Precision physics in the charm sector

Sara Collins

Universität Regensburg

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 813942.

German Japanese Workshop 2024, 25th-27th Sept. 2024

Overview

★ D and D_s decay constants. [RQCD-ALPHA,2405.04506] Kuberski, Joswig, Collins, Heitger, Söldner

eventually also D^* and D_s^* decay constants.

★ Lower lying charmonium spectrum and decay constants. Spiegel, Bali, Collins, Söldner.

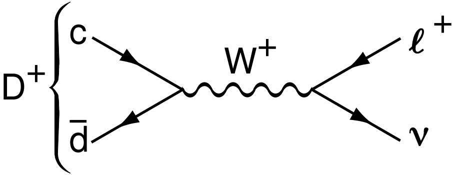
Present results for the 1S hyperfine splitting.

★ Not presented: singly and doubly charmed baryon spectrum. Radhakrishnan, Bali, Collins, Mathur, Söldner

D and D_s decay constants

$$\mathrm{i} f_{\mathrm{D}} p_\mu = \langle 0 | A_\mu^{\mathrm{dc}} | \mathrm{D}(p) \rangle \qquad \mathrm{i} f_{\mathrm{D_s}} p_\mu = \langle 0 | A_\mu^{\mathrm{sc}} | \mathrm{D_s}(p) \rangle$$

[PDG,Phys. Rev. D 110 (2024)]

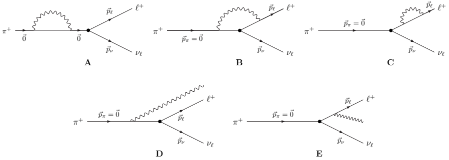


Leptonic decay width: tree-level

$$\Gamma(D^+ \to \ell^+ \nu) = \frac{G_F^2}{8\pi} f_D^2 m_\ell^2 M_D \left(1 - \frac{m_\ell^2}{M_D^2}\right)^2 |V_{cd}|^2$$

Similarly, $\Gamma(D_s^- \to \ell^- \nu)$ and $f_{\mathrm{D_s}} \to |V_{cs}|$.

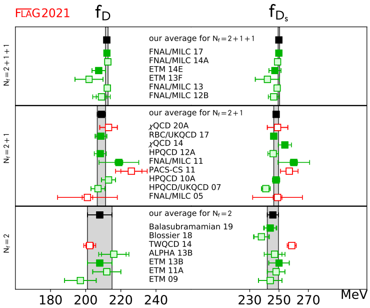
[Christ et al.,2304.08026]

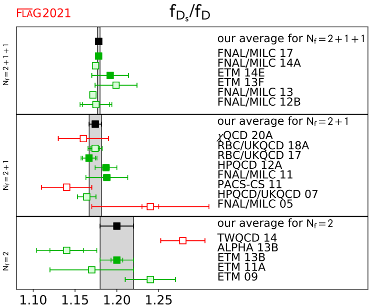


Radiative corrections involving virtual and real photons.

Nonperturbative treatment QCD+QED, see, e.g., [Giusti et al.,2302.01298], [Christ et al.,2304.08026], [Desiderio et al.,2006.05358], ...

D and D_s decay constants: current status [FLAG 21,2111.09849]





$N_f = 2+1+1$ [FNAL-MILC,1712.09262]: $f_{D_s} \sim 0.2\%$, $f_D \sim 0.3\%$, and $f_{D_s}/f_D \sim 0.1\%$.

CKM matrix elements

PDG: $|V_{cd}| = 0.221(4)$, dominated by $D \to \ell\nu$

$|V_{cs}| = 0.975(6)$, dominated by $D \to K\ell\nu$ (expt. more precise than $D_s \to \ell\nu$).

Unitarity of CKM matrix using individual determinations: rows, columns

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.0010 \pm 0.0120,$$
$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9971 \pm 0.0020,$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.0030 \pm 0.0120.$$

Most precise constraints from global fits, see, e.g., PDG, [CKMfitter,hep-ph/0104062], [UTfit,hep-ph/0501199] and updates, ...

Hadronic inputs from the lattice including $f_{D_{(s)}}$. $|V_{cd}| = 0.22487(68)$ and $|V_{cs}| = 0.97349(16)$.

Vector and tensor decay constants of $D^*_{(s)}$

$$m_{\mathrm{D}^*_{(q)}} f_{\mathrm{D}^*_{(q)}} \epsilon^\lambda_\mu = \langle 0 | V^{qc}_\mu | \mathrm{D}^*_{(q)}(p,\lambda) \rangle \qquad \mathrm{i} f^{\mathrm{T}}_{\mathrm{D}^*_{(q)}} (\epsilon^\lambda_\mu p_\nu - \epsilon^\lambda_\nu p_\mu) = \langle 0 | T^{qc}_{\mu\nu} | \mathrm{D}^*_{(q)}(p,\lambda) \rangle$$

Interest:

★ Heavy quark symmetry.

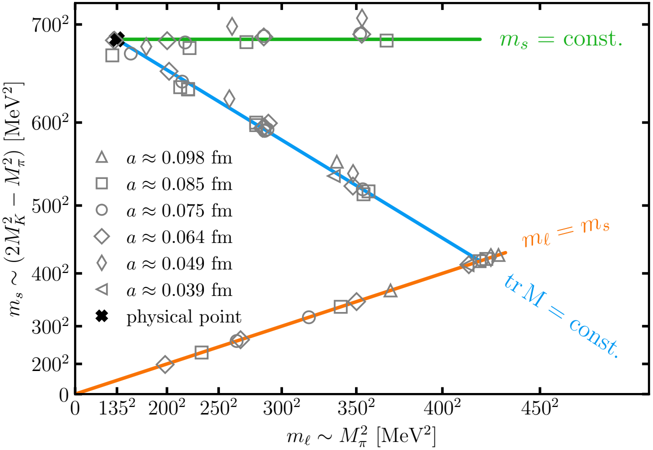
★ QCD factorization studies of charmed nonleptonic B meson decays (e.g. $B \to D^{(*)}_{(s)}\pi$, $B \to D^{(*)}_{(s)}D$).

★ Compare with model calculations.

★ . . .

First experimental measurement of $D^{*+}_s \to e\nu_e$ [BES III,2304.12159], $f_{D^{*+}_s} = 213.6^{+61.0}_{-45.8} \pm 43.9$ MeV.

CLS ensembles: quark mass plane



$N_f = 2 + 1$ NP $O(a)$ improved Wilson fermions

3 mass trajectories

Tight control of the quark mass dependence.

Many volumes, including $M_\pi L > 4$

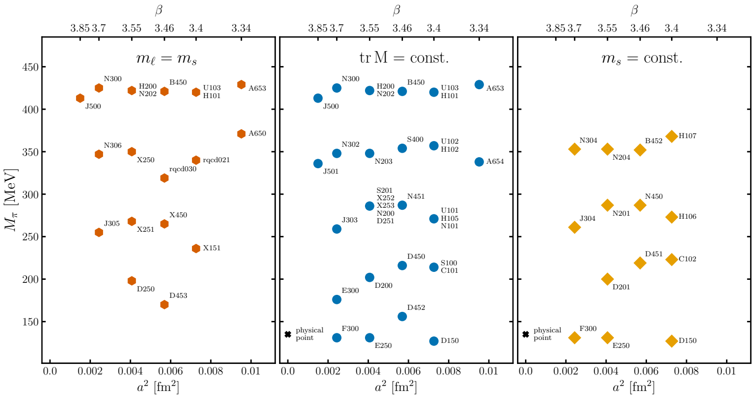
High statistics: usually a few 1000 MDUs

Additionally, 3-37 sources per configuration for the two-point functions.

CLS Ensembles used in the analysis

Around 50 ensembles, 2 m_π^{phys} ensembles, **6 lattice spacings**, a^2 varies by more than a factor of 6.

$a \leq 0.05$ fm open boundary conditions, $a > 0.05$ fm open and periodic b.c.s.



Charm quark (quenched flavour): two charm quark masses per ensemble bracketing the physical value.

$O(a^2 m_c^2)$ discretisation errors, $am_c \sim 0.1 - 0.5$.

Extraction of the decay constant

Bare decay constants obtained from fitting to $\boldsymbol{A_0 \tilde{P}}$ and $\boldsymbol{\tilde{P}\tilde{P}}$ two-point functions.

$$C^{qc}_{A_0\tilde{P}}(t) = \boldsymbol{A^{qc}_{A_0\tilde{P}}}\,\mathrm{e}^{-m_{\mathrm{D}_{(q)}}t} + \ldots\,,\; C^{qc}_{\tilde{P}\tilde{P}}(t) = \boldsymbol{A^{qc}_{\tilde{P}\tilde{P}}}\,\mathrm{e}^{-m_{\mathrm{D}_{(q)}}t} + \ldots$$

where $A^{qc}_{A_0\tilde{P}} = \langle 0 | A^{\mathrm{qc,I}}_{\mu} | \mathrm{D}_{(q)}(p)\rangle Z_{\tilde{P}}/2m_{D_{(q)}}$ and $A^{qc}_{\tilde{P}\tilde{P}} = Z^2_{\tilde{P}}/2m_{D_{(q)}}$. $A^{\mathrm{qc,I}}_{\mu} = A^{\mathrm{qc}}_{\mu} + a\boldsymbol{c_A}\frac{1}{2}(\partial_\mu + \partial^*_\mu)P^{\mathrm{qc}}$

In the large t limit: $\boldsymbol{f_{\mathrm{D}_{(q)}}} = \sqrt{2}\boldsymbol{A^{qc}_{A_0\tilde{P}}}/\sqrt{\boldsymbol{A^{qc}_{\tilde{P}\tilde{P}}} m_{\mathrm{D}_{(q)}}}$

$C^{qc}_{A_0\tilde{P}}(t)$ and $C^{qc}_{\tilde{P}\tilde{P}}(t)$ constructed from point-to-all propagators. Wuppertal (Gaussian) smearing with APE-smoothed links applied to the pseudoscalar operators ($\tilde{P}$).

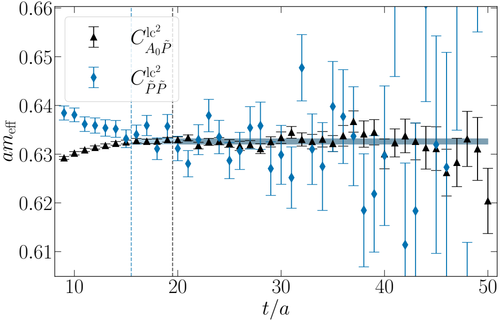
Matching and mass dependent $O(a)$ Symanzik improvement:

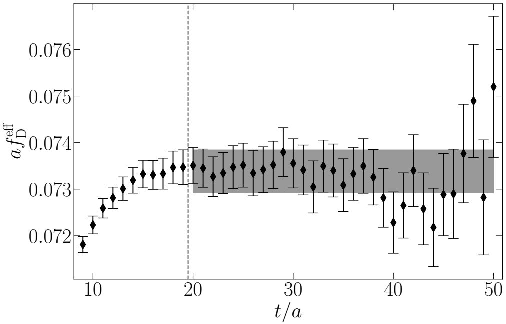
$$\mathbf{f}^{\mathrm{R}}_{\mathrm{D}_{(\mathbf{q})}} = \boldsymbol{Z_{\mathrm{A}}}\left[1 + a\left(\boldsymbol{b_{\mathrm{A}}}\, m_{qc} + \boldsymbol{\bar{b}_{\mathrm{A}}}\,\mathrm{Tr}\, M\right)\right] \boldsymbol{f_{\mathrm{D}_{(q)}}} + \mathrm{O}(a^2)$$

Non-perturbative $\boldsymbol{Z_{\mathrm{A}}}, \boldsymbol{b_{\mathrm{A}}}, \boldsymbol{c_{\mathrm{A}}}$: [ALPHA,1502.04999,1604.05827], [Korcyl and Bali,1607.07090], [Dalla Brida et al.,1808.09236], κ_{crit}: [RQCD,2211.03744].

Fitting analysis

E250: $a \approx 0.064$ fm, $m_\pi \approx 130$ MeV, $Lm_\pi = 4.05$, periodic b.c.s (average over all sources).





In spirit of [ALPHA,1004.2661], two-state fit determines t_{min} for one-state fit to extract $f_{D_{(q)}}$.

Right: effective decay constant, $f^{\rm eff}_{{\rm D}_{(q)}}(t) = \sqrt{2}C^{qc}_{A_0\tilde{P}}(t)/\sqrt{C^{qc}_{\tilde{P}\tilde{P}}(t)m_{{\rm D}_{(q)}}\exp(-m_{{\rm D}_{(q)}}t)}$

Ensembles with open b.c.: determine where boundary effects are significant $\rightarrow$ only use data in bulk region.

Quark mass interpolation and continuum extrapolation

All quantities rescaled by t_0 to form dimensionless combinations. Use the basis:

$$\bar{\mathbb{M}}^2 = \sqrt{8t_0}((2m_{\mathrm{K}}^2 + m_\pi^2) \propto 2m_{\mathrm{l}} + m_{\mathrm{s}}\,, \quad \delta\mathbb{M}^2 = \sqrt{8t_0}(m_{\mathrm{K}}^2 - m_\pi^2) \propto m_{\mathrm{s}} - m_{\mathrm{l}}\,, \quad \mathbb{M}_{\bar{\mathrm{D}}} = \sqrt{8t_0}M_{\bar{\mathrm{D}}} \propto m_{\mathrm{c}}$$

where $M_{\bar{\mathrm{D}}} = (2m_{\mathrm{D}} + m_{\mathrm{D_s}})/3$.

Leadings terms: inspired by NLO SU(3) heavy-meson ChPT [Goity,hep-ph/9206230] + $O(a^2)$ terms.

$$\sqrt{\mathbf{8t_0}}\mathbf{f}_{\mathrm{D_s}} = f_0 + c_1\,\bar{\mathbb{M}}^2 + \frac{2}{3}c_2\,\delta\mathbb{M}^2 + c_3\,(4\boldsymbol{\mu}_{\mathbf{K}} + \frac{4}{3}\boldsymbol{\mu}_{\boldsymbol{\eta}}) + c_4\,\mathbb{M}_{\bar{\mathbf{D}}} + c_5\mathbb{a}^2 + c_6\mathbb{a}^2\mathbb{M}_{\bar{\mathbf{D}}} + \ldots$$

$$\sqrt{\mathbf{8t_0}}\mathbf{f}_{\mathrm{D}} = f_0 + c_1\,\bar{\mathbb{M}}^2 - \frac{1}{3}c_2\,\delta\mathbb{M}^2 + c_3\,(3\boldsymbol{\mu}_{\boldsymbol{\pi}} + 2\boldsymbol{\mu}_{\mathbf{K}} + \frac{1}{3}\boldsymbol{\mu}_{\boldsymbol{\eta}}) + c_4\,\mathbb{M}_{\bar{\mathbf{D}}} + c_5\mathbb{a}^2 + c_6\mathbb{a}^2\mathbb{M}_{\bar{\mathbf{D}}} + \ldots$$

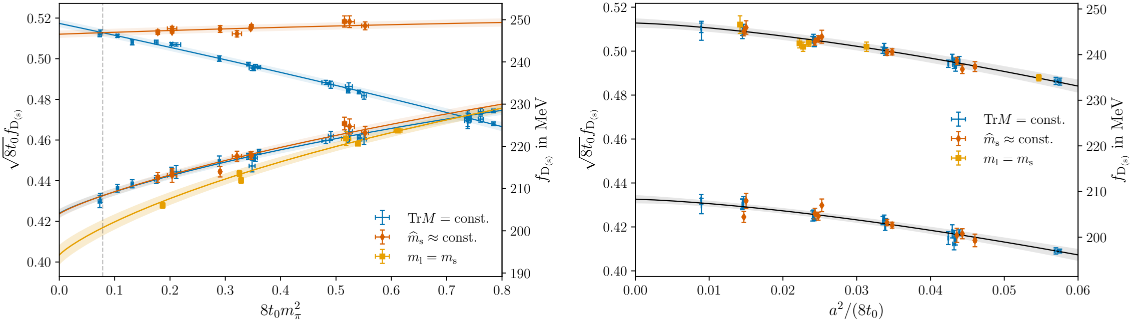
where $\mu_X = 8t_0 m_X^2 \log(8t_0 m_X^2)$, $X \in \{\pi, K, \eta\}$ and $\mathbb{a}^2 = \boldsymbol{a^2/8t_0}$.

482 models considered, including $(\bar{\mathbb{M}}^2)^2$, $(\delta\mathbb{M}^2)^2$, $\mathbb{M}_{\bar{\mathrm{D}}}^2$, $\bar{\mathbb{M}}^2\delta\mathbb{M}^2$, $\bar{\mathbb{M}}^2\mathbb{M}_{\bar{\mathrm{D}}}$, $\delta\mathbb{M}^2\mathbb{M}_{\bar{\mathrm{D}}}^2$, …terms,
and $\mathbb{M}$-dependent and $\mathbb{M}$-independent $\mathbb{a}^2$, $\mathbb{a}^3$, $\mathbb{a}^4$ terms.

Light and strange quark mass interpolation and continuum extrapolation

Simultaneous fit of f_{D} and $f_{\mathrm{D_s}}$ with all correlations taken into account.

Example of best fit with $\chi^2/d.o.f. = 0.92$ with ~ 160 d.o.f..



Data points projected using the fit to $m_{\bar{D}}^{phys}$ and the (left) continuum limit (right) also $m_{\pi,K}^{phys}$.

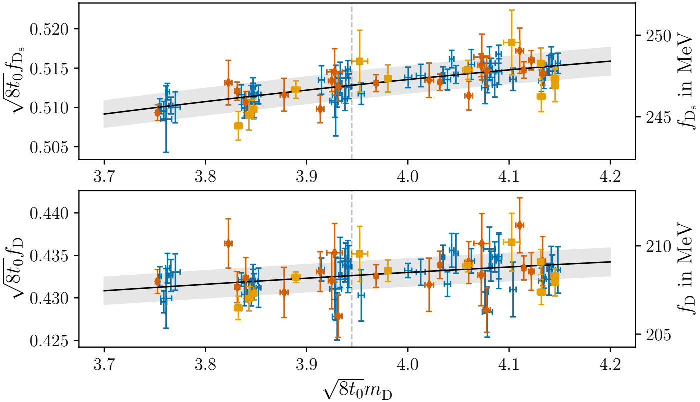
Fit includes a^3 terms. Cut-off effects of 5% from $a = 0.1$ fm to $a = 0$.

Charm quark mass dependence

Simultaneous fit of f_{D} and $f_{\mathrm{D_s}}$ with all correlations taken into account.

Example of best fit with $\chi^2/d.o.f. = 0.92$ with ~ 160 d.o.f..

Data points shifted using the fit to $m_{\pi,K}^{phys}$ and the continuum limit.



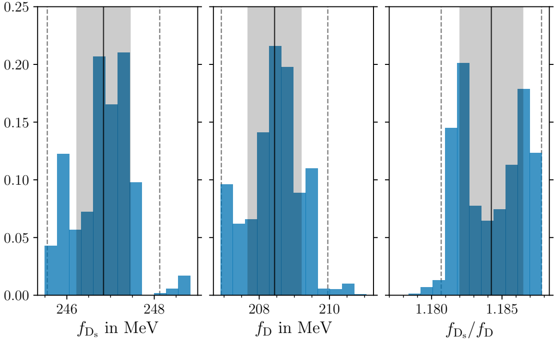
Global fit to two charm quark masses per ensemble $\rightarrow$ go beyond a linear interpolation.

Mild $m_{\bar{\mathrm{D}}}^2$ dependence is resolved.

Model average

Physical point for isoQCD: $\sqrt{t_{0,phys}}$ from [RQCD,2211.03744]
$m_\pi = 134.8(3)$ MeV and $m_K = 494.2(3)$ MeV from [FLAG 16,1607.00299],
$m_{\bar{D}} = 1899.4(3)$ MeV estimated in [RQCD,1706.01247] using
[Goity and Jayalath,hep-ph/0701245].

Data set fixed: $$\mathcal{O} = \sum_{k=1}^{482} w_k^{\mathrm{AIC}} \mathcal{O}_k \,, \qquad w_k^{\mathrm{AIC}} = N \exp(-\frac{1}{2}[\chi_k^2 + 2p_k]), \qquad \sum_{k=1}^{482} w_k^{\mathrm{AIC}} = 1$$



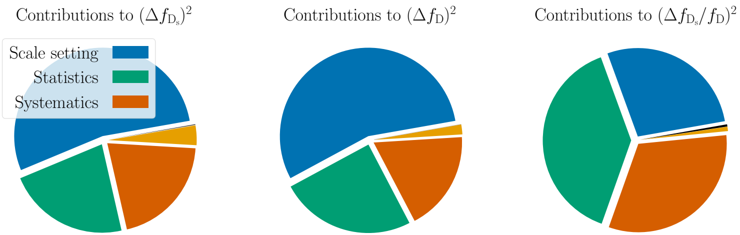
Variation of fit quality: $\leftarrow \chi^2/dof = 1.09 - 0.92$

Left: weighted histogram of central values of the fits.

Statistical error: from weighted average via standard error propagation.

Systematic error:
$$\sigma^2_{\mathrm{sys}} = \sum_{k=1}^{K} w_k^{\mathrm{AIC}} \mathcal{O}_k^2 - \left(\sum_{k=1}^{K} w_k^{\mathrm{AIC}} \mathcal{O}_k\right)^2 .$$

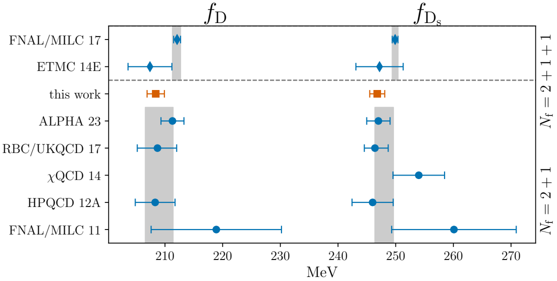
Final results and error budget

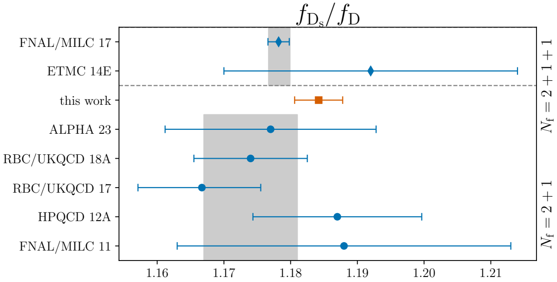


$f_{D_s} = 246.8(1.3)$ MeV $\quad f_D = 208.4(1.5)$ MeV $\quad f_{D_s}/f_D = 1.1842(36)$

- ★ Overall error of 0.5%, 0.7% and 0.3% in f_{D_s} , f_D and f_{D_s}/f_D.
- ★ Ratio computed from extrapolated f_D and f_{D_s}.
- ★ Uncertainty of f_D and f_{D_s} limited by the scale setting. Statistical error $\approx$ systematic error.
- ★ Systematic error dominated by the uncertainty due to the continuum limit extrapolation.

Comparison with other works





Grey bands: [FLAG 21,2111.09849] averages.

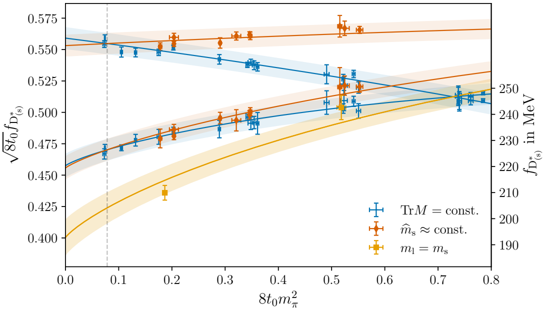
CLS ensembles: [ALPHA,2309.14154] 10 ensembles on $\mathrm{Tr}\, M =$ const trajectory, $m_\pi \geq 200$ MeV with twisted mass valence quarks.

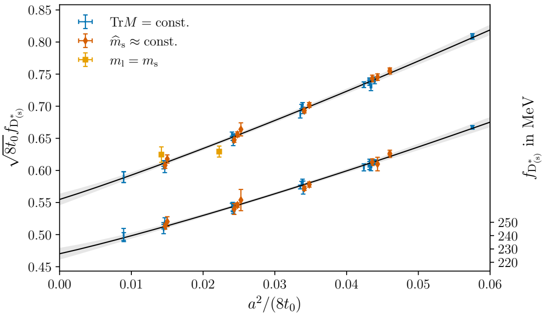
$N_f = 2+1+1$ [FNAL-MILC,1712.09262]: $f_{D_s} \sim 0.2\%$, $f_D \sim 0.3\%$, and $f_{D_s}/f_D \sim 0.1\%$.

RQCD-ALPHA $f_{D_{(s)}}$ results roughly 2σ below FNAL-MILC.

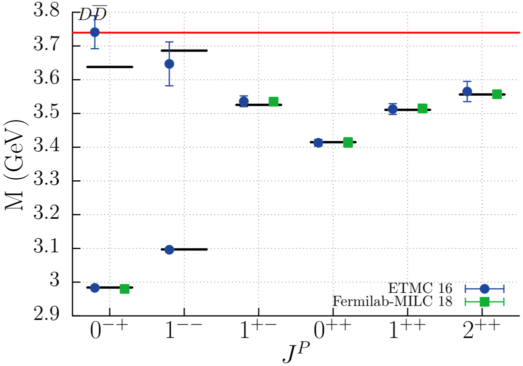
Charm sea effects on decay constants only studied in $c\bar{c}$ [ALPHA,2105.12278], cf. $N_f = 0, 2$, 0.5% effect.

Vector and tensor decay constants of $D^*_{(s)}$: **work in progress**





Lower lying charmonium spectrum and decay constants



Spectrum: test control of systematics

[ETMC,1603.06467] $N_f = 2+1+1$,
$a = 0.09, 0.08, 0.06$ fm,
$m_\pi = 224 - 468$ MeV.

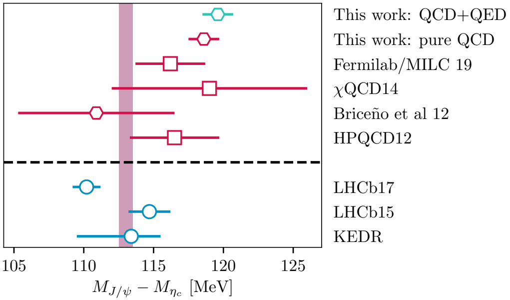
[Fermilab-MILC,1810.09983] $N_f = 2+1$,
$a = 0.14, 0.11, 0.08, 0.06, 0.04$ fm,
$m_\ell / m_s = 0.1, 0.2$.

Decay constants, $\langle 0|\bar{c}\Gamma c|X_{\bar{c}c}\rangle$: $\Gamma = \gamma_\mu\gamma_5$, $X_{\bar{c}c} = \eta_c$ (with assumptions) $\to \Gamma(\eta_c \to \gamma\gamma)$, $\Gamma(B \to \eta_c K)$
$\Gamma = \gamma_\mu$, $X_{\bar{c}c} = J/\psi \to \Gamma(J/\psi \to e^+e^-)$.

Also $\Gamma = \sigma_{\mu\nu}$, $X_{\bar{c}c} = J/\psi, h_c, \ldots$

Test of systematics and models. Compare to decay constants of (possible) non-quark model closed charm states.

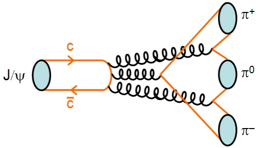
Charmonium 1S hyperfine splitting



[HPQCD,2005.01845]
High precision possible.

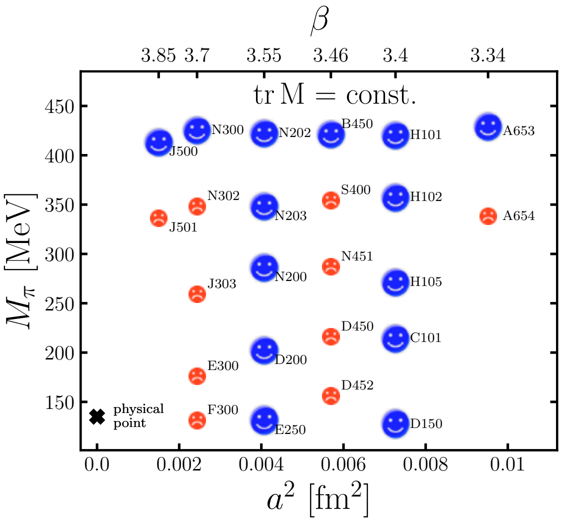
Discrepancy possibly due to omission of $\bar{c}c$ annihilation effects.

$c\bar{c}$ annihilation suppressed (OZI rule), $\Gamma_{J/\psi} \sim 93$ keV, $\Gamma_{\eta_c} = 32$ MeV.



Including $c\bar{c}$ annihilation effects: $c\bar{c}$ disconnected diagrams + mixing with light flavour singlet states and glueballs, see, e.g., [Urrea-Niño et al.,2312.16740], [Bali et al.,1110.2381], and also decays must also be taken into account.

Charmonium: ensembles analysed so far

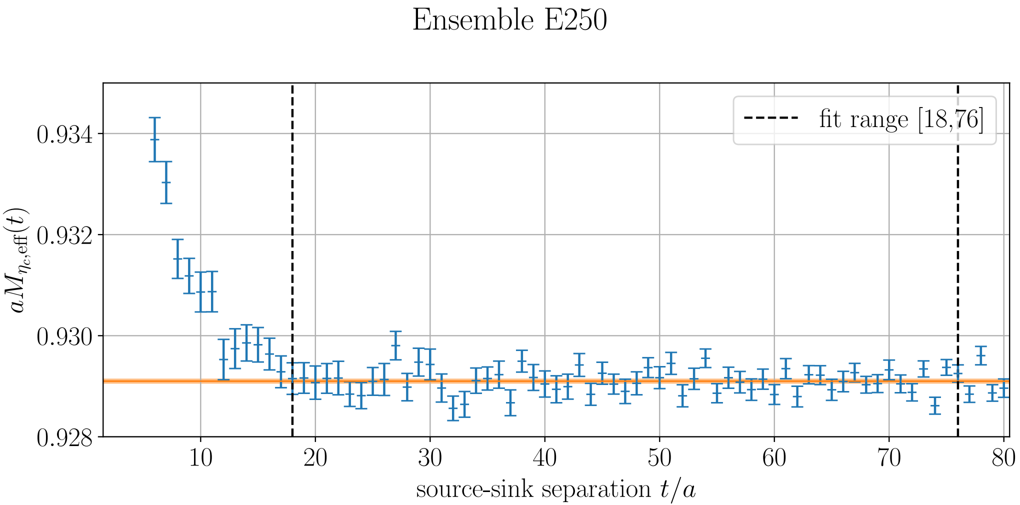


Omit $c\bar{c}$ disconnected diagrams.

Preliminary results on **14 ensembles** presented for $M_{J/\psi} - M_{\eta_c}$ only.

Data available for extracting the ground state $J = 0$ and $J = 1$ states.

Fitting analysis



$a \approx 0.064\,\mathrm{fm},\ M_\pi \approx 131\,\mathrm{MeV},\ V = 192 \cdot 96^3 a^4, \qquad aM(t) = \mathrm{arccosh}\left[\frac{C(t+a)+C(t-a)}{2C(t)}\right]$

Quark mass interpolation and continuum extrapolation

$\bar{\mathbb{M}}^{\mathbf{2}} = \sqrt{8t_0}((2m_{\mathrm{K}}^2 + m_\pi^2) \propto 2m_{\mathrm{l}} + m_{\mathrm{s}}\,, \quad \boldsymbol{\delta}\mathbb{M}^{\mathbf{2}} = \sqrt{8t_0}(m_{\mathrm{K}}^2 - m_\pi^2) \propto m_{\mathrm{s}} - m_{\mathrm{l}}\,, \quad \mathbb{M}_{\bar{\mathbf{D}}} = \sqrt{8t_0}M_{\bar{\mathrm{D}}} \to m_{\mathrm{c}}$

To leading non-trivial order in ChPT neither charmonium masses nor $M_{\bar{D}}$ depend on δM^2.

Fit form equivalent to:

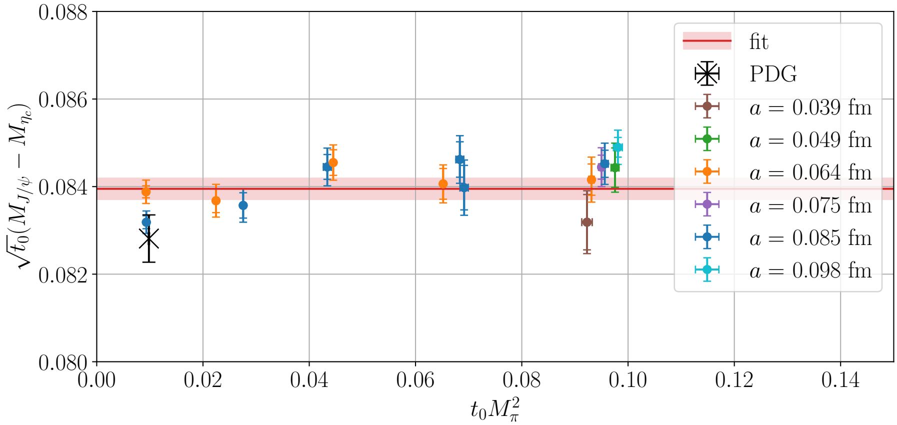
$$\sqrt{\mathbf{t_0}}\mathrm{M} = \mathrm{M}_0 + \bar{c}\bar{\mathbb{M}}^{\mathbf{2}} + c_c\mathbb{M}_{\bar{\mathbf{D}}} + c_a\mathbb{a}^{\mathbf{2}} + c_{ac}\mathbb{a}^{2}\mathbb{M}_{\bar{\mathbf{D}}} + \ldots$$

where $\mathbb{a}^{\mathbf{2}} = \boldsymbol{a}^{\mathbf{2}}/\mathbf{8}\boldsymbol{t}_{\mathbf{0}}^{*}$ and $12t_0^* M_\pi^2 = 1.11$ $(M_K = M_\pi)$.

Actual fit carried out differently.

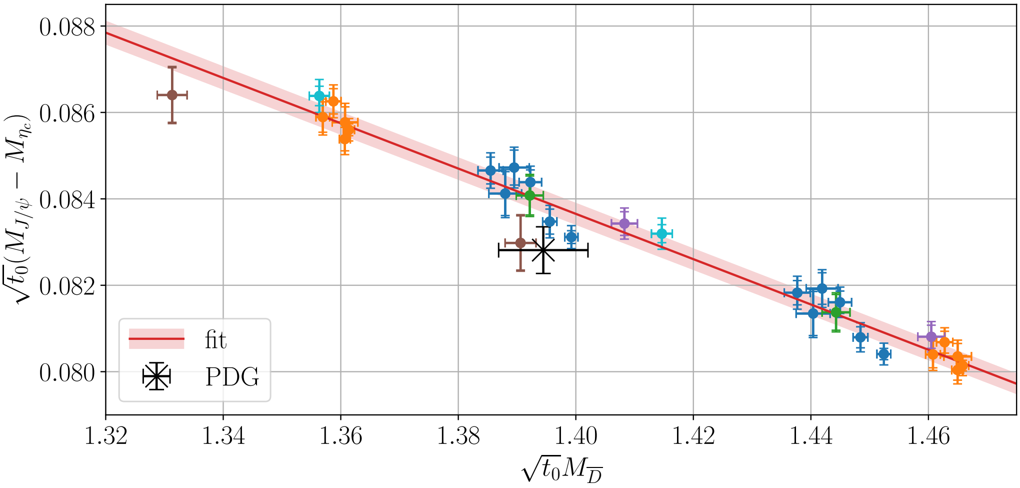
Additional terms will be considered in the future: a^3, $\mathrm{M}_{\bar{D}}^2$, $\bar{\mathrm{M}}^2\delta\mathrm{M}^2$, $\delta\mathrm{M}^4$ and other corrections.

$1S$ fine structure splitting versus the pion mass **(preliminary)**



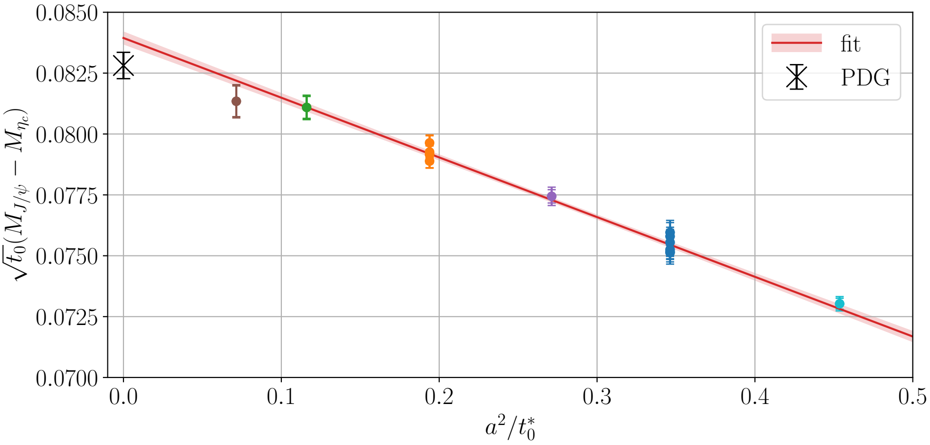
Uncertainty of $t_{0,\mathrm{ph}}$ was removed from the fit band and added to the experimental value.
Correlated $\chi^2/$**dof** $= 64.1/25$. Error inflated by $\sqrt{\chi^2/\mathrm{dof}}$. Extra fit parameters in the future.

$1S$ fine structure splitting versus the D meson mass **(preliminary)**



Uncertainty of $t_{0,\mathrm{ph}}$ was removed from the fit band and added to the experimental value.

The 1S fine structure splitting: continuum limit **(preliminary)**



Uncertainty of $t_{0,\text{ph}}$ only included in the “PDG” value. Cut-off effects of $\sim 15\%$ from $a = 0.1$ fm to $a = 0$.

Fine structure splitting in isoQCD **(preliminary)**

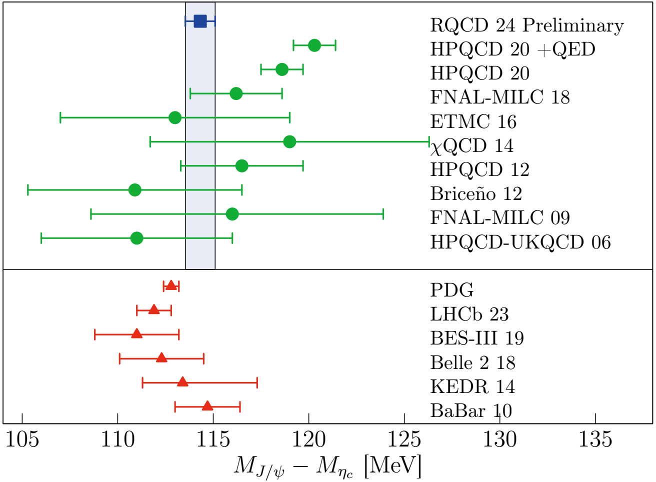
Light and strange sea quark effects are significant: cf. $N_f = 0$, e.g. $\Delta M = 77(2)(6)$ MeV [QCD-TARO,hep-lat/0307004].

Charm sea effects: [ALPHA,1905.12971] cf. $N_f = 0$ and $N_f = 2$, 2% effect in $(M_V - M_P)/M_P$ or 2 MeV.

From potential models and [HPQCD,2005.01845]: QED leads to an increase of up to 2 MeV.

[HPQCD,2005.01845]:
$\Delta M_{\mathrm{annihil.}} = +7.3(1.2)$ MeV.

M_{η_c} most affected by $c\bar{c}$ annihilation diagrams.



RQCD 24 (preliminary): $M_{\eta_c} = 2977(4)$ MeV cf. 2984 MeV (PDG).

Summary and outlook

★ Large set of high-statistics CLS ensembles enable tight control of quark mass and lattice spacing dependence (also of finite volume effects).

$\rightarrow$ precision calculation of open and closed charm observables.

★ Determination of f_{D} and $f_{\mathrm{D_s}}$ to sub-percent precision.

Discretisation effects are significant but moderate in magnitude, a^3 effects resolved.

Large number of models considered with high number of d.o.f. in the fit.

Further reduction in the error requires higher precision for $\sqrt{t_{0,phys}}$.

★ Charmonia below $D\bar{D}$ threshold: precision such that one can possibly resolve annihilation effects in 1S hyperfine splitting.

Future:

★ Vector and tensor D^* and D_s^* decay constants.

★ Charmonium: compute masses and decay constants of $J = 0$ and $J = 1$ states. Include more ensembles and carry out more sophisticated fits and analyses of systematics.