Summary: local short-range EFT

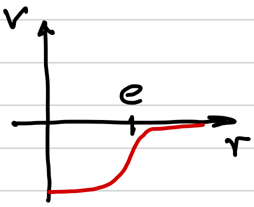
$$\mathcal{L}_{eff} = \psi^\dagger\left(i\partial_t + \frac{\vec{\nabla}^2}{2m}\right)\psi - \frac{g_2}{4}(\psi^\dagger\psi)^2 - \frac{g_{2,2}}{4}\vec{\nabla}(\psi^\dagger\psi)\cdot\vec{\nabla}(\psi^\dagger\psi) + \dots$$

$$- \frac{g_3}{36}(\psi^\dagger\psi)^3 + \dots$$

Naturalness: $\quad g_2 \sim \frac{\ell}{m}, \; g_{2,2} \sim \frac{\ell^3}{m}, \dots$

$\quad g_3 \sim \frac{\ell^4}{m}, \dots$

where $\ell \doteq$ length scale of underlying interaction

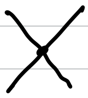


$a \sim r_e \sim \dots \sim \ell$

Scaling of diagrams: $\quad k^\nu$

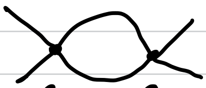
$$\nu = 3L + 2 + \sum_i (2i-2)V_{2i}$$

$\nu = 0$:



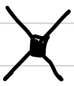
g_2

$\nu = 1$:



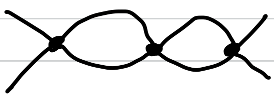
$g_2 \quad g_2$

$\nu = 2$:



$g_{2,2}$

$+$

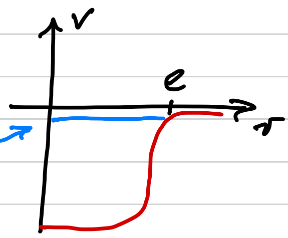


$g_2 \quad g_2 \quad g_2$

$\vdots$

Now consider fine tuned case with $|a| \gg \ell \sim r_e \sim \dots$

Shallow bound state
(or virtual state)



2.3 Two-body system with large scattering length

- $|a| \gg \ell \sim r_e \sim \ldots$ [requires fine tuning]

$\Longrightarrow$ cannot expand in (ka), but expansion in $(k\ell)$, (kr_e), ... still works well

- Physics:

$$T(k) = \frac{8\pi}{m} \frac{1}{-1/a + \frac{r_e}{2}k^2 + \ldots - ik}$$

$$= -\frac{8\pi a}{m} \frac{1}{1+ika} \left[1 - \frac{r_e a k^2/2}{1+ika}\right]^{-1}$$

$$= -\frac{8\pi a}{m} \left[\frac{1}{1+ika} + \frac{r_e a k^2/2}{(1+ika)^2} + \ldots\right]$$

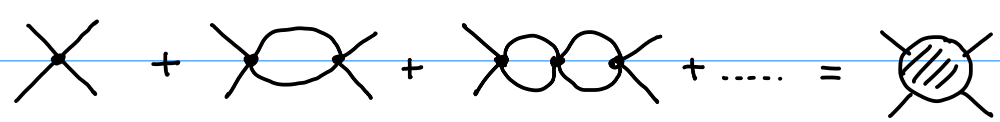
- count $k \sim 1/a \equiv Q$ $\longrightarrow$ expansion in $(Q\ell)$

Conjecture: $g_2 \sim \frac{1}{mQ}$, $g_{2,2} \sim \frac{\ell}{mQ^2}$, ..., $g_{2,2i} \sim \frac{\ell^i}{mQ^{i+1}}$

$\hookrightarrow$ Q^ν where $\nu = 5L - 2I + \sum_i (\underbrace{2i - (i+1)}_{i-1})$

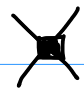
$\hookrightarrow$ $\boxed{\nu = 3L + 2 + \sum_i (i-3) V_{2i} \geq -1}$

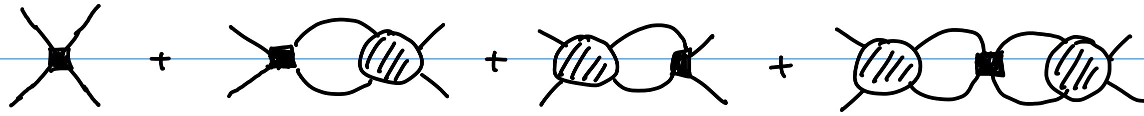
- What does this mean for Feynman diagrams?

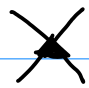
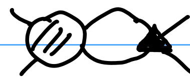
g_2:  $\sim Q^{-1}$

$\nu = -1$

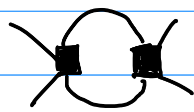
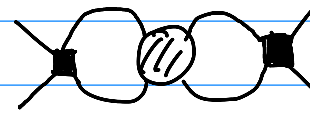
For each loop $\nu = \ldots \underbrace{+3}_{\Delta L} \ldots \underbrace{-(0-3)\cdot 1}_{\Delta V_{20}} = \nu$

$g_{2,2}$:  $\nu = 0 + 2 + (1-3)\cdot 1 = 0$

 $\nu = 0$

$g_{2,4}$:  +  + ...

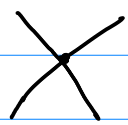
$\nu = 0 + 2 + (2-3)\cdot 1 = 1$

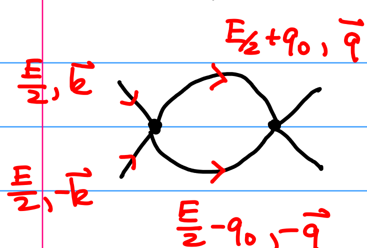
 +  + ...

$\nu = 3 + 2 + (1-3)\cdot 2 = 1$

....

- Now calculate the LO $(\nu = -1)$ contribution:

 $= -i g_2$

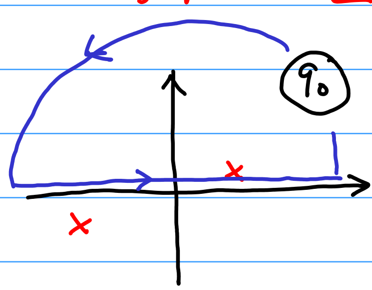


$\int dq_0 \int d^3\vec{q}$

$$= \frac{1}{2}(-\cancel{i} g_2)^2 \int \frac{d^4q}{(2\pi)^4} \frac{\cancel{i}}{E/2 + q_0 - \frac{\vec{q}^2}{2m} + i\epsilon} \frac{\cancel{i}}{E/2 - q_0 - \frac{\vec{q}^2}{2m} + i\epsilon}$$

symmetry factor

Regulator



$$= \frac{g_2^2}{2} \frac{\cancel{2}\pi i}{\cancel{2}\pi} (-1) \int^{\Lambda} \frac{d^3q}{(2\pi)^3} \frac{m}{mE - \vec{q}^2 + i\epsilon}$$

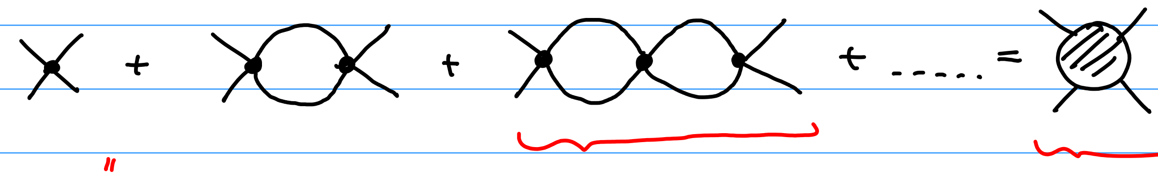
$$\int d^3q = \underbrace{\int d\Omega_q}_{4\pi} \int q^2 dq$$

$$= \frac{i m g_2^2}{2} \frac{1}{2\pi^2} \int_0^{\Lambda} dq \frac{q^2}{q^2 - mE - i\epsilon}$$

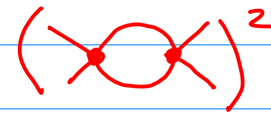
$$\int_0^{\Lambda} dq \, \frac{q^2}{q^2 - mE - i\epsilon} = \Lambda - \frac{\pi}{2}\sqrt{-mE - i\epsilon} + \cancel{\mathcal{O}(1/\Lambda)}$$

neglect, can be made arbitrarily small

• Need:



"Bubble sum" $\hat{=}$ geometric series

$\propto$  ^{2}

iT

$$iT = -ig_2 \left(1 - \frac{mg_2}{4\pi^2}\left(\Lambda - \frac{\pi}{2}\sqrt{-mE - i\epsilon}\right) + \left[\frac{mg_2}{4\pi^2}\left(\Lambda - \frac{\pi}{2}\sqrt{-mE - i\epsilon}\right)\right]^2 + \ldots \right)$$

$$= -i \left(1/g_2 + \frac{m}{4\pi^2}\left(\Lambda - \pi/2\sqrt{-mE - i\epsilon}\right)\right)^{-1}, \quad E = k^2/m$$

$$\overset{!}{=} i \, \frac{8\pi}{m}\left[-1/a - ik\right]^{-1}$$

matching/renormalization condition

$$\Leftrightarrow \quad \boxed{g_2(\Lambda) = \frac{8\pi a}{m}\left[1 - \frac{2a\Lambda}{\pi}\right]^{-1}}$$

Running coupling constant

• Expansion for $a \sim \ell$ reproduces natural case

• Change of $\Lambda \Rightarrow$ change of resolution scale (RG transformation)

• Ideas can be applied in nuclear & hadron physics