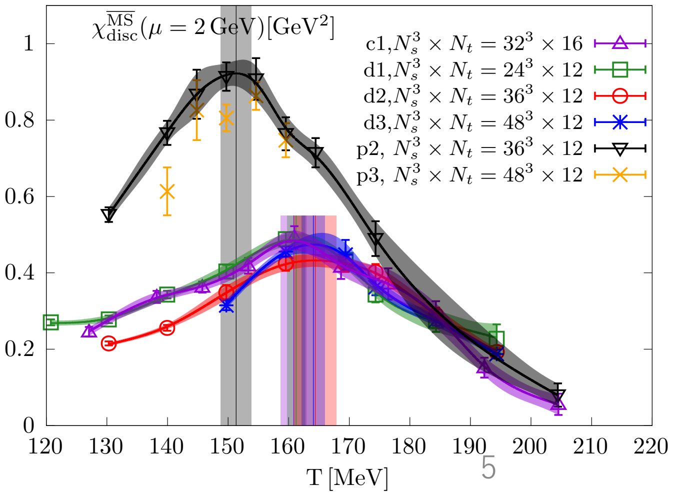






- N_f=2+1 - thermodynamics with LCP (mass = ms/10 = about 3 x physical ud quark mass)
  - 2 step renormalization for chiral condensate (power and log divergence) with an xm_{res} correction
  - 2 lattice spacings N_t=12, 16
  - 3 volumes N_s/N_t=2, 3, 4
  - *No phase transition !*
  - *T_{pc} determined $T_{pc} = 165(2)$ MeV*
  - PPR-Fugaku FY2020-2022
  - [PoS Lattice 2021, 2022]
- Physical point study
  - PPR-Fugaku 2023- preliminary results →



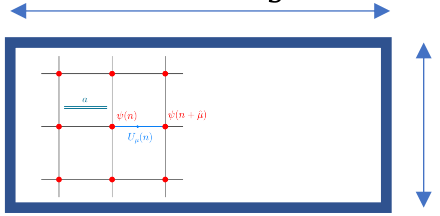


Modes of Simulations

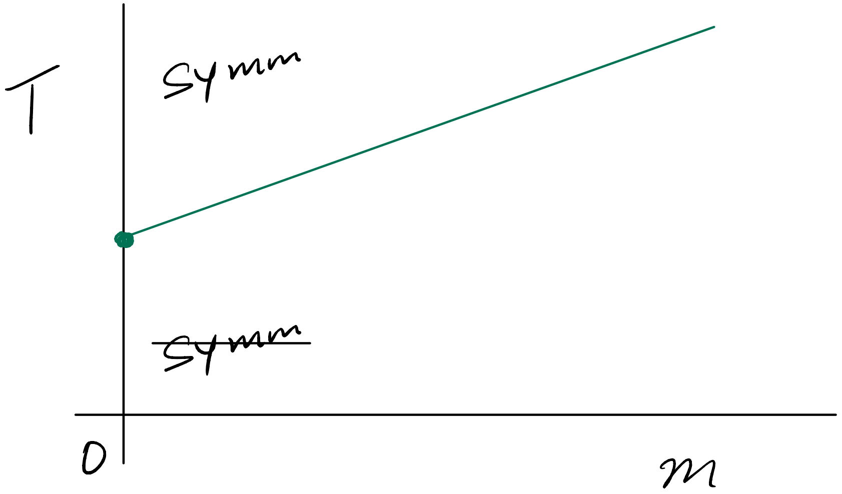
to locate phase transition

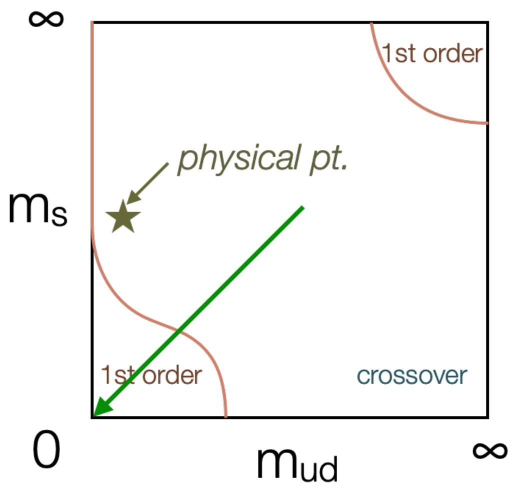
- tune parameters near transition
- ➢ T: fixed, change m
- ➢ m: fixed, change T

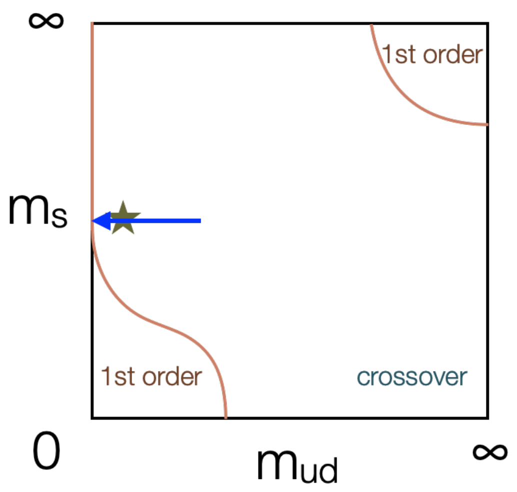
$L = aN_s$



$$\frac{1}{T} = aN_t$$







Modes of Simulations

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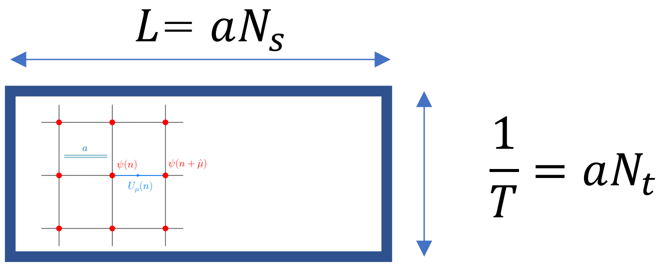
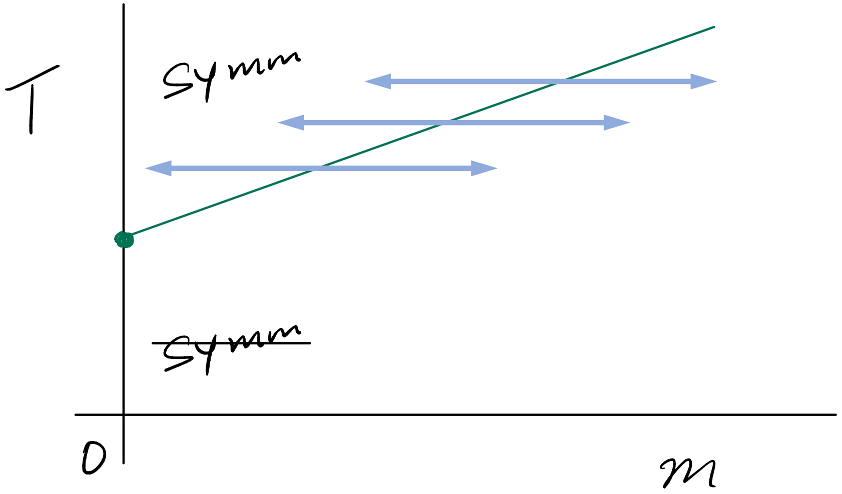
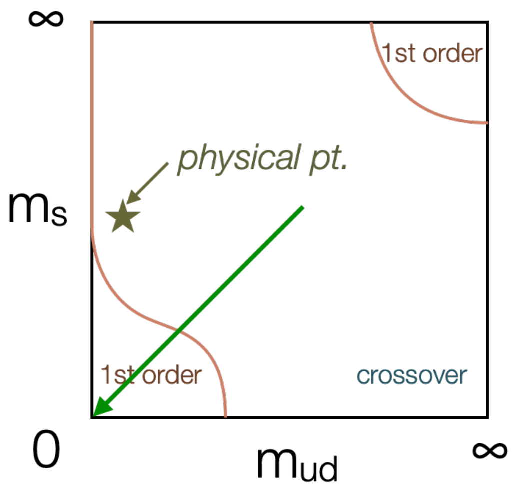
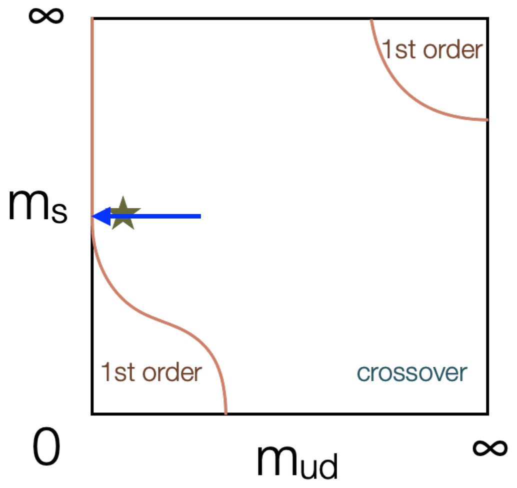








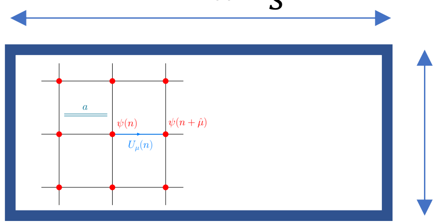
Modes of Simulations

Nf=2: Ward (Lattice 2024)
Nf=3: Zhang

to locate phase transition

- tune parameters near transition
- ➢ T: fixed, change m
- ➢ m: fixed, change T

$L = aN_s$



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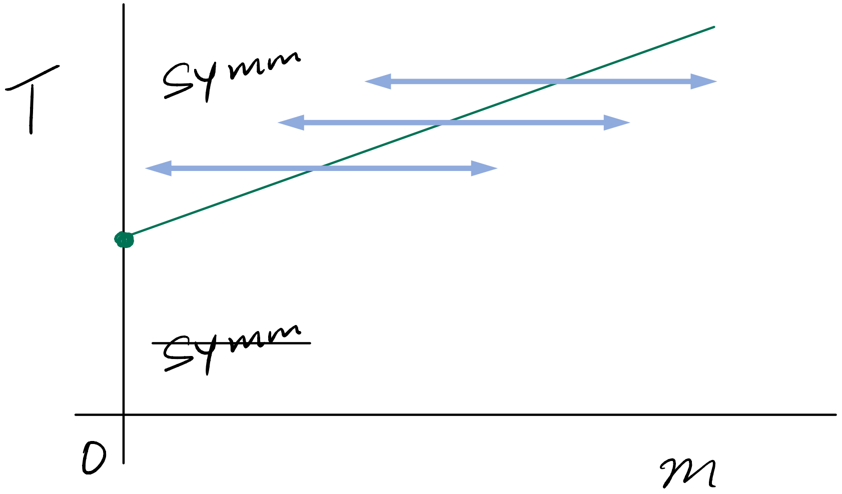
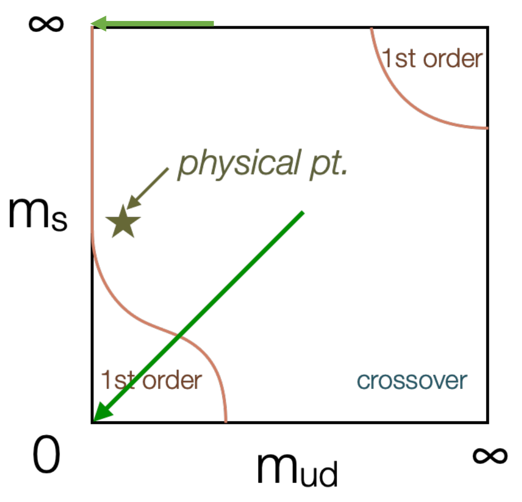
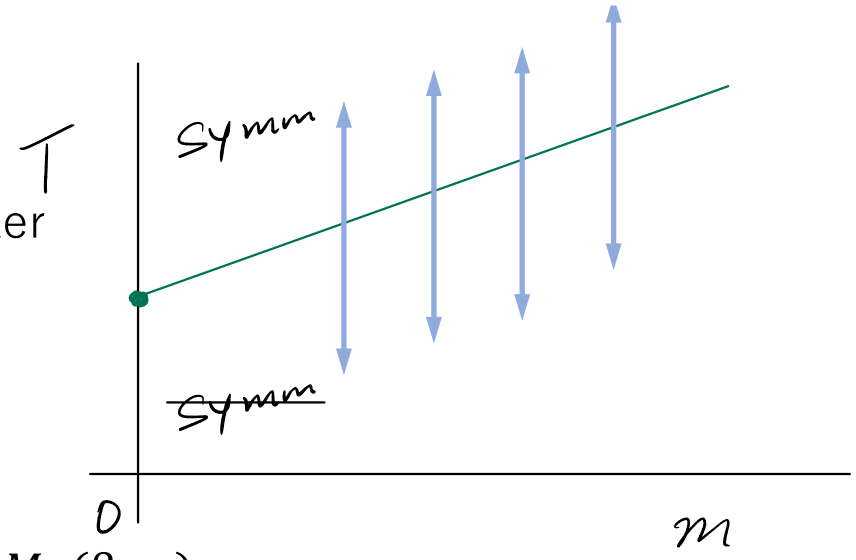
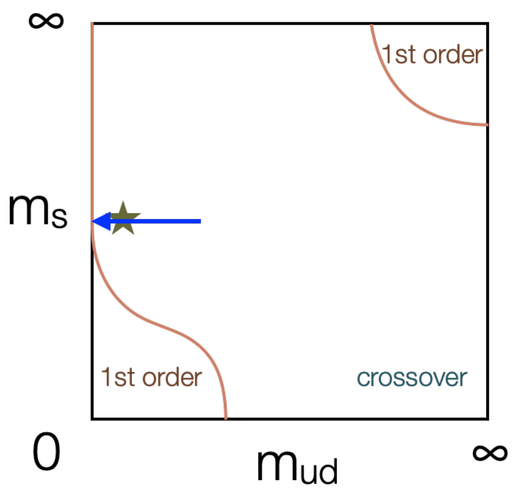








Fixing / changing the controlling parameter

- T: controled by
  - $a(\beta)$: controlled by β
  - N_t : discrete
- m: controlled by
  - input quark mass
  - $m(\beta) \leftarrow$ matching with hadronic scale: $M_H(\beta, m)$

N_f=2+1 Möbius DWF LCP for **2023-**

For the **L**ine of **C**onstant **P**hysics: $\boldsymbol{am_s(\beta)}$ with $\boldsymbol{a(\beta)}$

- Step 1: determine $\boldsymbol{a(\beta)}$ [fm] with t_0 (BMW) input
  - at $\beta = \mathbf{4.0}, \sout{4.1}^*, 4.17, 4.35, 4.47$

    * β=4.0 new data, to add support at small β

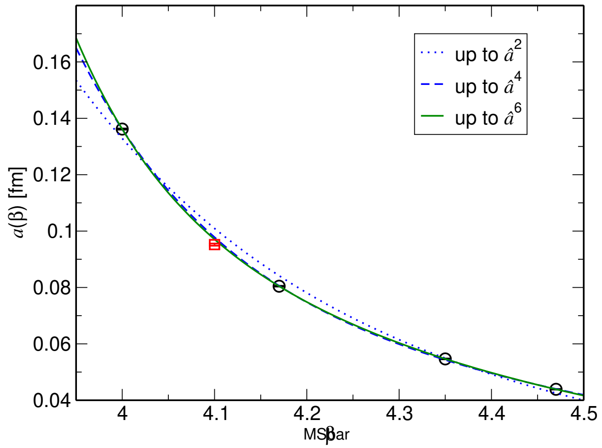
    * β=4.1 old pilot study data, removed - small volume and statistics

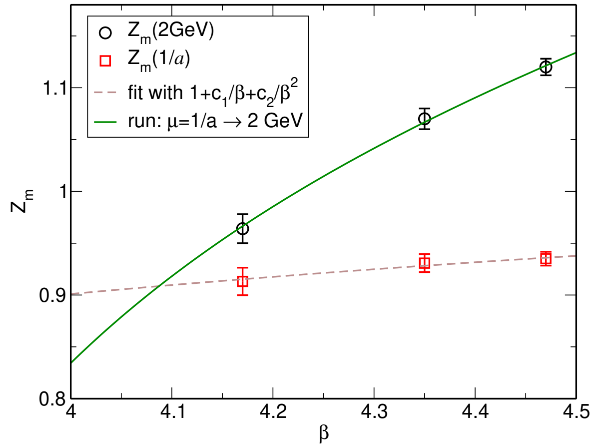
- Step 2: determine $Z_m(\beta)$ using Non-Perturbative Renormalization results
  - at $\beta =$ $4.17, 4.35, 4.47$; Z_m with $\overline{MS}$ 2 GeV are available
  - NNNLO running: $\mu = 2\,GeV \to 1/a$ & β polynomial fit & running back
  - use $Z_m(\beta)$ so obtained for $\beta \geq 4.0$: $\beta < 4.17$ region is extrapolation
  - $1/Z_m(\beta)$ will be used to renormalize scalar operator, **chiral condensate**
- Step 3: solve $\boldsymbol{am_s(\beta)}$ with input (*quark mass input*):
  - $m_s^R = Z_m \cdot am_s^{latt} \cdot a^{-1} = 92$ MeV
  - $\frac{m_s}{m_{ud}} = 27.4$ (See for example FLAG 2019)
- See for details in Lattice 2021 proc by S.Aoki et al.

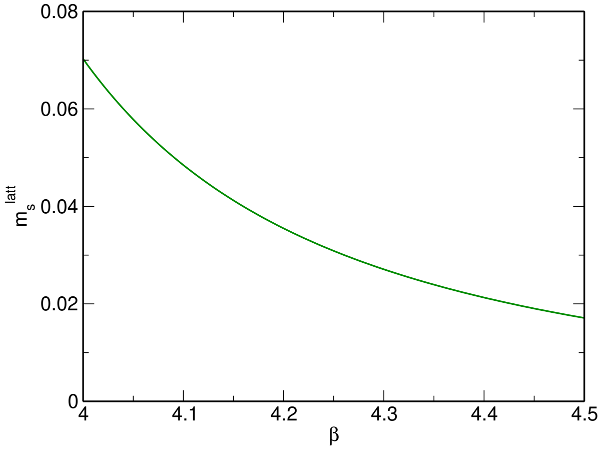
Do simulation

- Step 4: proper tuning of input mass: correct m_{res}

Do simulation 2nd round / correction with reweighting + valence meas.



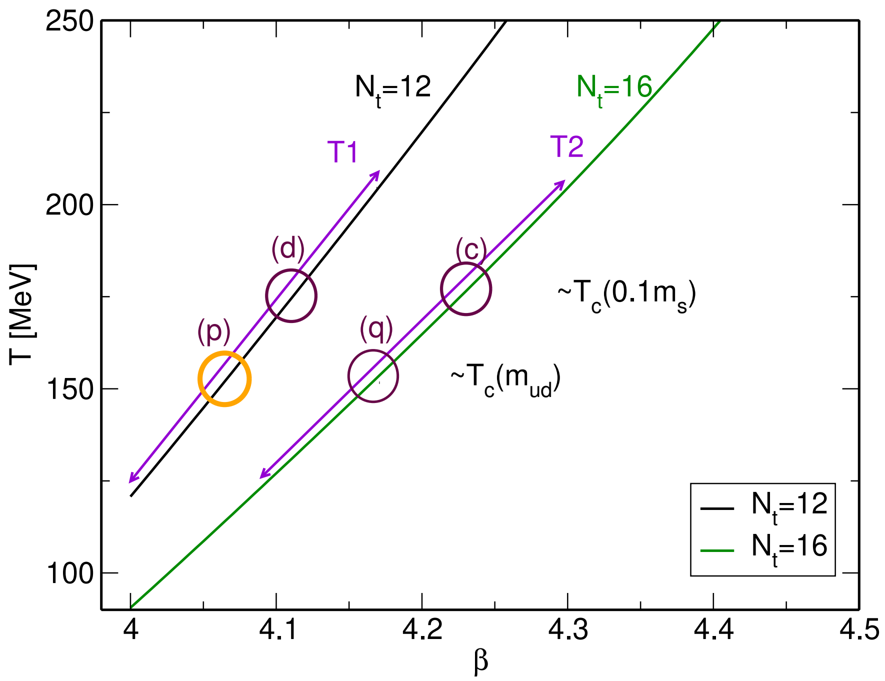


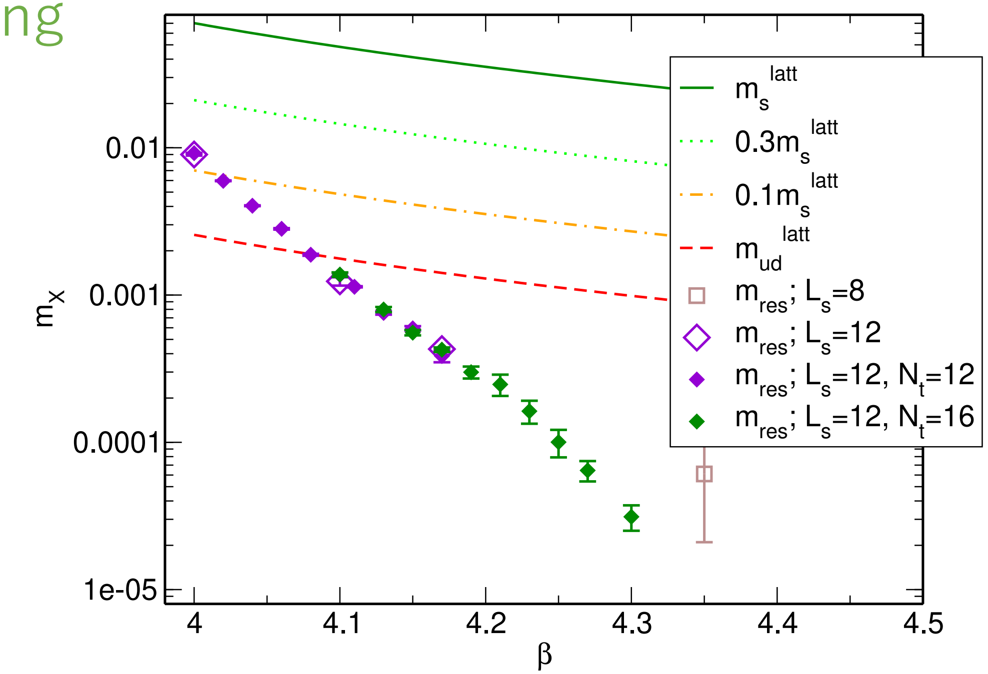


Simulation plan: 2nd round w/ treatment of m_{res} effect

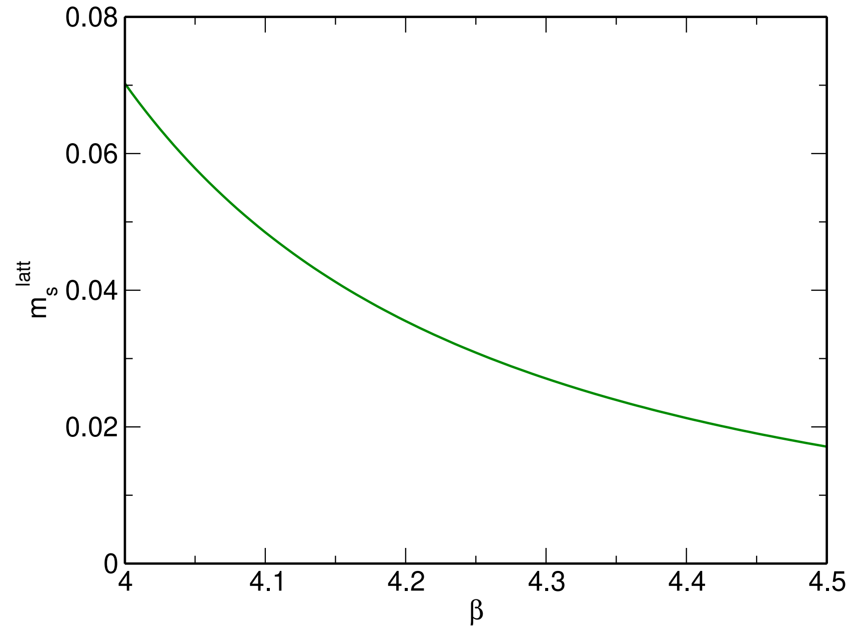
$L_s = 12$ fixed throughout this study

- T1-(d)
  - $N_t = 12$
  - $m_l = 0.1 m_s$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 24^3, 36^3$
- T2-(c)
  - $N_t = 16$
  - $m_l = 0.1 m_s$
  - m_{res} shift by reweighting
  - $V_s = 32^3$
- T1-(p)
  - $N_t = 12$
  - $m_l = m_{ud}$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 36^3, 48^3$
- T2-(q)
  - $N_t = 16$
  - $m_l = m_{ud}$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 48^3$





LCP remarks for **FT2023-**

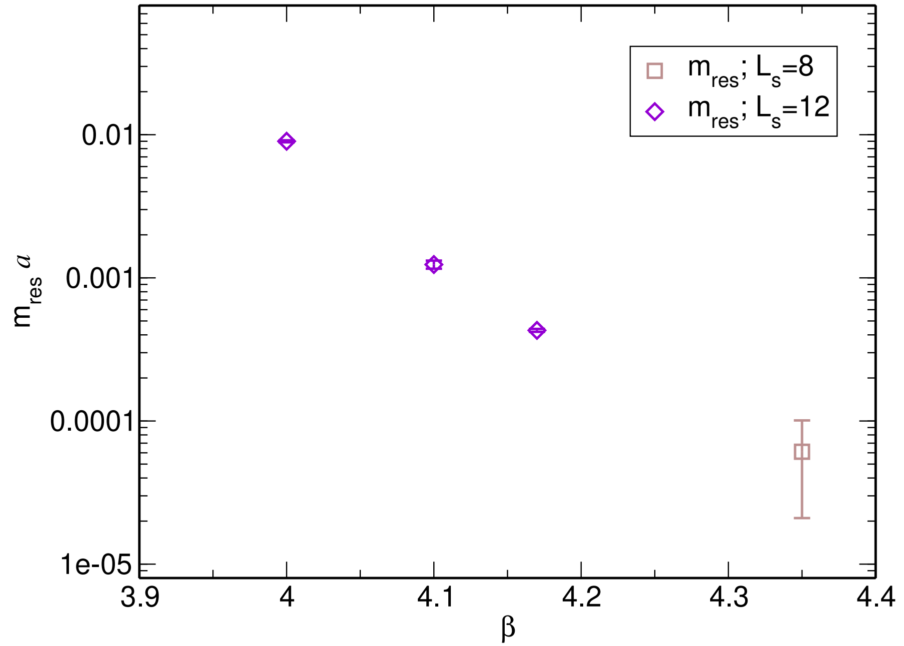


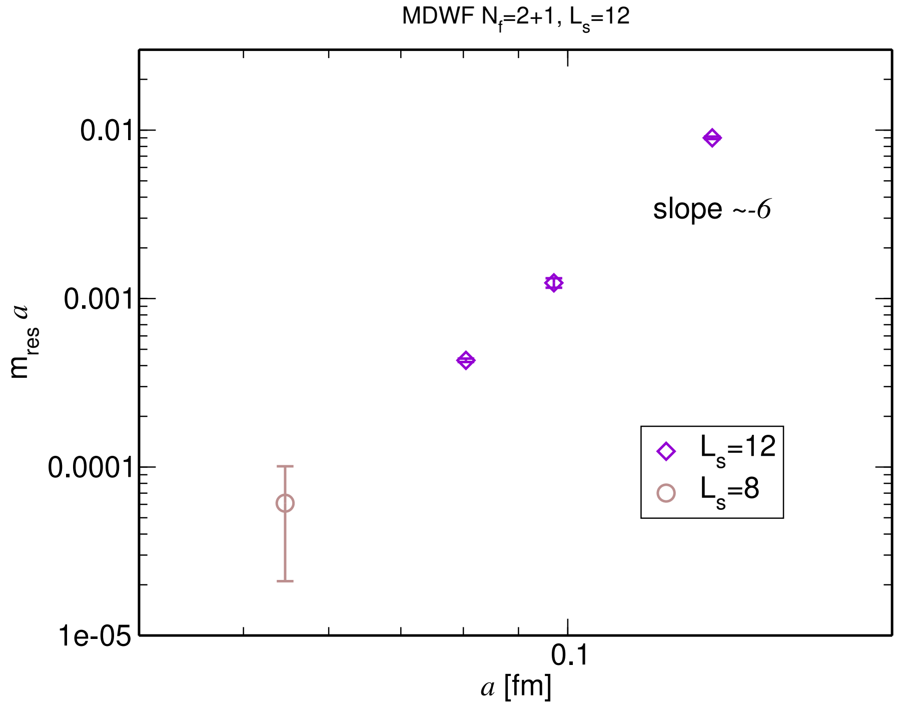
Features

- Fine lattice: use of existing results ($\mathbf{0.04 \le a \le 0.08}$ fm)
  - Granted preciseness towards continuum limit
- Coarse lattice parametrization is an extrapolation
  - Preciseness might be deteriorated
  - Newly computing Z_m e.g. at $\beta = 4.0$ (lower edge) might improve, but not done so far
    - NPR of Z_m at $a^{-1} \simeq$1.4 GeV may have sizable error (window problem) anyway
- Smooth connection from fine to coarse should not alter leading $O(a^2)$
  - Difference should be higher order
- Error estimated from Kaon mass (at physical point)
  - Δm_K ~ ~~10 %~~ at $\beta = 4.0$ ($a \simeq 0.14$ fm) → Δm_K ~ a few %
  - Δm_K ~ a few % at $\beta = 4.17$ ($a \simeq 0.08$ fm)

Domain wall fermions

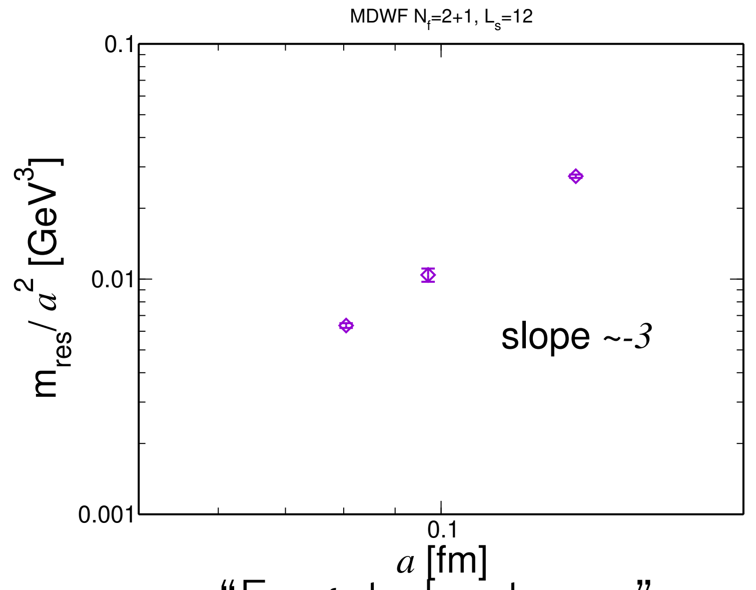
- Möbius DWF → OVF by reweighting
  - Successful (w/ error growth) at $\beta = 4.17$ ($a \simeq 0.08$ fm)
    - See Lattice 2021 JLQCD (presenter: K.Suzuki)
  - Questionable for
    - Coarser lattice: rough gauge, DWF chiral symmetry breaking
    - Finer lattice: larger V (# sites)
- Chiral fermion with continuum limit
  - A practical choice is to stick on DWF
- Controlling chiral symmetry breaking with DWF
  - WTI residual mass m_{res}: $m_\pi^2 \propto (m_f + m_{res})(1 + h.o.)$
  - Understanding $m_{res}(\beta)$ with fixed L_s (5-th dim size)
- $m_{res}[MeV] \sim a^X$, where $X \sim 5$
  - Vanishes quickly as $a \to 0$
  - 1st (dumb) approximation: forget about m_{res}
  - Better : $m_f^{cont} \leftrightarrow (m_f + m_{res})$ but, this is not always enough





Light quark $\Sigma = -\langle\bar{\psi}\psi\rangle$: conventional and residual power divergence

- $\Sigma|_{DWF} \sim C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)
  - $m_{res} \neq x m_{res}$; $x = O(1) \neq 1$
    - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.
- cf: $m_\pi^2 \propto (m_f + m_{res})$ [1+h.o.]
- $\Sigma|_{DWF} \to C_D \frac{x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$; $(m_f \to 0)$
- $\Sigma|_{DWF} \to C_D \frac{-(1-x) m_{res}}{a^2} + \Sigma|_{cont.}$; $(m_f \to -m_{res})$



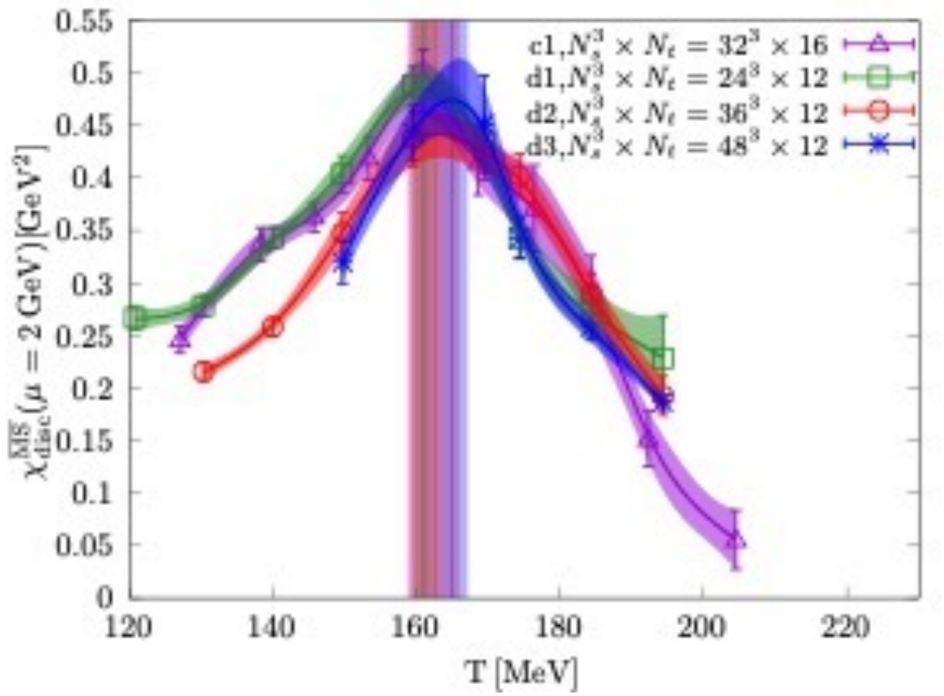
"Forget about m_{res}" is dumber for Σ, but…

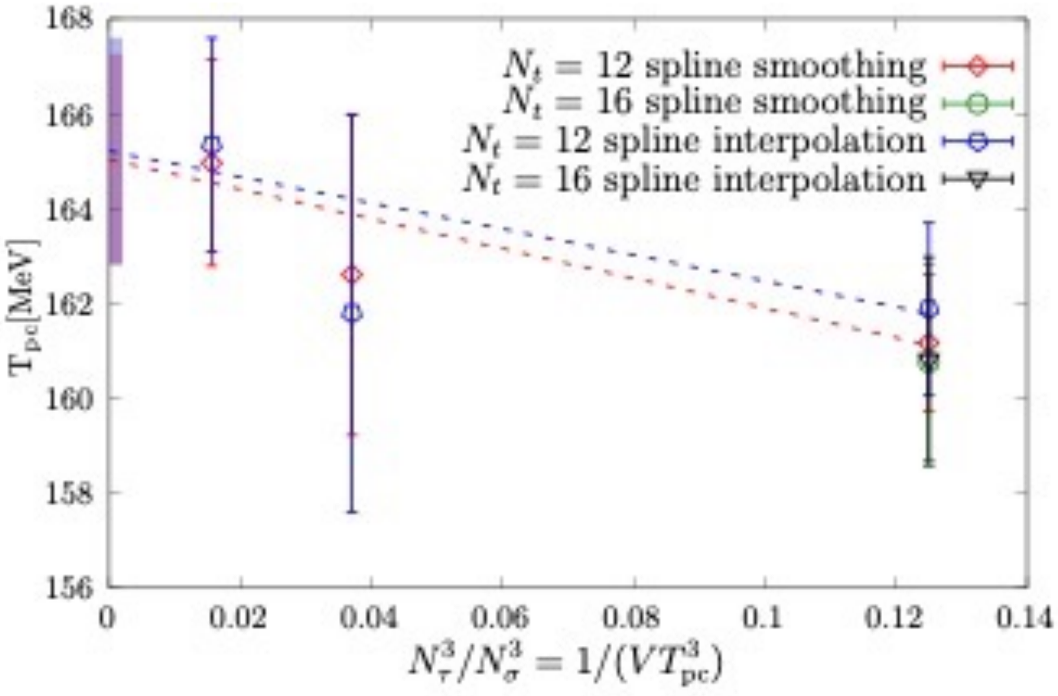
Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$: no power div. in disconnected susceptibility

- $\chi_{disc} = \langle\overline{u}u \cdot \overline{d}d\rangle - \langle\overline{u}u\rangle\,\langle\overline{d}d\rangle$
  - power divergence in $\langle\overline{\psi}\psi\rangle$ cancels out
  - no new divergence over Σ because no new contact terms
  - needs multiplicative renormalization for logarithmic divergence
  - $Z_S(\beta) = 1/Z_m(\beta)$
  - we stick for now on this quantity
- $\chi_{total} = \langle\overline{\psi}\psi \cdot \overline{\psi}\psi\rangle - \langle\overline{\psi}\psi\rangle\,\langle\overline{\psi}\psi\rangle$
  - has power divergence everywhere
  - needs to understand the power divergence of $\Sigma = -\langle\overline{\psi}\psi\rangle$ first

Chiral susceptibility (disconnected)

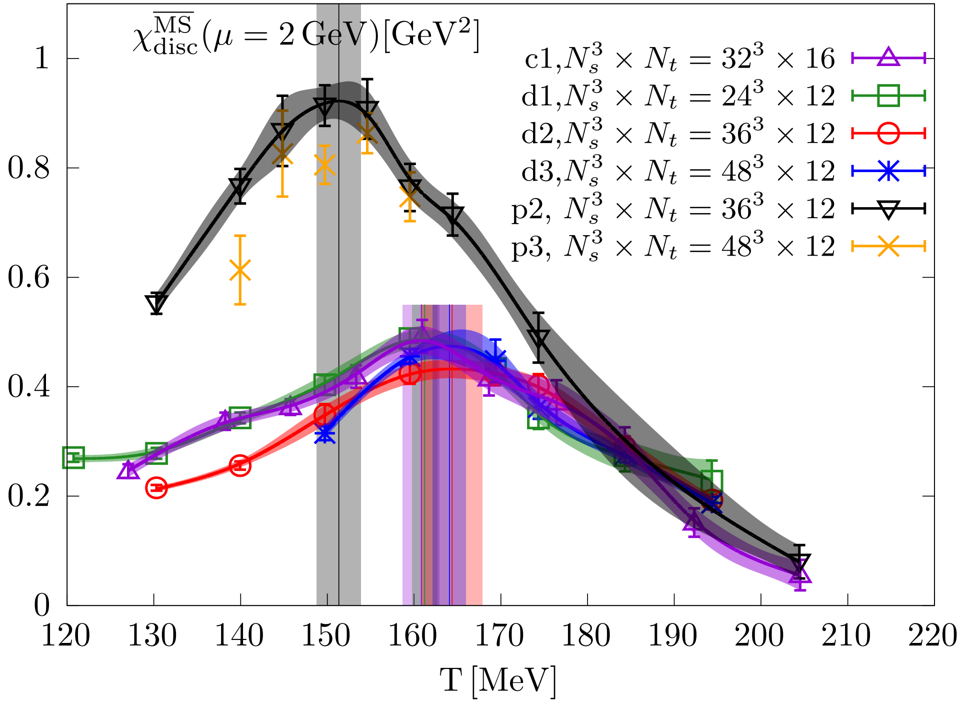
$m_l = 0.1 m_s$ (about 3 time larger than physics u,d mass)





- no subtraction needed in addition to vacuum subtraction
- peak position : mild volume dependence → infinite volume limit
- observing no dependence for N_t=12 and 16 (LT=2)
- $\boldsymbol{T_{pc} = 165\,(2)}$ MeV from the disconnected chiral condensate

Disconnected chiral susceptibility at average **physical** u and d quark mass



Likely NO phase transition at physical point with chiral fermions.
No surprise happened so far..

$m_l = m_s/10$

- d1,d2,d3 : $N_t = 12$, LT=2,3,4
- c1 : $N_t = 16$, LT=2
- good scaling $N_t = 12$ -16 observed for LT=2

$m_l = m_{ud}$

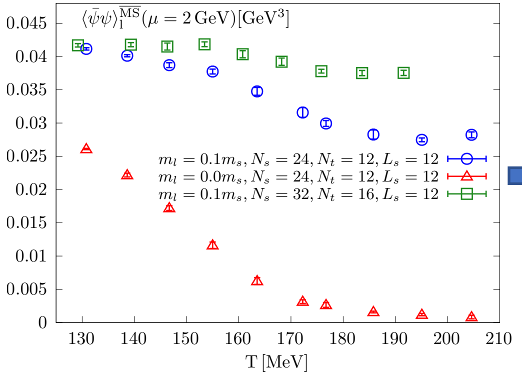
- p2,p3: N_t=12, aspect ratio LT = 3, 4
  - Statistics is ~20,000 MDTU for LT=3, sampled every 10 MDTU
  - LT=4 very preliminary, currently running to get to planned satat.
- $T_{pc} = \mathbf{151\ (3)}$ MeV (preliminary) on $\mathbf{36^3{\times}12}$, compared with
  - $T_{pc} = \mathbf{155\ (1)(8)}$ w/ DWF (**N_t=8**) by HotQCD (2014)
  - $T_{pc} = \mathbf{156.5\ (1.5)}$ w/ HISQ by HotQCD (2019) (≃disconnected)
  - $T_{pc} = \mathbf{158.0\ (0.6)}$ w/ stout staggered by Budapest-Wuppertal (2020)

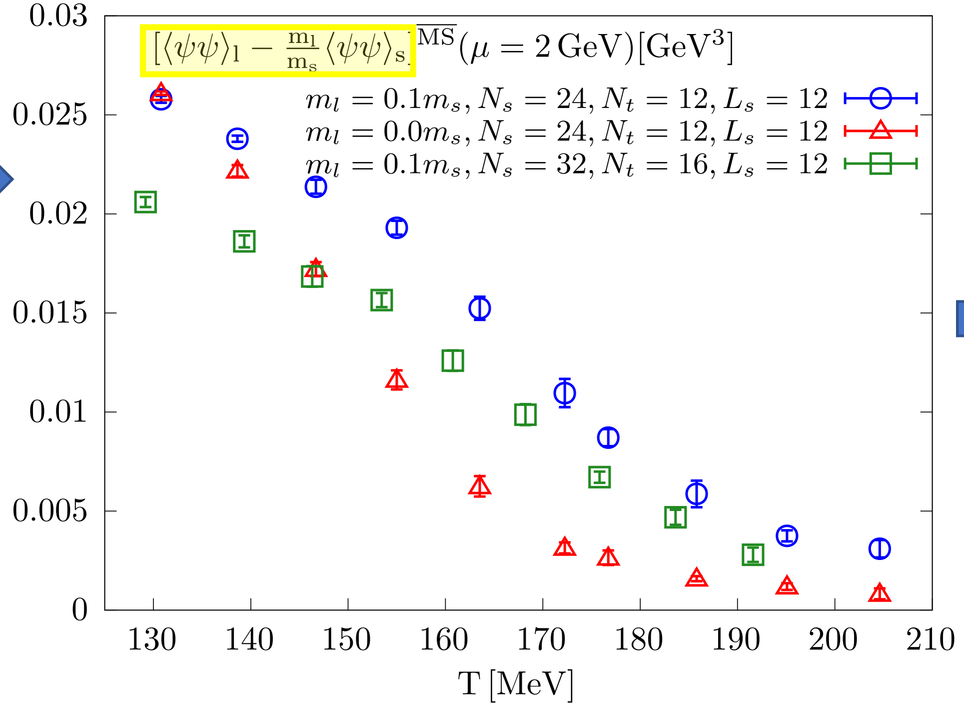
Light quark $\Sigma = -\langle\bar{\psi}\psi\rangle$

- Two step UV renormalization necessary (naively)
  - Logarithmic divergence (multiplicative): $Z_S(\overline{MS}, 2\text{ GeV})$
  - Power divergence (additive): $\propto m_f\, a^{-2}$
    - Subtracted using $\langle\bar{s}s\rangle$

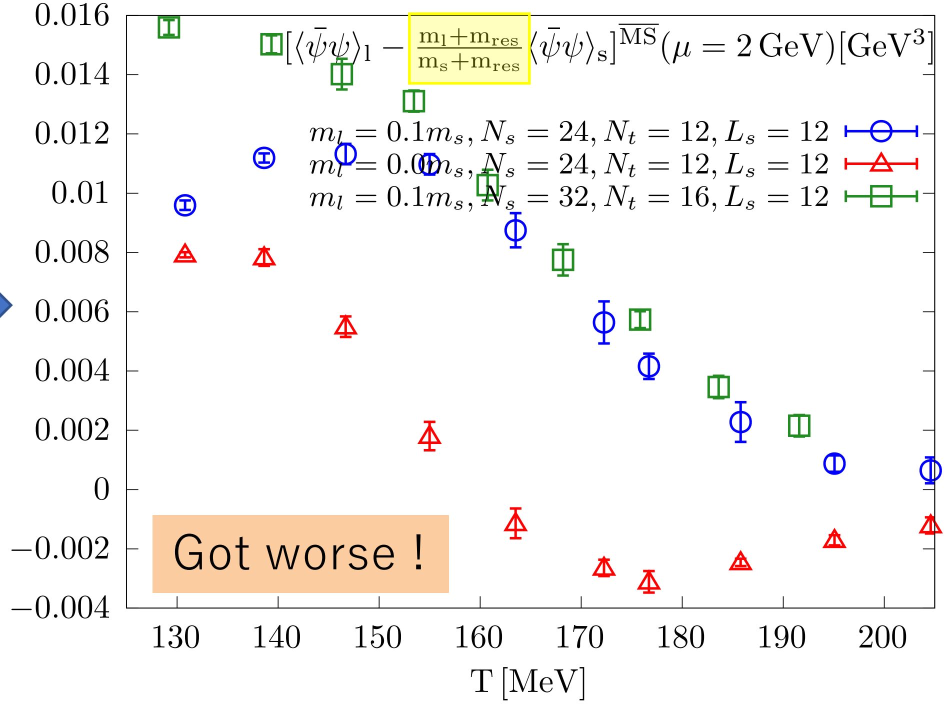
$$m_{res} = \frac{\langle 0|J_{5q}|\pi\rangle}{\langle 0|P|\pi\rangle}$$

$$m_\pi^2 \propto (m_f + m_{res})$$





Origin of subtraction: $m_l = 0$



Origin of subtraction: $m_l = -m_{res}$

Light quark $\Sigma = -\langle\bar{\psi}\psi\rangle$: residual power divergence

- $\Sigma|_{DWF} = C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)

  $m_{res} \neq x m_{res}$; $x = O(1) \neq 1$

  - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.

- (we proposed another way to utilize m'_{res}, which end up mixing T=0 C_R into high T)
- Yet another way of subtraction including $x m_{res}$ using $\boldsymbol{N_f = 3, T = 0}$ & $\boldsymbol{T > T_c}$ information

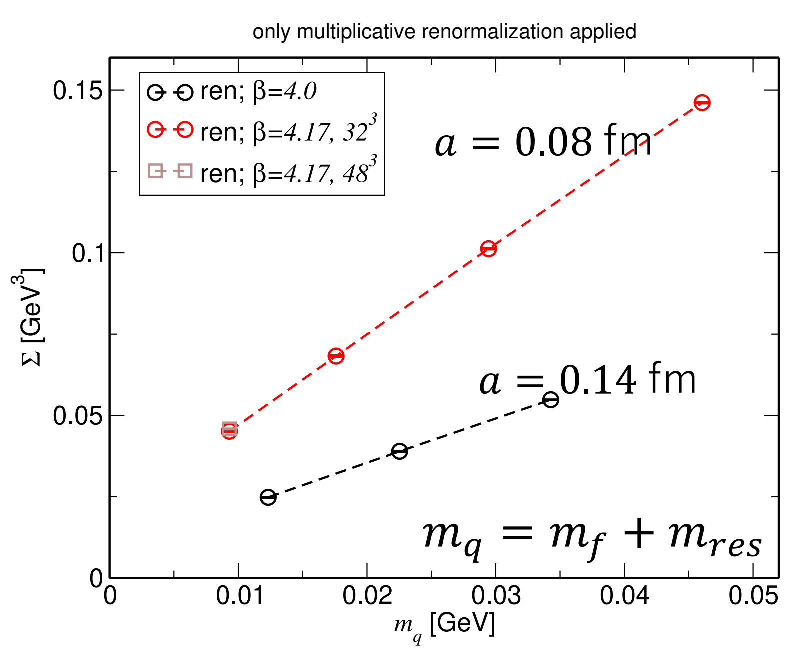
  →see the talk by Yu Zhang

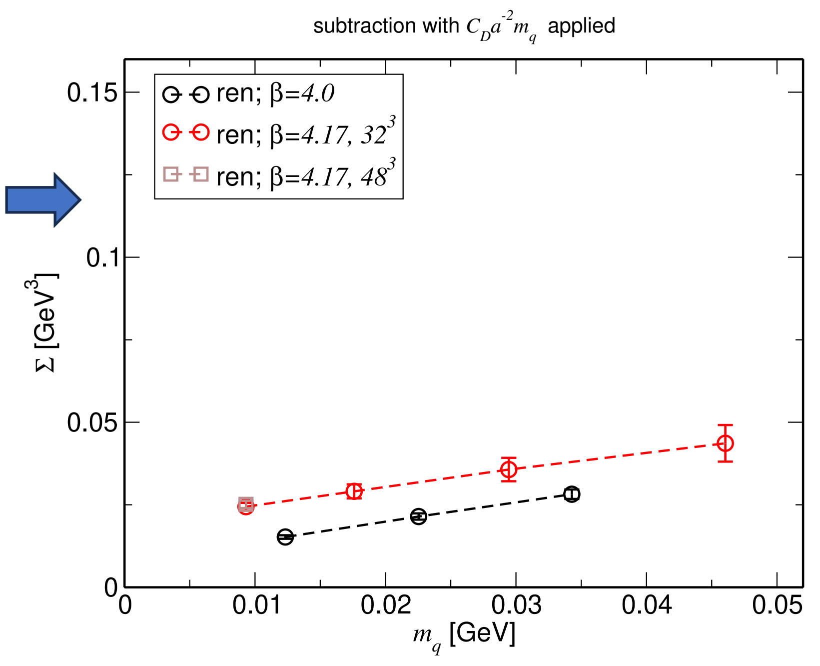
1. Prepare several different lattice spacing for $\boldsymbol{T = 0}$
2. Compute coefficient linear in m_f : $\Sigma|_{DWF} \sim const. + (\frac{C_D}{a^2} + C_R) m_f + \cdots$
3. Separate divergent term : *linear fit in* a^2 *of.* $C_D + a^2 C_R \rightarrow \boldsymbol{C_D = 0.37(2)}$
4. Estimate x using $\boldsymbol{T > T_c}$ through $\Sigma|_{DWF} \rightarrow \frac{-C_D(1-x) m_{res}}{a^2} = 0$ $(m_f \rightarrow -m_{res})$ [ren.cond. $\Sigma|_{cont.} = 0$]

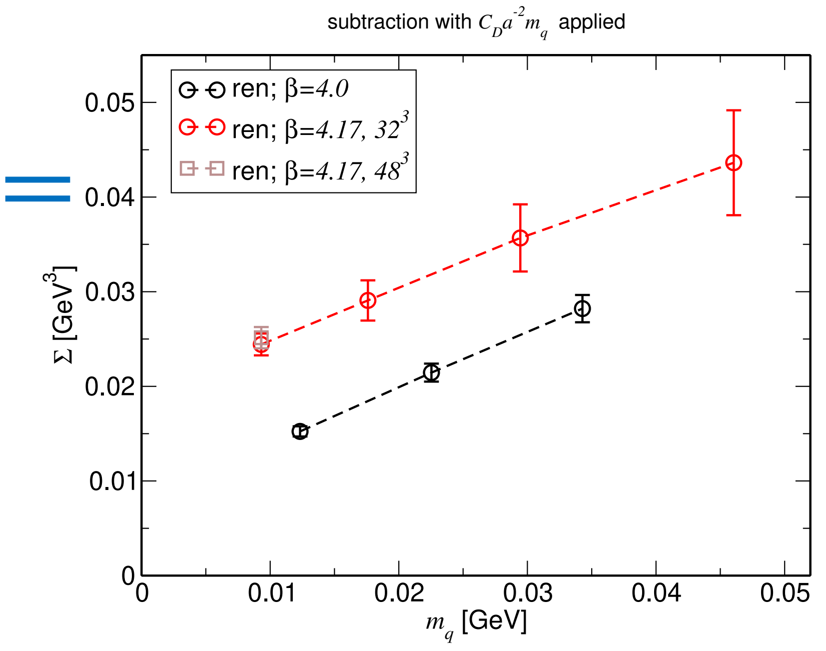
→ $\boldsymbol{N_f = 3; \beta = 4.0}$ estimate: $\boldsymbol{x = -0.6(1)}$

- In general, $\boldsymbol{x}$ may depend on $\boldsymbol{\beta}$, for now use this value as a reference for all $\boldsymbol{\beta}$
- We also use C_D (single flavor normalization) of $N_f = 3$ for $N_f = 2 + 1$

test on $N_f = 2+1, T = 0$ measurements





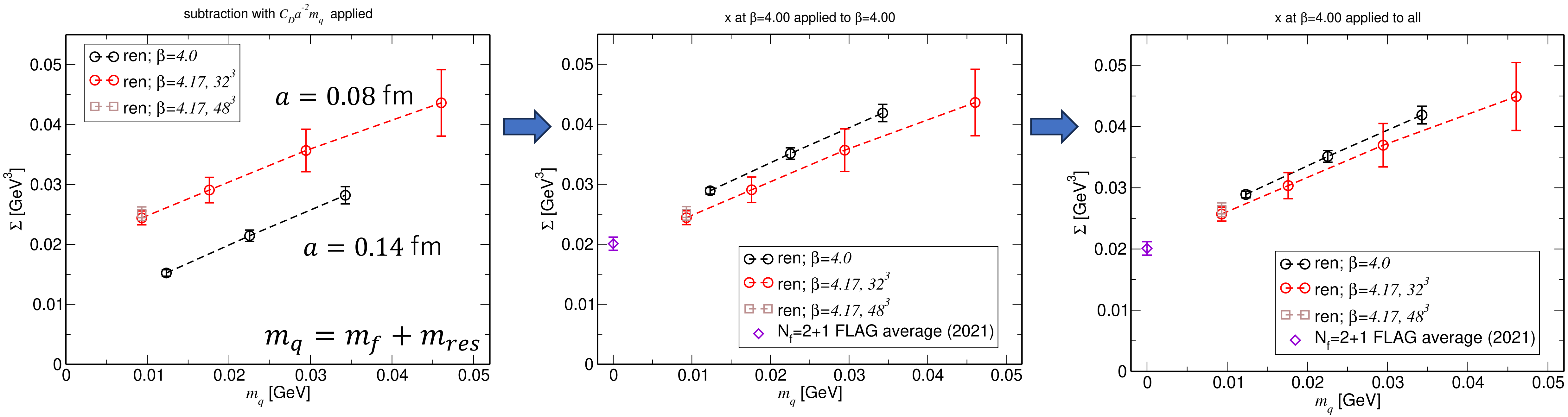


only multiplicative renormalizations applied

$C_D a^{-2} m_q$ subtraction applied

changing y-axis range

test on $N_f = 2+1, T = 0$ measurements



$C_D a^{-2} m_q$ subtraction applied

$C_D a^{-2}(1-x) m_{res}$ subtraction applied only to $\beta = 4.0$

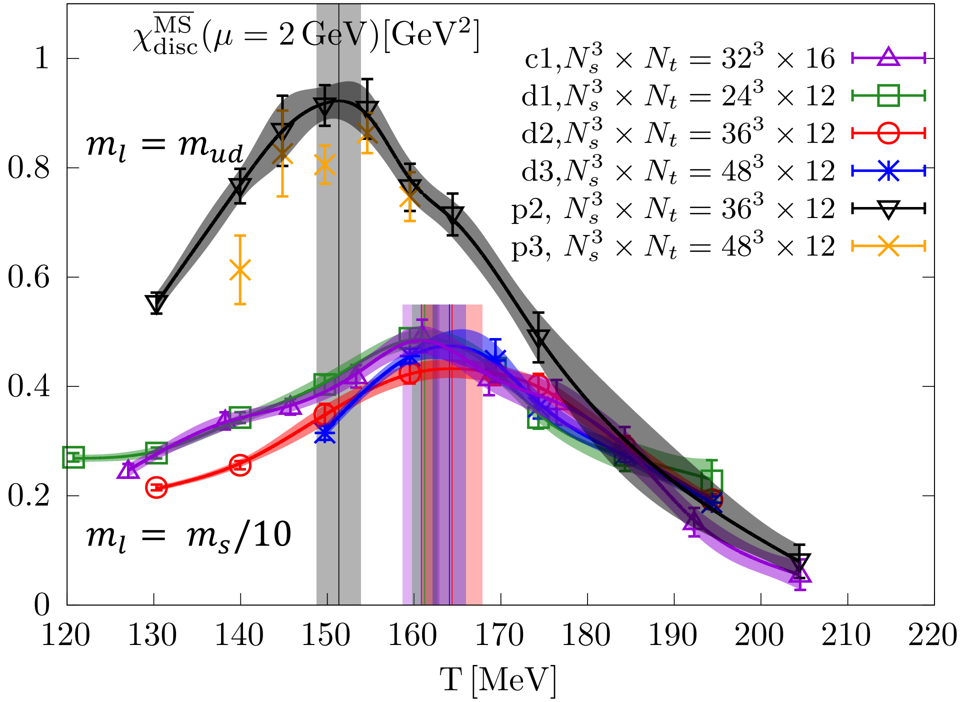
$C_D a^{-2}(1-x) m_{res}$ subtraction applied to all assuming x is universal

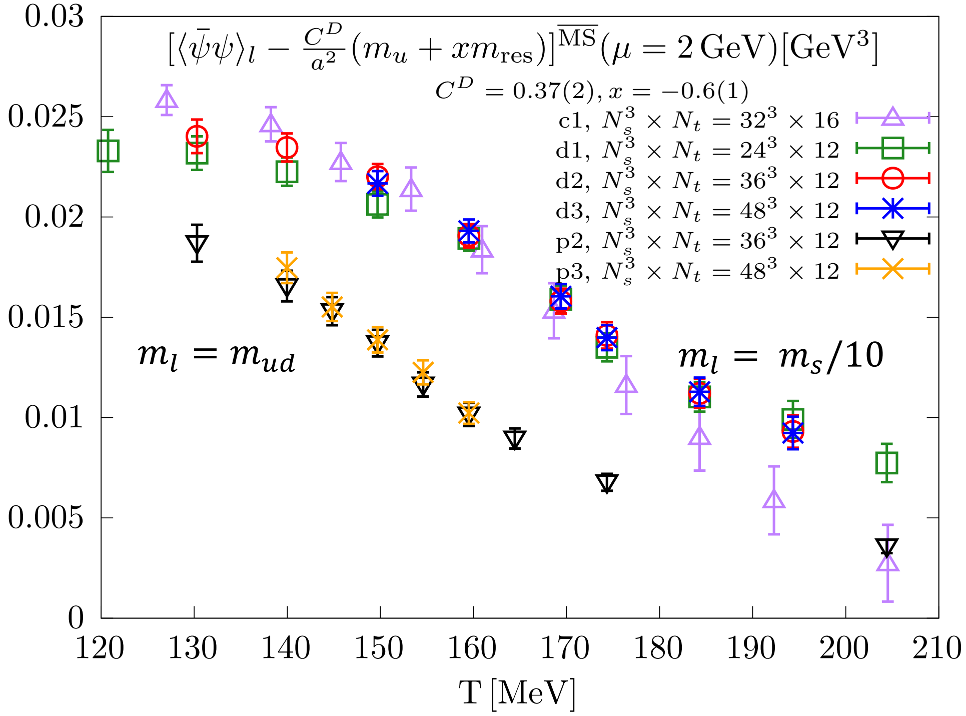
Seemingly, both conventional and residual divergence are controlled, but

- need to check if x does not depend much on β
- refinement of precision and check applicability range of C_D necessary

Disconnected chiral susceptibility and chiral condensate

all divergences subtracted assuming x is universal



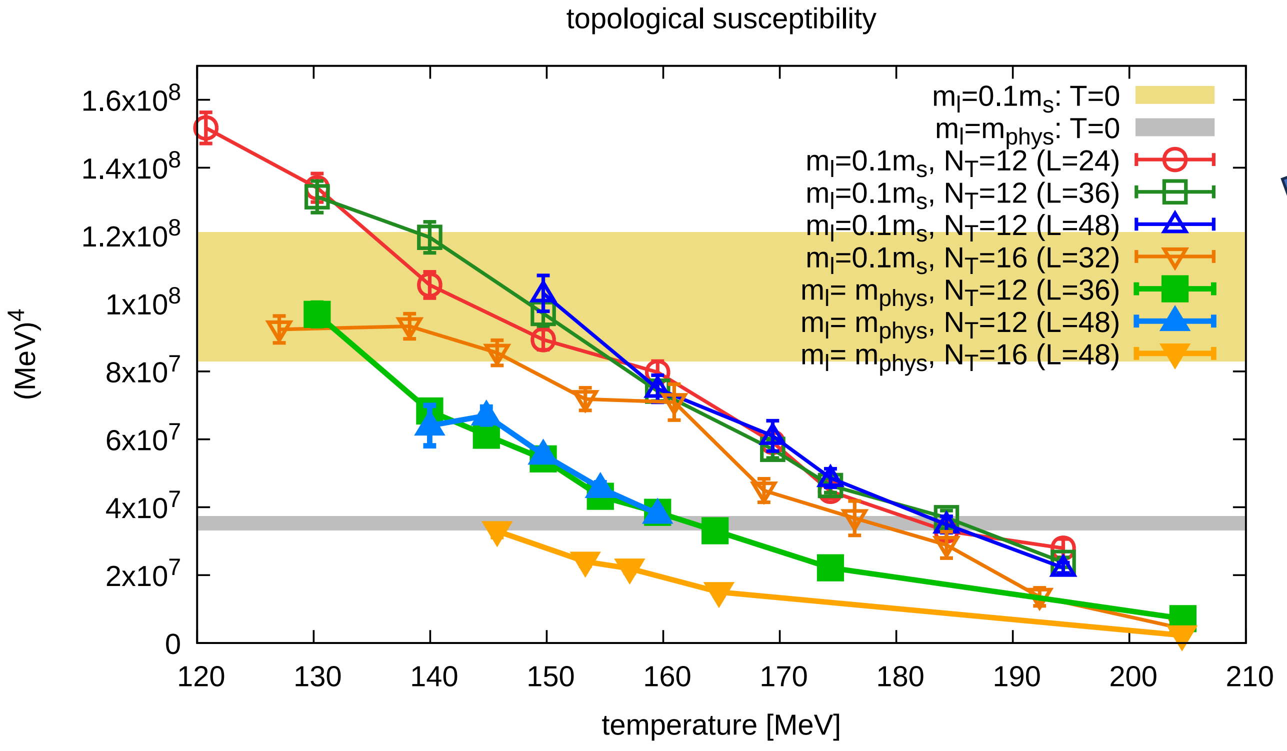


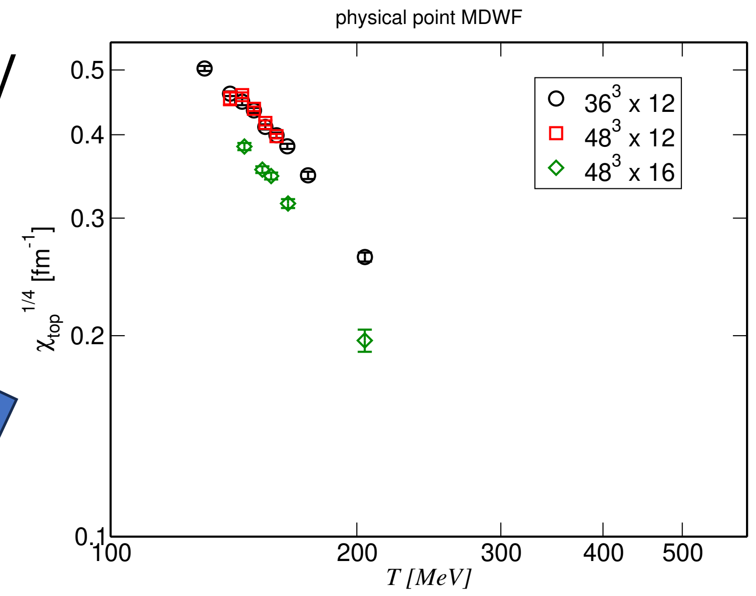
Likely NO phase transition at physical point with chiral fermions.
No surprise happened so far..

$m_l = m_{ud}$

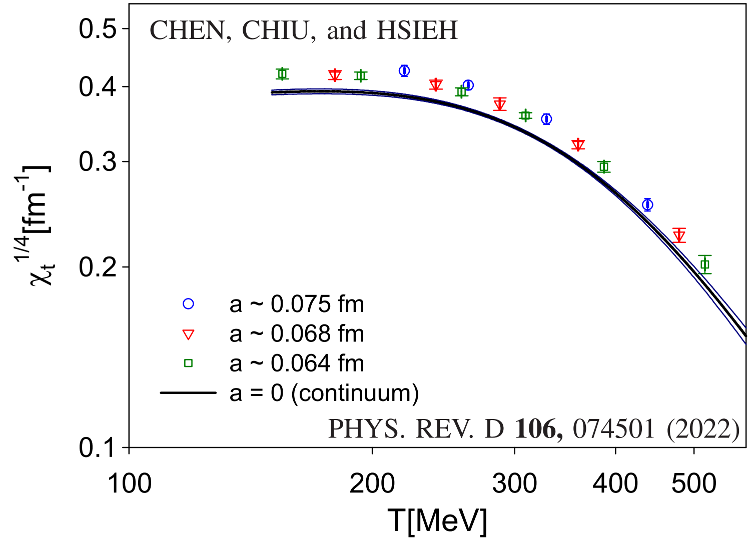
- p2,p3: N_t=12, aspect ratio LT = 3, 4
  - Statistics is ~20,000 MDTU for LT=3, sampled every 10 MDTU
  - LT=4 very preliminary, currently running to get to planned satat.
- $\boldsymbol{T_{pc} = 151\,(3)}$ MeV (preliminary) on $\boldsymbol{36^3{\times}12}$

topological susceptibility





MDWF
$N_f=2+1$

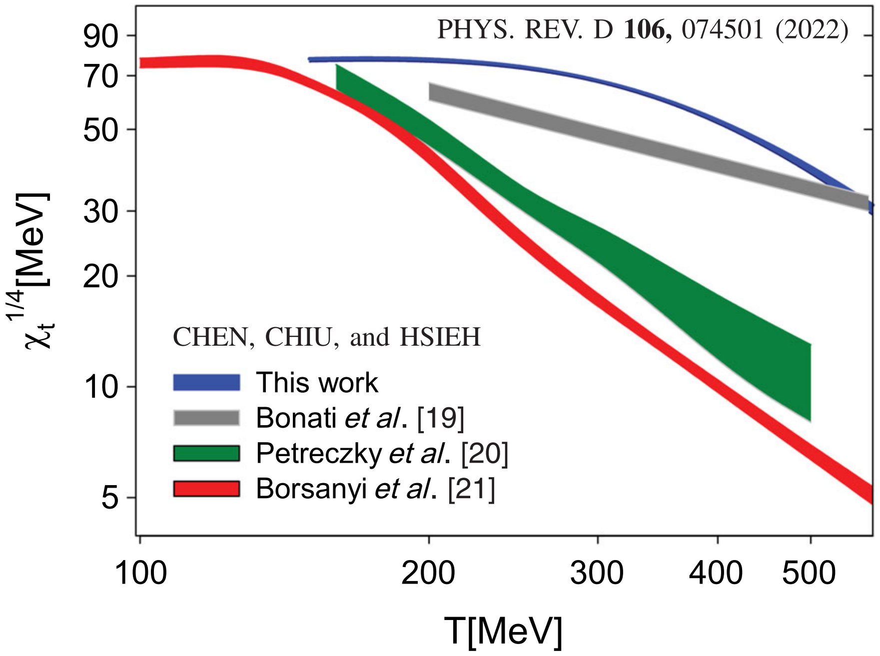


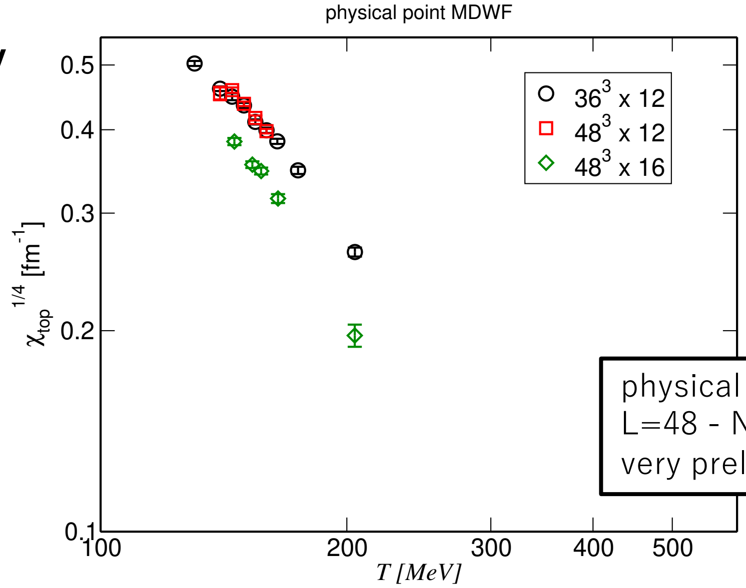
optimal DWF
$N_f=2+1+1$

physical point
L=48 - Nt=12 and 16 are very preliminary (low statistics)

topological susceptibility

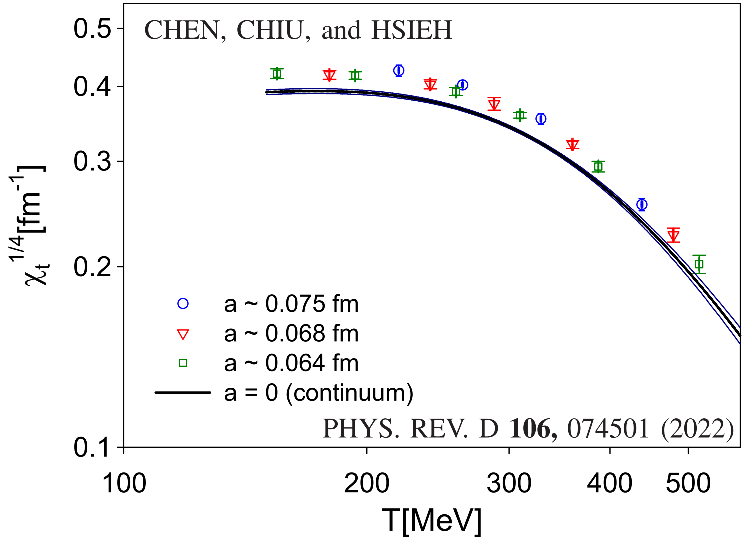
Summary by Chen et al (TWQCD)





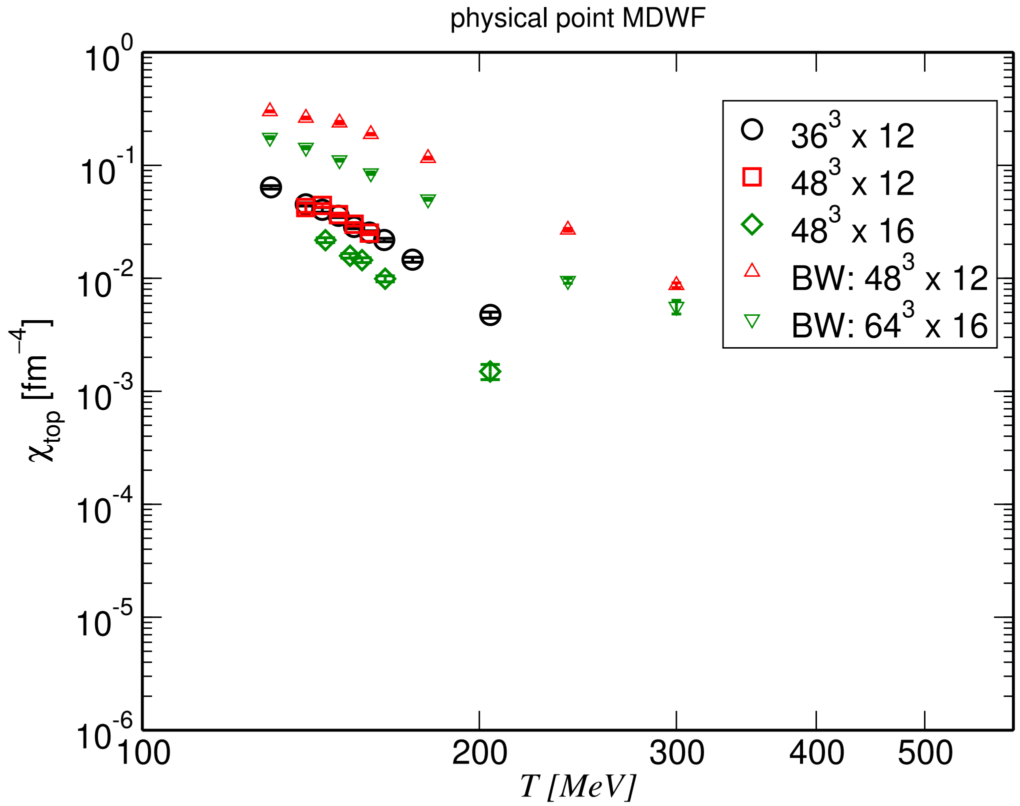
MDWF
N_f=2+1

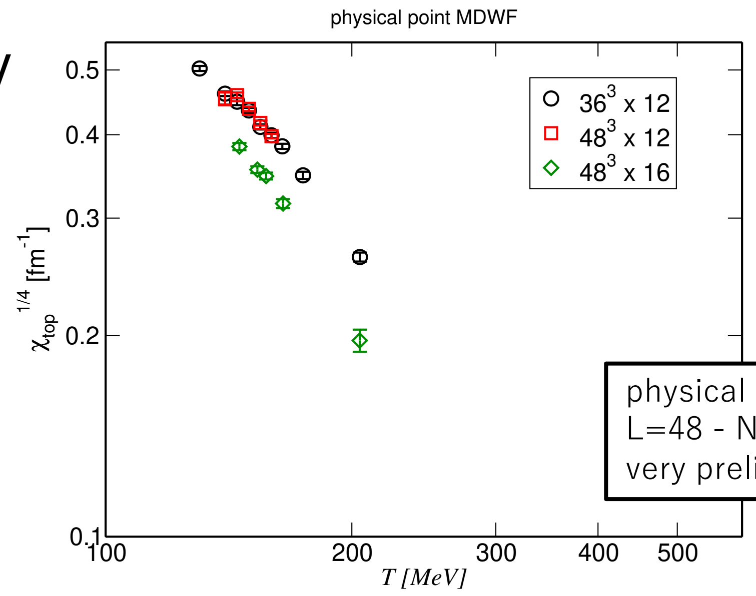
physical point
L=48 - Nt=12 and 16 are
very preliminary (low statistics)



optimal DWF
N_f=2+1+1

topological susceptibility





MDWF
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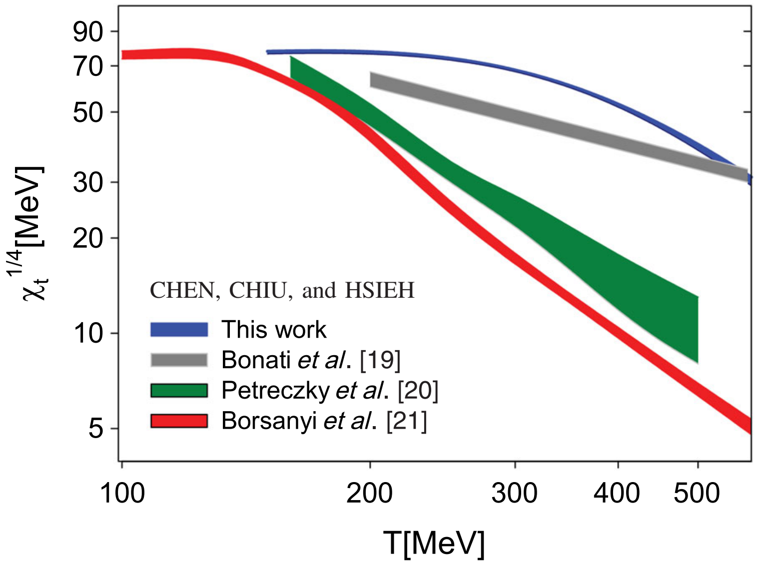
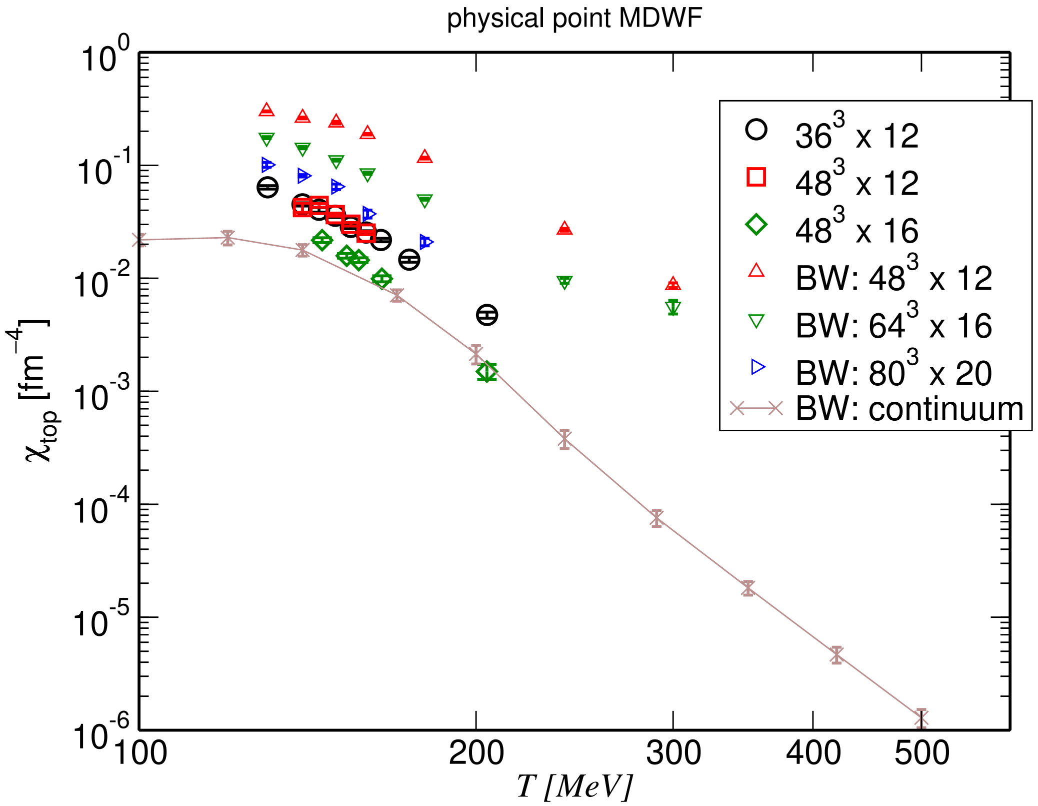
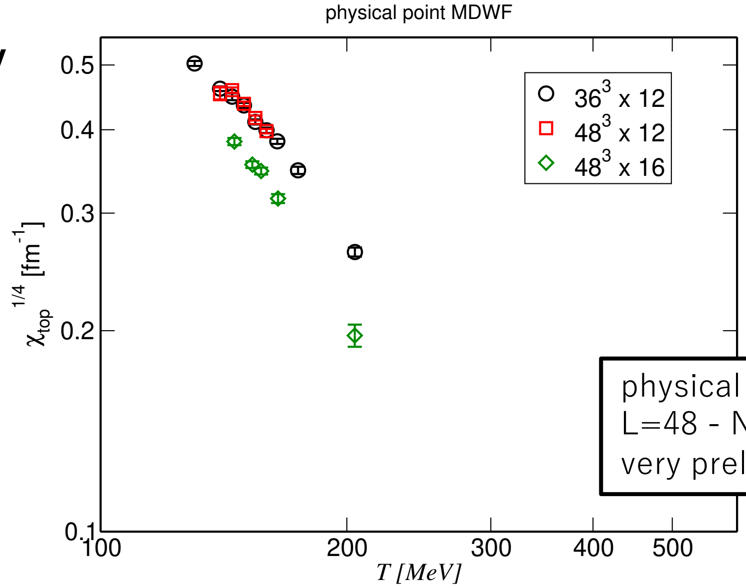


topological susceptibility



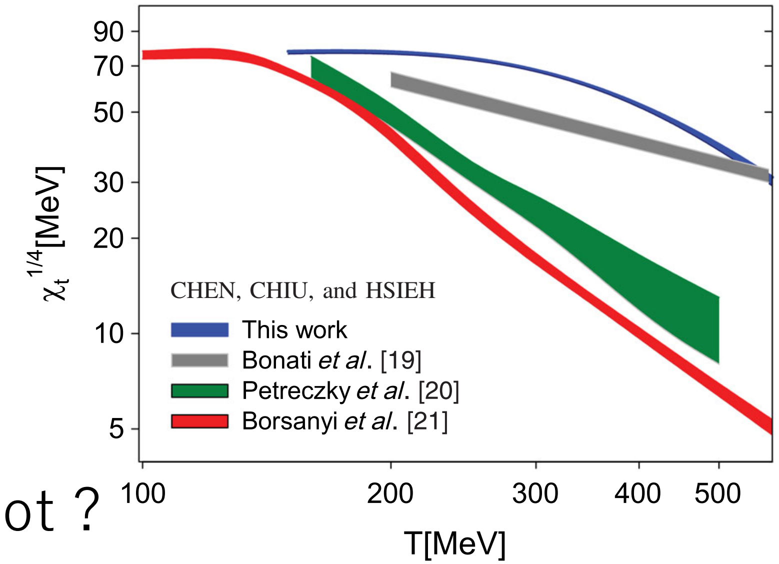


MDWF
N_f=2+1

physical point
L=48 - Nt=12 and 16 are
very preliminary (low statistics)

MDWF(JLQCD) χ_t at physical point

- inconsistent with Chen et al (optimal DWF)
- getting closer to BW[continuum] for $a \to 0$
- N_t=16 already ~continuum or even undershoot ?



- more detailed study needed

Summary and Outlook

Nf=2+1 Physical point computation of QCD thermodynamics with Möbius DWF

- use LCP, determined with T=0 JLQCD knowledge
- no surprise on the existence/non-existence on the transition
- machinery to treat power divergence, residual chiral symmetry effect is being finalized
- seemingly the both types of “divergence” are under control using Nf=3 results
- further improvement underway
- Disconnected chiral susceptibility show no hint of phase transition for Nt=12
  - $T_{pc} \simeq T_{pc}$(staggered)
  - no surprise so far with chiral fermions
- Topological susceptibility showing large lattice artifact for Nt=12. Nt=16 promising.

Outlook

- refinement of power divergence subtraction using T=0 information of very fine MDWF
- 48^3 for Nt=12 and 16 are being run on Fugaku
- plan to be completed by the end of FY2025 with a few additional points on 64^3x16.
- use of these configuration underway
  - ➢ see eg. talk by Goswami on charge fluctuation

thank you for your attention

Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$: residual power divergence

- $\Sigma|_{DWF} \sim \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)

  $m_{res} \neq x m_{res}$; $x = O(1) \neq 1$

  - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.

- We propose another way to estimate $x m_{res}$ using m'_{res}

  ~~If chiral symmetry is restored $\to \Sigma|_{cont.} = 0$~~

  ~~$\to m_f = -x m_{res}$ is a **zero** of $\Sigma|_{DWF}$ which is **related** with~~

  ~~$m'_{res} = \frac{\sum_x \langle J_{5q}(x) P(0)\rangle}{\sum_x \langle P(x)\, P(0)\rangle}$ ($\leftrightarrow$ $m_{res} = \frac{\sum_{\vec{x}} \langle J_{5q}(\vec{x},t) P(0)\rangle}{\sum_{\vec{x}} \langle P(\vec{x},t)\, P(0)\rangle} \to \frac{\langle 0|J_{5q}|\pi\rangle}{\langle 0|P|\pi\rangle}$~~ (large t))

  $m_f = -m_{res}'$ is a zero of $\Sigma|_{DWF}$ ($\leftrightarrow$ $m_f = -m_{res}$ is a zero of , m_π^2)

  Due to Axial WT identity: $(m_f + m'_{res}) \sum_x \langle P(x)\, P(0)\rangle = \Sigma$

  From: $\Delta_\mu \langle A_\mu(x)\, P(0)\rangle = 2 m_f \langle P(x)\, P(0)\rangle + 2\langle J_{5q}(x)\, P(0)\rangle - 2\,\Sigma\,\delta_{x,0}$

Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$: residual power divergence

- $\Sigma|_{DWF} = C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)

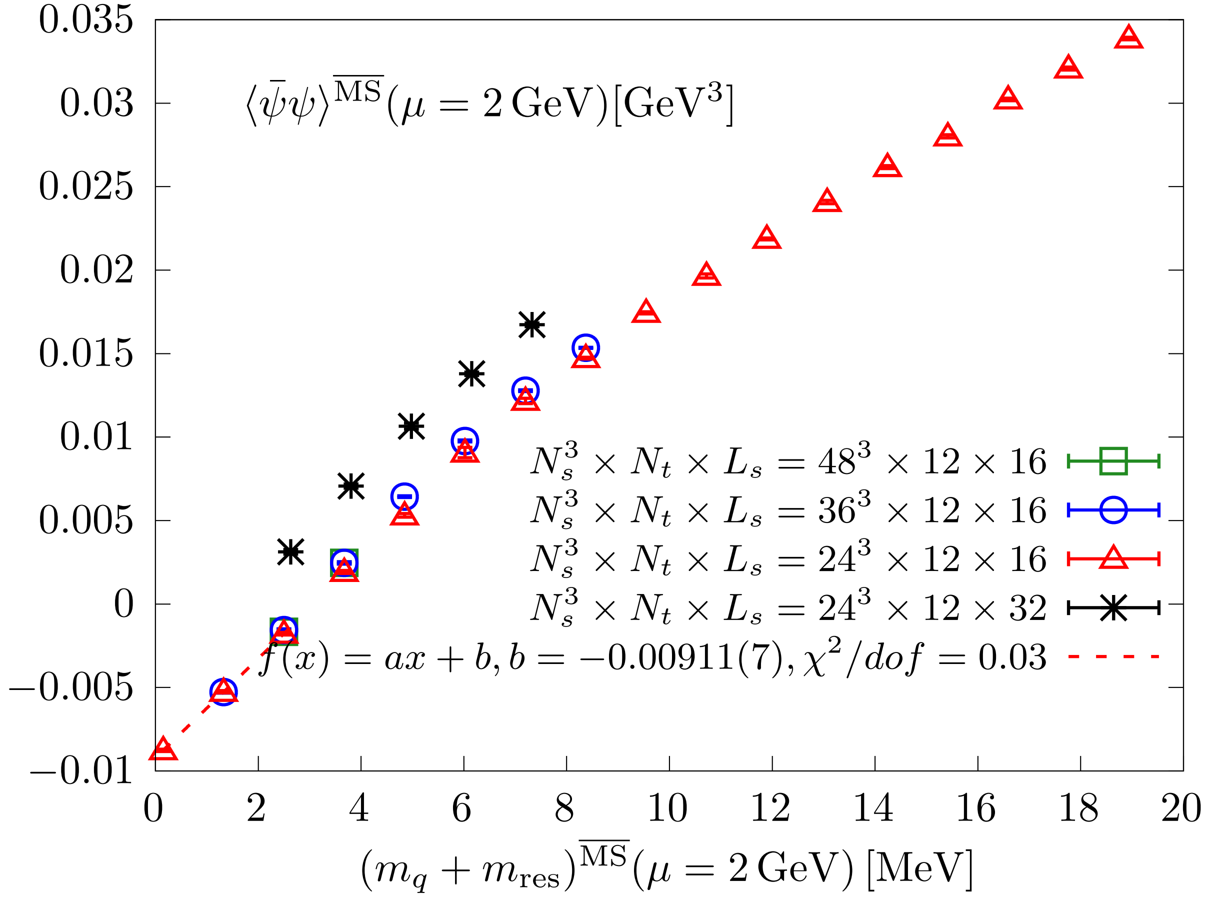
  $m_{res} \neq x m_{res}; \quad x = O(1) \neq 1$

  - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.

- $\boldsymbol{N_f = 3}$ case

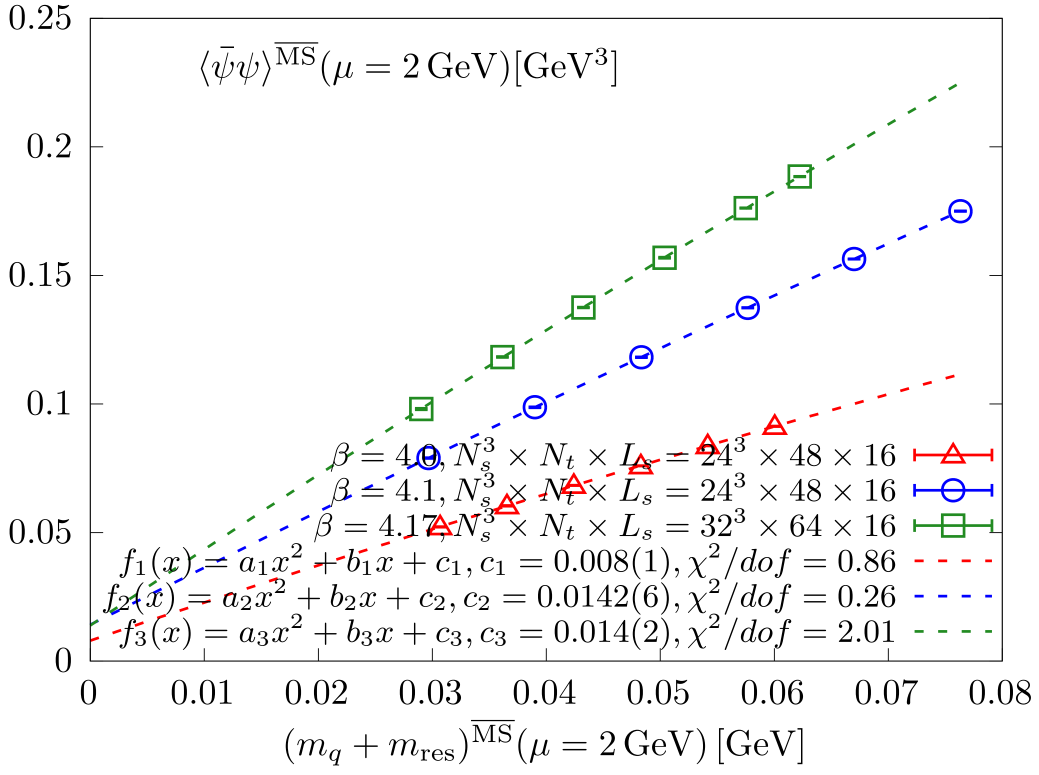
  - T>0 problem @ β =4.0
  - T=0 exercise @ β =4.0, 4.1, 4.17
  - T>0 div free Σ

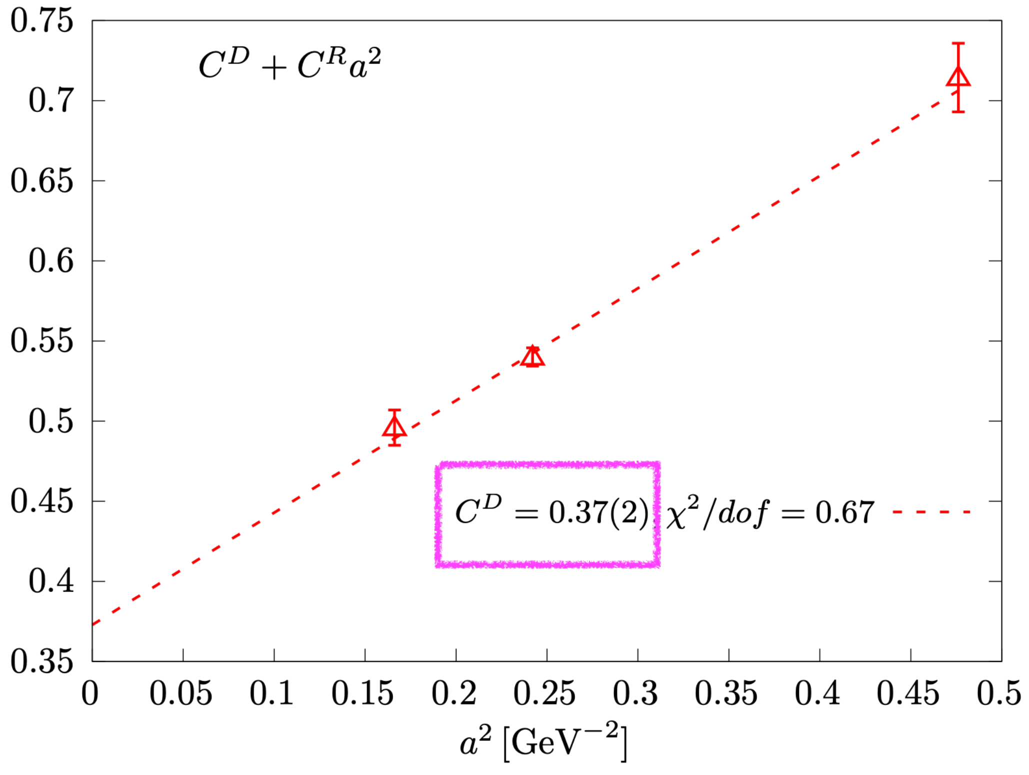
N_f=3, N_t=12 chiral condensate



- only multiplicative renormalization applied
- quark mass: m_{res} shift applied
- @T=0: $m_\pi \to 0$, $(m_q + m_{res}) \to 0$
- L_s=16
  - three volumes: 24^3, 36^3, 48^3
- L_s=32
  - smaller m_{res}, 24^3
- Intercept = $C_D \dfrac{-(1-x)m_{res}}{a^2} < 0$
  - need to be subtracted

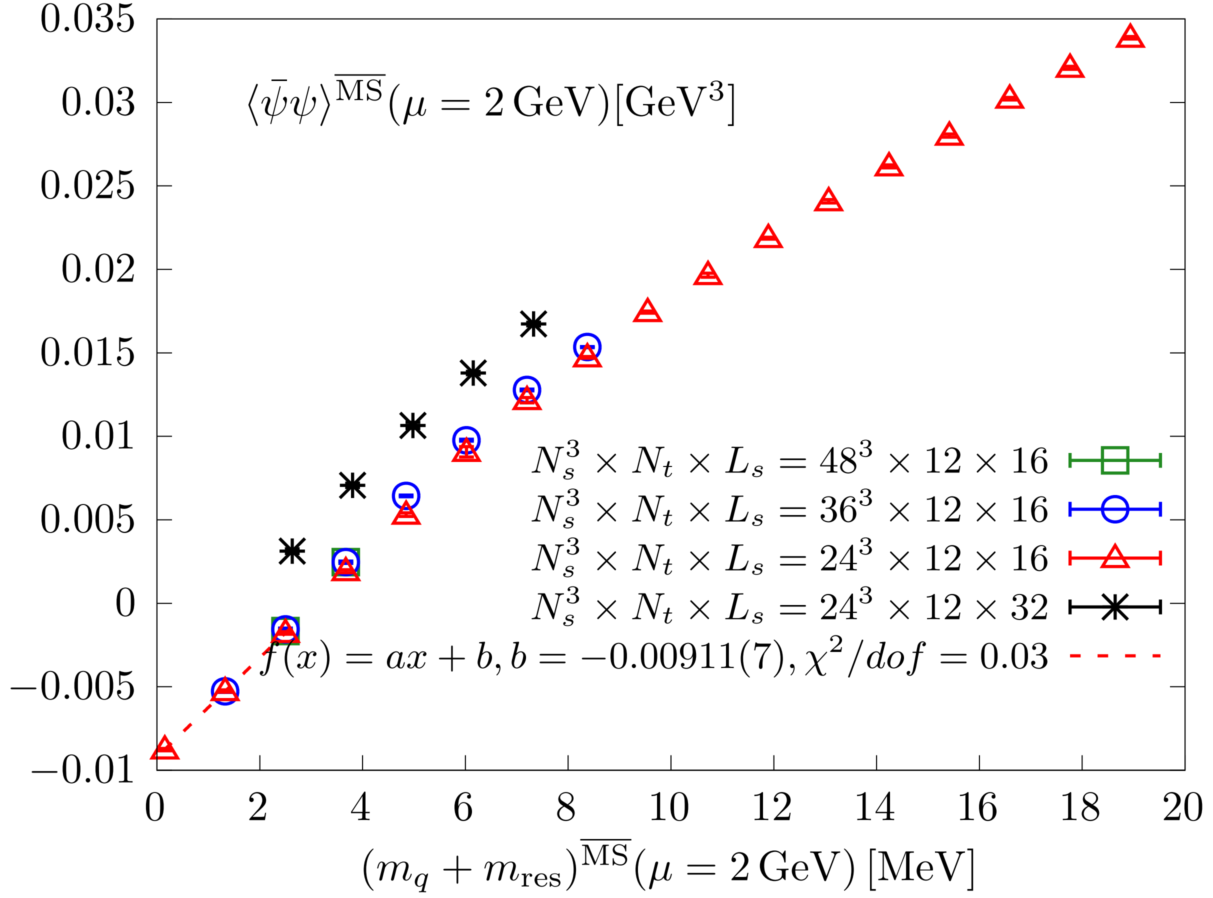
Nf=3, T=0 chiral condensate

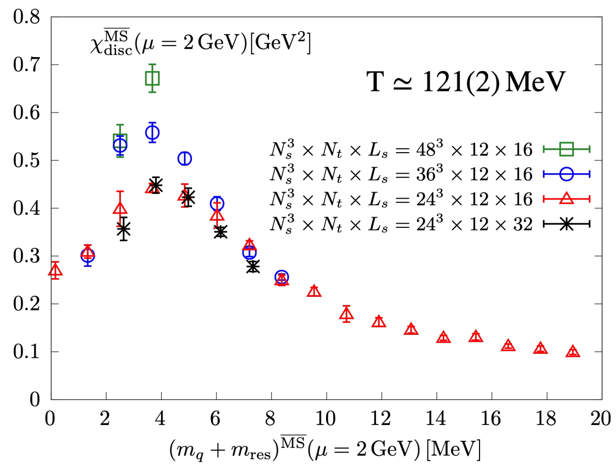




- $\Sigma(m) = C_0 + C_1\, m + C_2\, m^2$ fit
- $C_1 = \frac{C_D}{a^2} + C_R$
  - C_D/a^2: divergent, C_R: regular
- $\Sigma|_{DWF} = C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$
- $C_D + C_R a^2$
  - $C_D = 0.37(2)$ from linear fit

N_f=3, N_t=12 chiral condensate

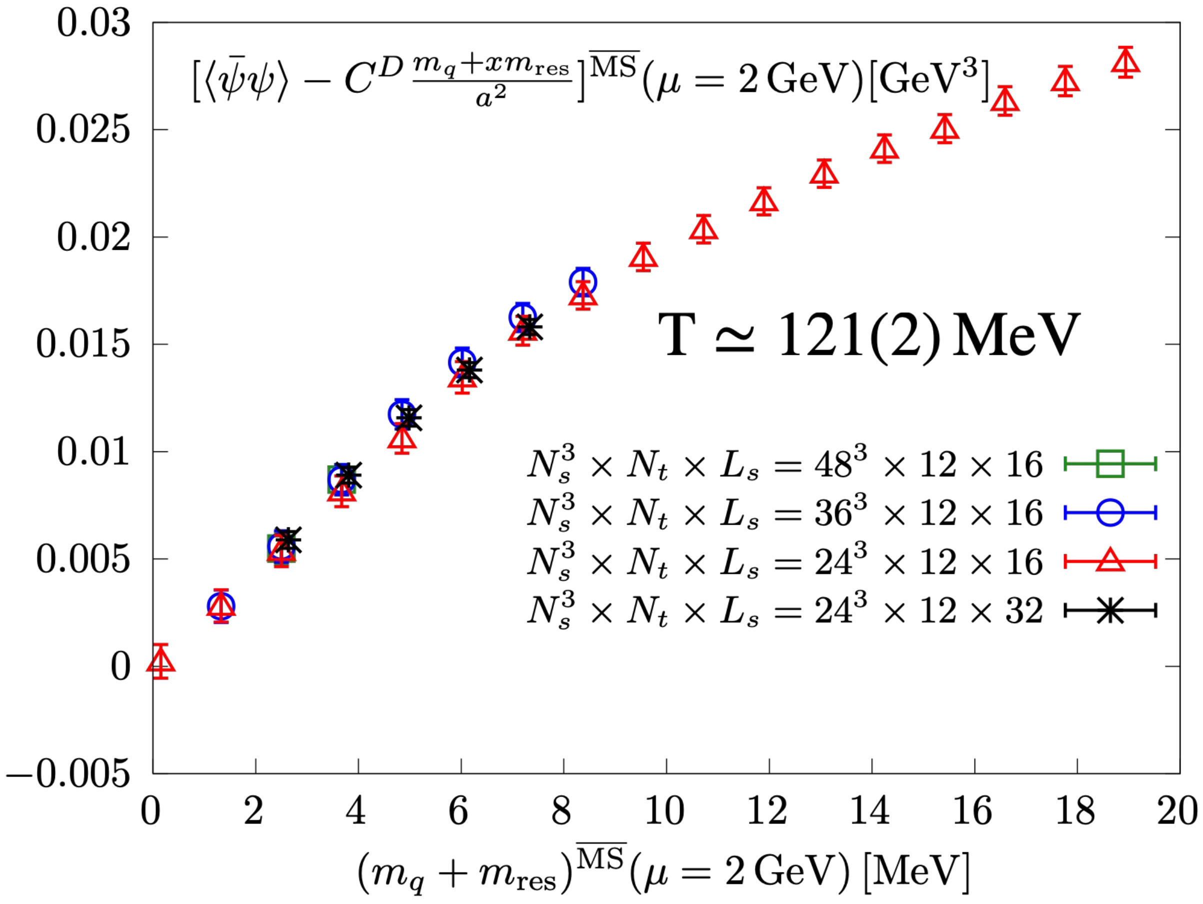


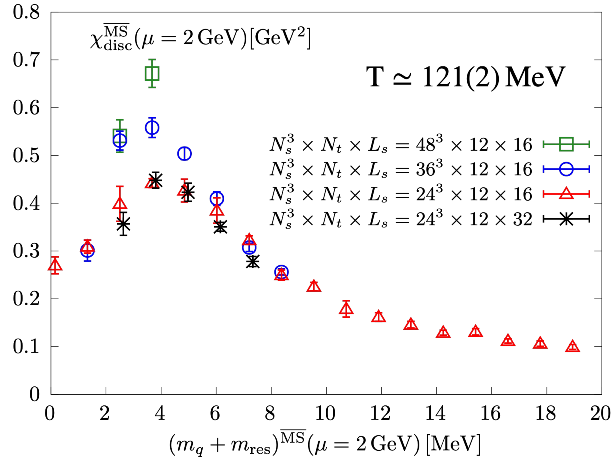


- $m_{pc} \simeq 4$ MeV
- $m < m_{pc}$: high T "phase"
- $\Sigma|_{DWF} \to C_D \frac{-(1-x)m_{res}}{a^2} + \Sigma|_{cont.}$; $(m_f \to -m_{res})$

- $\Sigma|_{cont.}=0$: renormalization cond.
  - applied to determine x
  - x=-0.6(1) from 24^3x12x16

N_f=3, N_t=12 chiral condensate

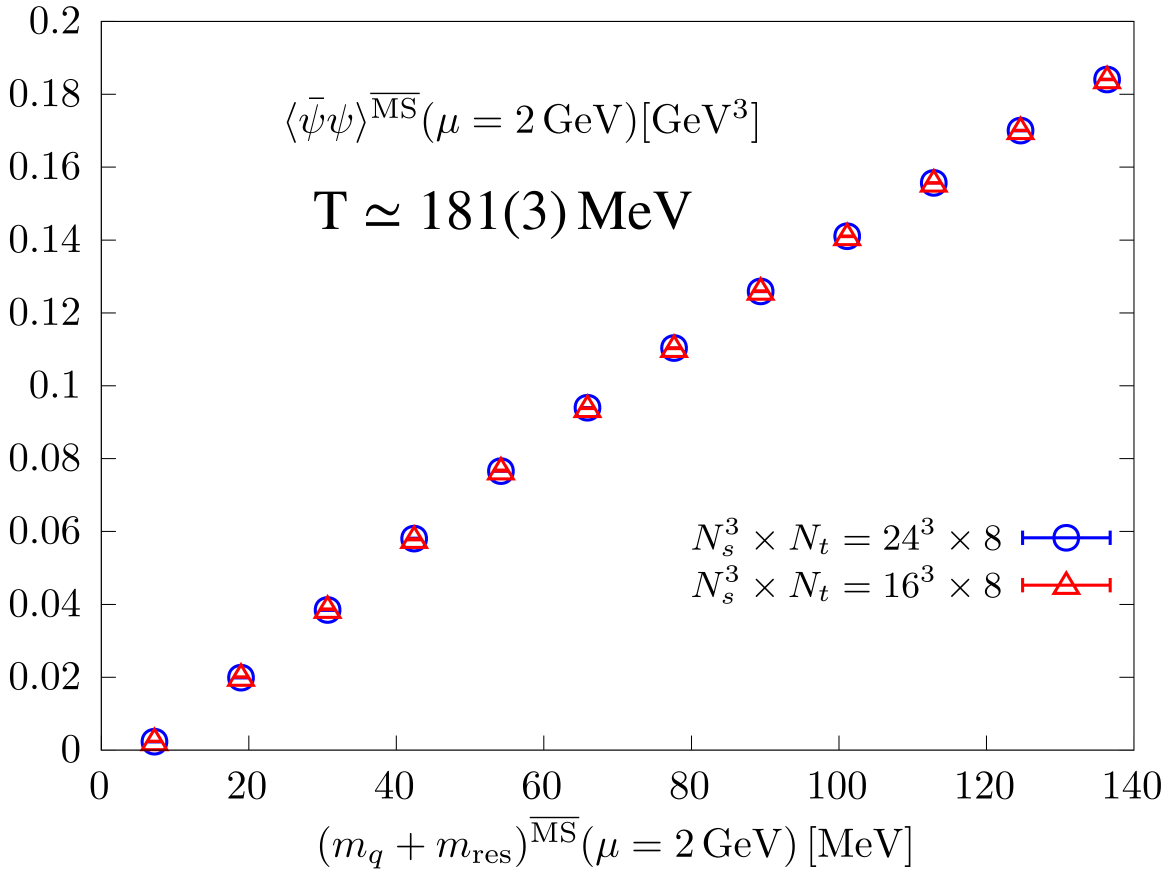




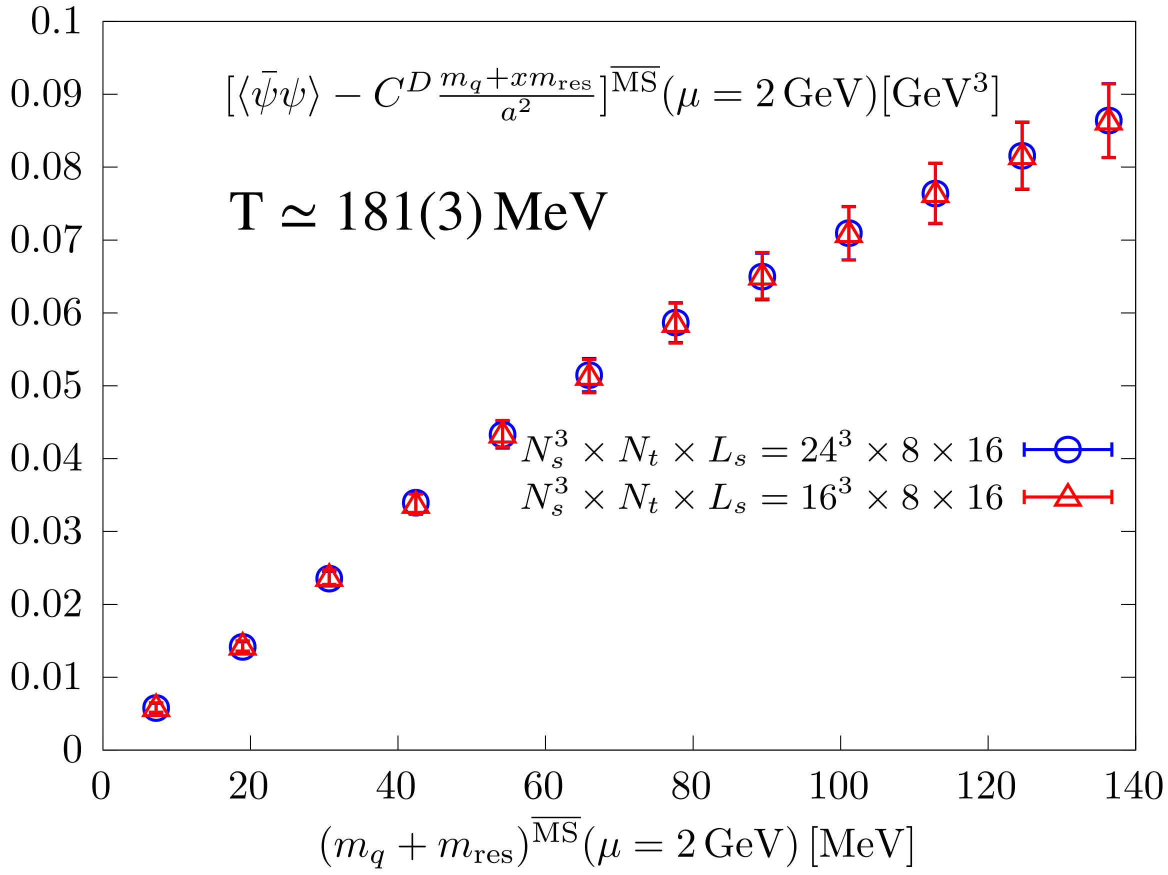
- $m_{pc} \simeq 4$ MeV
- $m < m_{pc}$: high T "phase"
- $\Sigma|_{DWF} \to C_D \frac{-(1-x)m_{res}}{a^2} + \Sigma|_{cont.}$; $(m_f \to -m_{res})$
- $\Sigma|_{cont.}$=0 : renormalization cond.
  - applied to determine x
  - x=-0.6(1) from 24^3x12x16
- **subtraction using these to all sets**
  - **note: consistency L_s=16 <-> 32**

Renormalized chiral condensate

Multiplicatively renormalized chiral condensate



Additive and multiplicatively renormalized chiral condensate



Subtracted chiral condensate vanishes in the chiral limit as expected since T> T_c

X