Effective Field Theory I

Hans-Werner Hammer
Technische Universität Darmstadt

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Agenda

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1. Effective Field Theory I: Basic ideas
2. Effective Field Theory II: Nonrelativistic short-range EFT
3. Effective Field Theory III: Applications in hadrons and nuclei

Literature

G.P. Lepage, “What is renormalization”, TASI Lectures 1989, arXiv:hep-ph/0506330

D.B. Kaplan, “Effective Field Theories”, NNPS Lectures 1995, arXiv:nucl-th/9506035

G.P. Lepage, “How to renormalize the Schrödinger equation”, Swieca Lectures 1997, arXiv:hep-ph/0506330

E. Braaten, HWH, “Universality in few-body systems with large scattering length”, Phys. Rep. **428**, 259 (2006), arXiv:cond-mat/0410417

Resolution of a Microscope

- Resolution of microscope is limited
- Diffraction effects:

  Light is EM wave ($\lambda \approx 550$ nm)
- Resolution: $d = \dfrac{\lambda}{A_N}$
- Aperture: $A_N \approx 0.95 ... 1.5$

  $$\longrightarrow d_{max} \approx 0.4\,\mu\text{m}$$

- Quantum mechanics:

  particles have wave character $\Rightarrow$ matter waves

$\lambda = \dfrac{h}{p}$ (de Broglie, 1924) $\Rightarrow$ QM microscope

$\Rightarrow$ intrinsic resolution scale determined by typical p

Scales in Physics

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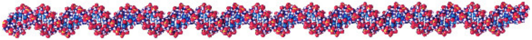
Difference between typical biological and physical Systems?

- DNA molecule 

  many energy scales of comparable size
- Hydrogen atom: few separated scales
  - Hierarchy: $m_e \ll m_p \ll m_{Z_0} \ll \ldots$
- Physical quantities have units $\longrightarrow$ dimensional analysis

  (vgl. M. Planck, Sitzungsb. Preuß. Akad. Wiss. **5**, 479 (1899)), “Planck units”)

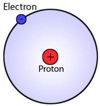
$$E_0 = m_e \times f\left(\alpha, \frac{m_e}{m_p}, \frac{m_e}{m_{Z_0}}, \ldots\right)$$

  - f contains dynamics
- How to exploit scale separation?

$\Longrightarrow$ Effective (Field) Theories $\quad (\Longleftrightarrow \quad$ Calculation of $f)$

Hydrogen Atom

- Quantum bound state of e^- and p



- Energy levels (lowest order): Schrödinger-Eq. for $V(r) = -e^2/r$

$$E = E_0 = -\frac{m_e \alpha^2}{2n^2}, \quad \alpha = \frac{e^2}{4\pi}$$

- Corrections beyond leading order: $E = E_0 \left[1 + \mathcal{O}(\alpha, \frac{m_e}{m_p}, \ldots) \right]$
  - corrections from EM interaction: fine structure $\boldsymbol{L} \cdot \boldsymbol{S} \sim \alpha^4$, ...
  - corrections from proton structure: finite proton mass m_p

    hyperfine structure: μ_p

    finite proton size: r_p

Dimensional Analysis

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- Estimate effect of r_p on E_0
- Natural scales: $a_0 = 1/(m_e\alpha) \sim 0.5$ Å (Bohr radius)
- Proton charge radius: $G_E(q^2) = 1 + q^2 \overbrace{G_E'(0)}^{r_p^2/6} + \ldots$

  guess: $G_E'(0) \sim 1/m_p^2, \quad q \sim 1/a_0 \quad \Longrightarrow \quad \left(\frac{m_e\alpha}{m_p}\right)^2 \sim 10^{-11}$
- Size of correction to E_0: $10^{-11}E_0 \to 10^{-11} \times 3 \cdot 10^{15}$ Hz ~ 30 kHz
- Estimated enhancement in muonic hydrogen by factor $(m_\mu/m_e)^2 \approx 200^2$

  $\Rightarrow$ higher precision in muonic atoms possible
  (R. Pohl et al., Nature **466**, 213 (2010))
- How to construct an effective theory to calculate the corrections from m_p, μ_p, r_p ?

Construction of an Effective Theory

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- How to construct an effective theory to calculate these corrections?
- Need to identify:
  - Low and high scales in the problem: M_{lo}, M_{hi}, ...
  - Active degrees of freedom: coordinates, particles, ...
  - Symmetries to constain interactions
  - Power counting: organize the theory through expansion in M_{lo}/M_{hi}
- Guiding principle:

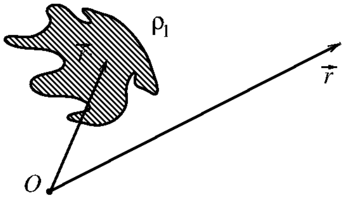
  Include long-range physics explicitly, parametrize short-range physics in low-energy constants

- Consider an example from classical electrodynamics

Example: Multipole Expansion

- Separation of scales: $\langle r' \rangle \ll r$
- Short-range physics: expand in $\frac{\langle r' \rangle}{r}$

  $\longrightarrow$ "power counting"
- Breakdown scale: $\langle r' \rangle \sim r$



$$\phi(\mathbf{r}) = \sum_{l,m} \frac{C_l Y_{lm}(\Omega)}{r^{l+1}} \overbrace{\int d^3r' \rho_1(\mathbf{r}') Y^*_{lm}(\Omega')(r')^l}^{q_{lm}}$$

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{l,m} C_l Y_{lm}(\Omega) Y^*_{lm}(\Omega') \frac{r^l_<}{r^{l+1}_>}$$

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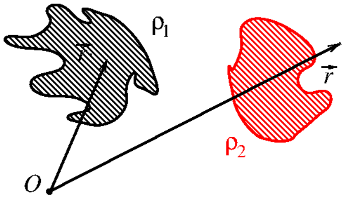

$$\phi(\boldsymbol{r}) \quad = \quad \sum_{l,m} \frac{C_l Y_{lm}(\Omega)}{r^{l+1}} \overbrace{\int d^3r' \rho_1(\boldsymbol{r'}) Y^*_{lm}(\Omega')(r')^l}^{q_{lm}} + \int d^3r'' \frac{\rho_2(\boldsymbol{r''})}{|\boldsymbol{r} - \boldsymbol{r''}|}$$

- Long-range physics: include explicitly
- Use these ideas in the framework of Quantum Field Theory

Weinbergs Conjecture

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Quantum Field Theory is "only" way to satisfy quantum mechanics, Lorentz invariance, and cluster decomposition

$\Rightarrow$ to calculate the most general S-matrix consistent with given symmetries for any theory below some scale simply use the most general effective Lagrangian $\mathcal{L}_{eff}$ consistent with these principles in terms of the appropriate asymptotic states

(Weinberg, 1979)

- Nonrelativistic systems: Lorentz invariance $\longrightarrow$ Galilei invariance

$\Longrightarrow$ QM framework possible

- Need "power counting scheme" to organize the infinitely many terms in $\mathcal{L}_{eff}$

$\Longrightarrow$ predictive power

Construction of an EFT

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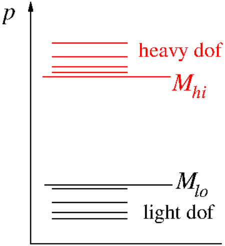
- Construct most general $\mathcal{L}_{eff}$ respecting underlying symmetries

- Exploit separation of scales: $p, M_{lo} \ll M_{hi}$
- Non-renormalizable theory, but only finite number of operators in $\mathcal{L}$ contribute at each order

  $\Rightarrow$ Low-energy constants (LEC)
- Symmetries and light dof's must be known



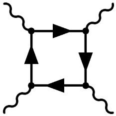
- Work at low energies: $p \ll M_{hi}$
- Fix LEC's from matching (to experiment, other theory, ...)
- Calculate observables in expansion in $p/M_{hi}, M_{lo}/M_{hi}$

  $\Longrightarrow$ limited range of applicability

Light-By-Light Scattering

- Classic Example (Euler, Heisenberg, 1936)
- Photon energy ω, electron mass m_e
- Separation of scales: $\omega \ll m_e$
- Only photons are active dof $\Rightarrow$ theory simplifies

  $\Rightarrow \mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \to \mathcal{L}_{eff}[A_\mu]$



- Invariants: $F_{\mu\nu}F^{\mu\nu}$, $(F_{\mu\nu}F^{\mu\nu})^2$, $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$, ...,

  with field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, dual tensor $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$

$$\boxed{\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{c_1}_{LEC}(F_{\mu\nu}F^{\mu\nu})^2 + \underbrace{c_2}_{LEC}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \ldots}$$

- Estimate size & energy dependence of cross section $\longrightarrow$ exercise
- Observation in Pb-Pb Collisions at LHC

  (ATLAS Kollaboration, Nature Physics **13**, 852 (2017))

Fermi Theory

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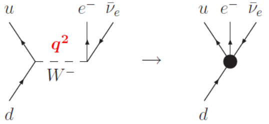
- Weak decays
  - mediated by $W^{\pm}$ bosons ($M_W \approx 80$ GeV)
  - energy release in neutron β-decay ~ 1 MeV [$n \to pe^-\bar{\nu}_e$]
  - energy release in kaon decays ~ 300 MeV [$K \to \pi e \nu$]



$$\frac{e^2}{8\sin^2\theta_W} \times \frac{1}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \frac{e^2}{8M_W^2 \sin^2\theta_W}\left\{1 + \frac{q^2}{M_W^2} + \dots\right\} = \frac{G_F}{\sqrt{2}} + \dots$$

$\Longrightarrow$ Fermi Theory of Weak Interaction

Structure of EFTs

- Energy/momentum expansion (derivatives acting on fields)
- Dimensional analysis (breakdown scale M_{hi})
  - derivatives $\rightarrow$ powers of typical momentum q
    - one derivative: $\partial \sim q/M_{hi}$
    - vertex w/ N derivatives: $\partial^N \sim (q/M_{hi})^N$
    - terms with more derivatives are suppressed if $q \ll M_{hi}$
  - Loops: additional powers of momenta from vertices and loop momenta

    $\Longrightarrow$ loops are generally suppressed compared to trees (exceptions!)

 $\sim (q/M_{hi})^N$

 $\sim (q/M_{hi})^{2N} \underbrace{\int d^4p\, f(p,q)}_{(q/M_{hi})^M}$

Additional Transparencies

Construction of an QFT

1. Construct $\mathcal{L}$ respecting symmetries (e.g. gauge invariance of QED)

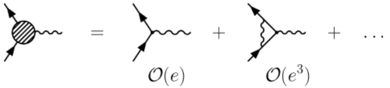
   $\psi \to \psi' = e^{i\alpha(x)}\psi, \quad A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \alpha_\mu(x)$

2. Retain renormalizable interactions ($D \leq 4$): $\quad [\psi] = 3/2,\ [A] = 1$

   keep $\underbrace{\bar{\psi}\gamma_\mu\psi A^\mu}_{D=4}$, $\quad$ but drop $\underbrace{\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}}_{D=5}$, $\quad \underbrace{(F_{\mu\nu}F^{\mu\nu})^2}_{D=8}$

   $$\boxed{\Longrightarrow \qquad \mathcal{L} = \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}}$$

3. Calculate Feynman diagrams:



4. Fix parameters from data → predictions

   Example: $\quad \mu_e = -\frac{e g_e \boldsymbol{s}_e}{2m_e}, \quad g_e = 2\left[1 + \frac{e^2}{8\pi^2} + \mathcal{O}(e^4)\right]$

Renormalizability

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- Renormalization: method to make sense of infinities/UV sensitivities in QFT
  - Some observables are sensitive to short-distances $\Rightarrow$ match to experiment and predict other observables
  - Classical renormalizability: redefinition of a finite number of terms sufficient to all orders (QED: e, m_e, Z_3, Z_2)
  - EFT: new structures appear at every order
- Renormalizable theories have been very successful $\Longrightarrow$ Standard Model (SM)
- Non-renormalizable interactions ($D > 4$) are suppressed at low energies $\Longleftarrow$ factors $1/M_{hi}^{D-4}$
- Modern understanding: SM is also low-energy EFT, physics beyond SM can be included through non-renormalizable interactions