Effective Field Theory III

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Agenda

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1. Effective Field Theory I: Basic ideas
2. Effective Field Theory II: Nonrelativistic short-range EFT
3. Effective Field Theory III: Applications in hadrons and nuclei

Literature

G.P. Lepage, “What is renormalization”, TASI Lectures 1989, arXiv:hep-ph/0506330

D.B. Kaplan, “Effective Field Theories”, NNPS Lectures 1995, arXiv:nucl-th/9506035

G.P. Lepage, “How to renormalize the Schrödinger equation”, Swieca Lectures 1997, arXiv:hep-ph/0506330

E. Braaten, HWH, “Universality in few-body systems with large scattering length”, Phys. Rep. **428**, 259 (2006), arXiv:cond-mat/0410417

Effective Field Theory

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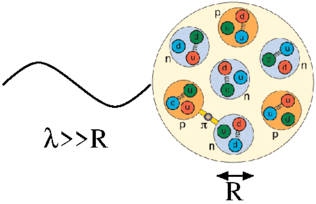
- Separation of scales:

  $1/Q = \lambda \gg R$

- Limited resolution at low energy:

  $\longrightarrow$ expand in powers of QR



Effective Field Theory

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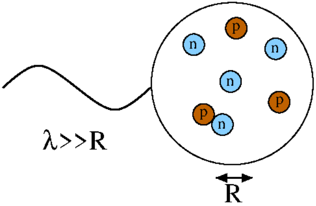


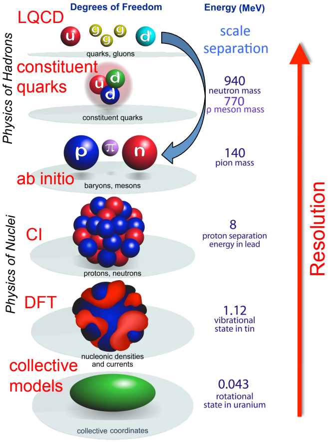
- Short-distance physics not resolved

  $\longrightarrow$ capture in low-energy constants using renormalization

  $\longrightarrow$ include long-range physics explicitly

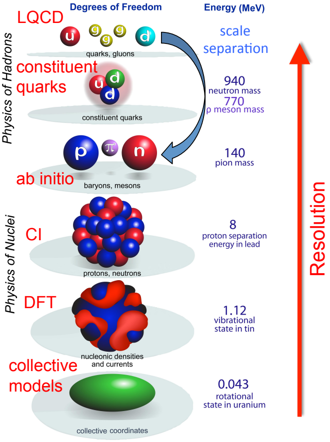
- Systematic, model independent $\rightarrow$ error estimates

Nuclear EFTs



from Bertsch, Dean, Nazarewicz (2007)

Nuclear EFTs



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EFT exploits separation of scales

$\Downarrow$

EFT depends on resolution scale

- Chiral EFT: N, π, (Δ)
- Pionless EFT: N
- Halo EFT: N, clusters
- EFT for deformed nuclei: collective dof
- EFT at Fermi surface: quasi-N
- ...

Nuclear forces from chiral EFT

$$e^{iZ[J]} = \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}G\, e^{i\int d^4x\,\mathcal{L}_{QCD}[q,\bar{q},G;J]} \Leftrightarrow \int \mathcal{D}\pi\,\mathcal{D}N e^{i\int d^4x\,\mathcal{L}_{eff}[\pi,N;J]}$$

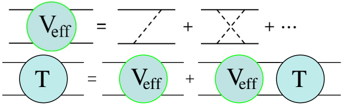
- Write down most general $\mathcal{L}_{\text{eff}}$ consistent with chiral symmetry

$$\mathcal{L}_{\text{QCD}} \to \mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

- Compute S-Matrix elements perturbatively in chiral expansion
- Fix low-energy constants (LECs) & make predictions...

Complication for $\geq 2N$: large scattering length ($\Rightarrow$ ^{2}H, nuclei)

- Weinberg approach: (Weinberg, 1990, 1991; Ordonez and van Kolck, 1992; ...)
  - Use $\mathcal{L}_{eff}$ to compute V_{eff}
    (syst. expansion in Q/Λ_χ and m_π/Λ_χ)
  - Solve LS equation with V_{eff}
  - UV regulator: cutoff Λ



Nuclear forces from chiral EFT

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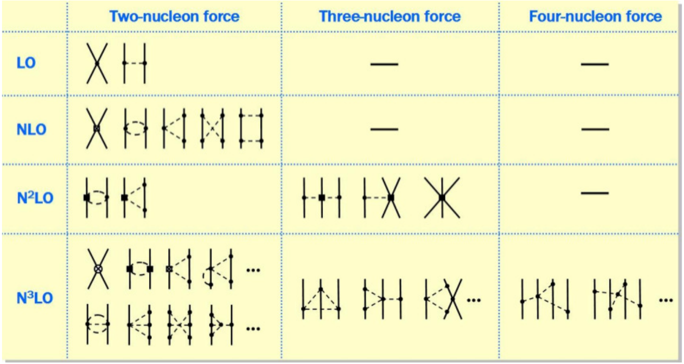
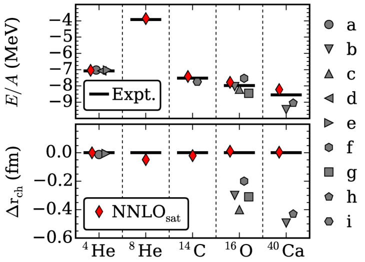


- Hierarchy of nuclear forces: $V_{2N} \gg V_{3N} \gg V_{4N} \gg \ldots$
- Successful phenomemology using FY, NCSM, CC, IMSRG, Lattice EFT, ...

Nuclear forces from chiral EFT



- Successful phenomenology

  (e.g., Ekström et al., 2015)

- But some issues under discussion:
  - Narrow range of cutoffs ($\Lambda \approx 450...600$ MeV)

    $\Rightarrow$ Is exact RG invariance even possible or desired?
  - Some observables show dependence on type of regulator
    (local vs. non-local, *r*-space vs. *p*-space etc.)
- Alternative approaches available but rarely used for $A > 3$
  (Nogga, Timmermans, van Kolck, 2005; ....)

Nuclear Scales and the Unitary Limit

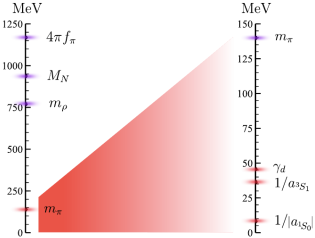
- Chiral EFT: $Q \sim m_\pi$

  $\Rightarrow$ contact interactions and pion exchange between nucleons

- Pionless EFT: $Q \sim \gamma_d \sim 1/a$

  $\Rightarrow$ contact interactions only

  $\Rightarrow$ expansion around unitary limit



- Unitary limit: $a \to \infty$, $R \sim r_e, \ldots \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \left[-1/a + r_e k^2/2 + \ldots - ik\right]^{-1} \quad \Longrightarrow \quad i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound

- Standard pionless EFT: include scattering lengths exactly at LO
- Alternative: strictly expand around $1/a = 0$ and include finite a in perturbation theory (S. König et al., Phys. Rev. Lett. **118**, 202501 (2017))

Physics Near the Unitary Limit

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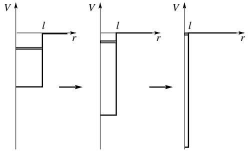
- Consider short-ranged, resonant S-wave interactions $\Longrightarrow$ halo state
- Unitary limit: $a \to \infty$, $\ell \sim r_e \to 0$

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \sim i/k$$



  - Scattering amplitude scale invariant, saturates unitarity bound

- System has also (non-relativistic) conformal symmetry/Schrödinger symmetry

Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...

- Exploit approximate conformal symmetry for nuclear reactions with neutrons

$$a \approx -18.6 \text{ fm}, \quad r_e \approx 2.8 \text{ fm} \quad \Rightarrow \quad \underbrace{1/(ma^2)}_{0.1\text{ MeV}} \ll E_n^{cms} \ll \underbrace{1/(mr_e^2)}_{5\text{ MeV}}$$

Schrödinger symmetry

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- Non-relativistic conformal symmetry: Schrödinger symmetry
  - Galilei symmetry

    space + time translations

    rotations

    Galilei boosts
  - Scale transformations

$$\boldsymbol{x} \to e^{\lambda}\boldsymbol{x}, \qquad t \to e^{2\lambda}t, \qquad \psi \to e^{-\lambda\Delta}\psi$$

  - Special conformal transformations

$$\boldsymbol{x} \to \frac{\boldsymbol{x}}{1+\xi t}, \qquad t \to \frac{t}{1+\xi t}, \qquad \psi \to \psi' = \dots$$

  $\Rightarrow$ preserves angles
- 12 Parameters
- Generators: $H, \boldsymbol{P}, \boldsymbol{L}, \boldsymbol{K}, D, C,$ satisfy Schrödinger algebra

Unitary limit: RG perspective

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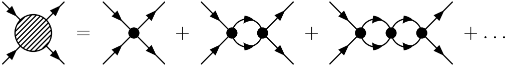
- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e \sim \ell$)



- Renormalization group equation: $$\Lambda \frac{d}{d\Lambda}\tilde{g}_2 = \tilde{g}_2(1+\tilde{g}_2)$$
- Two fixed points:

  – $\tilde{g}_2 = 0 \Leftrightarrow a = 0 \quad \Rightarrow$ no interaction, free particles

  – $\tilde{g}_2 = -1 \Leftrightarrow 1/a = 0 \quad \Rightarrow$ unitary limit $\quad \Rightarrow$ conformal/Schrödinger symmetry

- Neutrons: $a \approx -18.6$ fm, $\quad r_e \approx 2.8$ fm $\quad \Rightarrow$ approximately conformal

$\Longrightarrow$ multineutron system can be described by conformal field

Conformal field theory constraints

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- Two-point function of (primary) conformal field operator $\mathcal{U}$

$$G_{\mathcal{U}}(t,\boldsymbol{x}) = -i\langle T\mathcal{U}(t,\boldsymbol{x})\mathcal{U}^\dagger(0,\boldsymbol{0})\rangle = C\,\frac{\theta(t)}{(it)^\Delta}\exp\left(\frac{iM\boldsymbol{x}^2}{2t}\right)$$

- Determined by symmetry up to overall constant C
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega,\boldsymbol{p}) = -C\left(\frac{2\pi}{M}\right)^{3/2}\Gamma\left(\frac{5}{2}-\Delta\right)\left(\frac{\boldsymbol{p}^2}{2M}-\omega\right)^{\Delta-\frac{5}{2}}$$

  - pole only for $\Delta = 3/2$ (free field)
  - branch cut for $\Delta > 3/2$

Conformal field theory constraints

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- Imaginary part of propagator

$$\mathrm{Im}\, G_{\mathcal{U}}(\omega, \boldsymbol{p}) \sim \begin{cases} \delta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta = \frac{3}{2}\,, \\ \left(\omega - \frac{\boldsymbol{p}^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

- General $\mathcal{U}$ does not behave like a particle (“unparticle/unnucleus”)
  $\Rightarrow$ continuous energy spectrum
- Examples of unnuclei
  - N free fields: $\mathcal{U} = \psi_1 \ldots \psi_N$, $M = N m_\psi$, $\Delta = 3N/2$
  - N interacting fields: $\mathcal{U} = \psi_1 \ldots \psi_N$, $M = N m_\psi$, $\Delta > 3/2$
- Corrections from finite a and r_0 (S. Dutta, R. Mishra, D.T. Son, Phys. Rev. D **109**, 016001 (2024))

$$\mathrm{Im}\, G_{\mathcal{U}}(\omega, 0) \sim \omega^{\Delta - \frac{5}{2}} \theta(\omega) \left(1 + \frac{c_1}{a\sqrt{m\omega}} + c_2 r_0 \sqrt{m\omega}\right)\,, \quad c_2 = 0$$

Unparticle physics

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- (Relativistic) unparticle (Georgi, Phys. Rev. Lett. **98**, 221601 (2007))
  - field ψ in relativistic conformal field theory
  - ψ characterized by scaling dimension Δ, massless
  - hidden conformal symmetry sector beyond Standard model (weakly coupled)
  - no evidence at LHC so far
    (CMS Coll., EPJC **75**, 235 (2015), PRD **93**, 052011, JHEP **03**, 061 (2017))
- (Non-relativistic) unparticle/unnucleus
  - non-relativistic analog of Georgi's unparticle
  - field ψ in non-relativistic conformal field theory
    (cf. Nishida, Son, Phys. Rev. D **76**, 086004 (2007))
  - ψ characterized by scaling dimension Δ and mass M
  - free field has $\Delta = 3/2 \quad \Longleftrightarrow \quad$ mass dimension
    $\Rightarrow$ lowest possible value (unitarity)
  - N neutrons are (approximate) unparticle with mass Nm_N and scaling dimension $\Delta = ?$

Scaling dimension $\Leftrightarrow$ critical exponents

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- How to calculate scaling dimension Δ? not given by symmetry
  $\Longrightarrow$ non-perturbative problem

  (1) Δ can be obtained from field theory calculation

  (2) Δ can be obtained from operator state correspondence

$$\boxed{\Delta \text{ of primary operator = (Energy of state in HO)}/\hbar\omega}$$

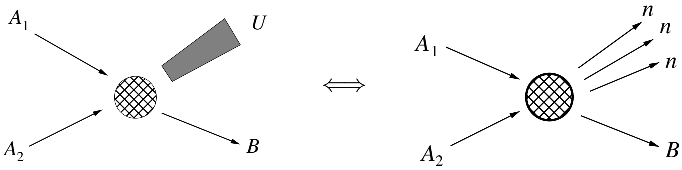
(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

N	S	L	$\mathcal{O}$	Δ	Δ_{free}
2	0	0	$\psi_1\psi_2$	2	3
3	1/2	1	$\psi_1\psi_2\nabla_j\psi_2$	4.27272	5.5
3	1/2	0	$\psi_1\nabla_j\psi_2\nabla_j\psi_2$	4.66622	6.5
4	0	0	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$	5.07(1)	8
5	1/2	1	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2\nabla_i\psi_1$	7.6(1)	10.5

- Connection between Δ and energy of particles in a trap
- Predictions for $N \leq 70$ available (M.G. Endres et al., PRA **84**, 043644 (2011))

Reactions with neutrons

- High-energy nuclear reaction with multi-neutron final state
  (HWH, Son, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021))



$$\Delta M c^2 + E_{A_1} + E_{A_2} = E_B + E_{\mathcal{U}}$$

- Assumption: energy scale of primary reaction $\gg E_{\mathcal{U}} - \frac{\boldsymbol{p}^2}{2M_{\mathcal{U}}} = E_n^{cms}$
- Factorization: $\frac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \, \mathrm{Im}\, G_{\mathcal{U}}(E_{\mathcal{U}}, \boldsymbol{p})$
- Reproduces Watson-Migdal treatment of FSI for $2n$

Conformal symmetry and reactions with neutrons

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- Two ways to do experiments
  - (a) detect recoil particle B $\quad \frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta - 5/2}, \qquad E_0 = (1 + M_B/M_{\mathcal{U}})^{-1} E_{\text{kin}}$
  - (b) detect all final state particles including neutrons: $\quad \frac{d\sigma}{dE} \sim (E_{xn}^{cms})^{\Delta - 5/2}$

Conformal symmetry and reactions with neutrons

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- $2n$ case can be understood from dimer propagator in halo/pionless EFT ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

- See also (M.Göbel et al., Phys. Rev. C **104**, 024001 (2021)) for $^6\mathrm{He}(p, p\alpha)2n$

Predictions for relative energy distributions

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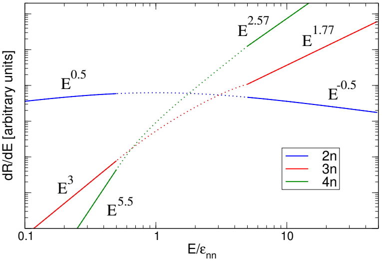
- Power law behavior at low energies



Braaten, HWH, Phys. Rev. D **107**, 034017 (2023)

- $N = 5: \quad E^8 \longrightarrow E^{5.1}$
- $N = 6: \quad E^{10.5} \longrightarrow E^{6.2}$
- $\ldots$
- Predictions for $N \leq 70$ available
  (M.G. Endres et al., PRA **84**, 043644 (2011))
- No peak except for $2n$

$3n$ distributions from experiment

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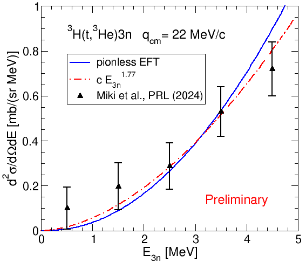
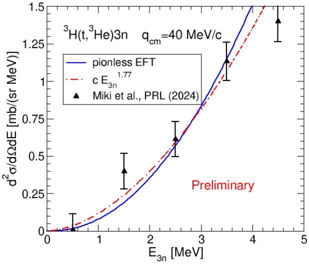
- $3n$ case consistent with previous experiments for $^3\mathrm{H}(\pi^-,\gamma)3n$
  (Miller et al., Nucl. Phys. A **343**, 347 (1980))
- $3n$ distributions from $^3\mathrm{H}(^3\mathrm{H},^3\mathrm{He})3n$ (Miki et al., Phys. Rev. Lett. **133**, 012501 (2024))

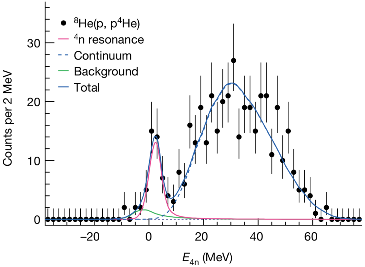


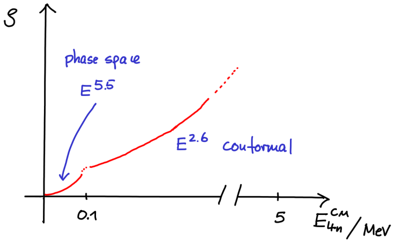


- Small q_{cm} data consistent

$4n$ distribution from experiment

- Search for tetraneutron resonances in $^{8}\mathrm{He}(p, p\alpha)4n$
  (M. Duer et al., Nature **606**,678 (2022))





- Structure from initial state?
- Dineutron correlations can produce peak
  (Lazauskas, Hiyama, Carbonell, Phys. Rev. Lett. **130**, 102501 (2023))
- CFT result confirmed in pionless EFT (T. Backert, HWH, S. König, in preparation)

Neutral charm mesons and $X(3872)$

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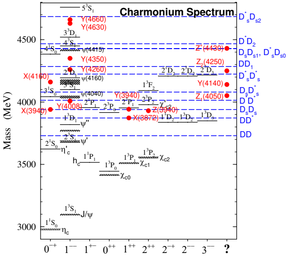
- New $c\bar{c}$ states at B factories: X, Y, Z
  (cf. Godfrey, arXiv:0910.3409)
  - Challenge for understanding of QCD
  - Unitary limit relevant?
- $X(3872)$ (Belle, CDF, BaBar, D0, LHCb)
- Nature of $X(3872)$?
  - $\bar{D}^0 D^{0*}$-molecule, tetraquark, charmonium hybrid, ...



$$m_X = (3871.65 \pm 0.06)\ \text{MeV}, \quad \Gamma = (1.19 \pm 0.21)\ \text{MeV}, \quad J^{PC} = 1^{++} \quad \text{(PDG 2023)}$$

- Assumption: $X(3872)$ is weakly-bound D^0-$\bar{D}^{0*}$-molecule

$\Rightarrow \quad |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}\ , \qquad B_X = (0.07 \pm 0.12)\ \text{MeV} \approx 1/(2\mu_{DD^*}a^2)$

$\Rightarrow$ universal properties (Braaten et al., 2003-2008; ...)

Neutral charm mesons and $X(3872)$

- Approximate unparticles of three D^0/D^{0*} mesons
- Interaction of $X(3872)$ with D^0, $\bar{D}^0$, D^{0*}, $\bar{D}^{0*}$ determined by large a

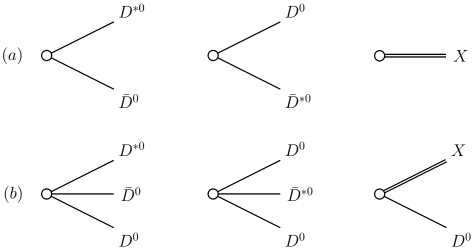
(Canham, HWH, Springer, PRD **80**, 014009 (2009))

$$a_{D^0X} = -9.7a \qquad a_{D^{*0}X} = -16.6\,a$$

- Richer structure because of $X(3872)$ (bound state)

two charm mesons

three charm mesons



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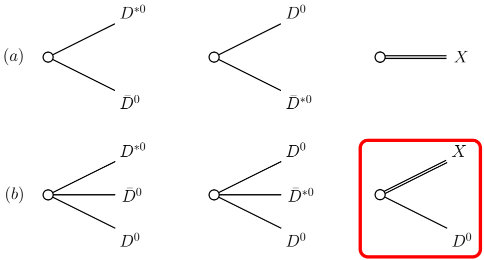

Scaling Behavior

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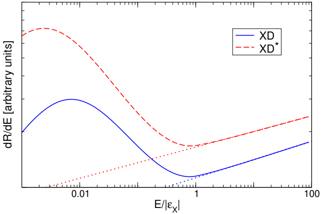
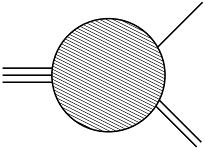
- Universal scaling for unparticles of three neutral charm mesons

  (Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022))





XD point production

$$\frac{dR}{dE} \sim (E^{-(\Delta_1+\Delta_2-\Delta_3)/2})^2\sqrt{E} \approx E^{0.1} \qquad \text{for } E/|\varepsilon_X| \gg 1$$ from conformal 3-pt. function

$\Delta_1{=}3/2, \quad \Delta_2{=}2, \quad \Delta_3{\approx}3.10119/3.08697$ (Braaten, HWH, Phys. Rev. D **107**, 034017 (2023))

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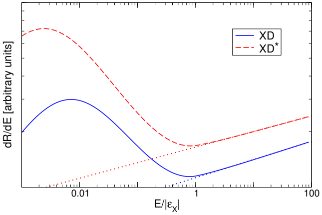

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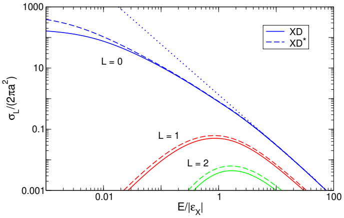

XD elastic scattering

$$\frac{dR}{dE} \sim (E^{-(\Delta_1+\Delta_2-\Delta_3)/2})^2\sqrt{E} \approx E^{0.1}, \qquad \sigma \sim E^{-1.6} \qquad \text{for } E/|\varepsilon_X| \gg 1$$

$\Delta_1 = 3/2, \quad \Delta_2 = 2, \quad \Delta_3 \approx 3.10119/3.08697$

Summary and Outlook

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- Universality in strongly interacting quantum systems
- High-energy nuclear reactions with final state neutrons
  - $\Rightarrow$ (approximate) conformal symmetry
  - $\Rightarrow$ power law behavior of observables determined by scaling dimension Δ
- Model-independent constraints on (certain) nuclear reactions
- Connection between reactions & properties of trapped particles

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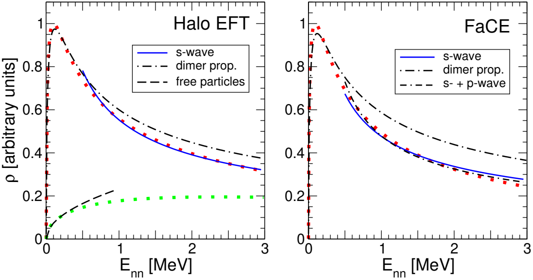
- Model-independent constraints on (certain) nuclear reactions
- Connection between reactions & properties of trapped particles
- Understand difference between $3n$ and $4n$, initial state dependence
- Other applications & extensions
  - Neutral charm mesons
  - Two-component fermions in ultracold atom physics
  - Remnant in systems with the Efimov effect?
    - $\Rightarrow$ bosonic atoms, nucleons, α particles
    - $\Rightarrow$ only discrete scale symmetry, complex scaling dimensions

Additional transparencies

Hard knockout reaction calculations

- Two-neutron spectrum for ${}^6\mathrm{He}(p,p\alpha)2n$ (Göbel et al., Phys. Rev. C **104**, 024001 (2021))

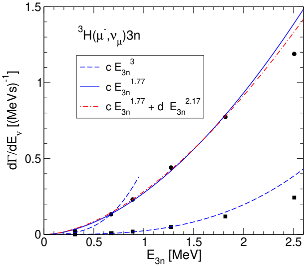


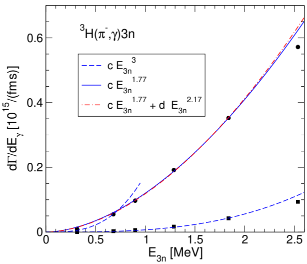
- Can be understood from dimer propagator ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

$3n$ reaction calculations

- Radiative muon/pion capture on the triton (AV18 + UIX)





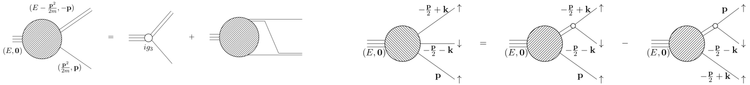
Golak et al., PRC **98**, 054001 (2018)

Golak et al., PRC **94**, 054001 (2016)

- Conformal prediction:
$d\Gamma/dE \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}, \qquad 0.1\text{ MeV} \ll E_{3n} \ll 5\text{ MeV}$

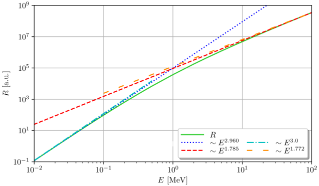
3n system in pionless EFT

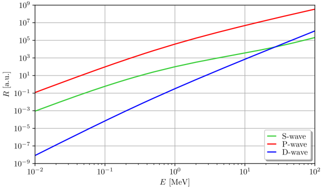
- Point-production amplitude in pionless EFT



- Reproduces conformal behavior (Backert, Dietz, HWH, König, Son, preliminary)

P-Wave





- Free and unitary limit exponents reproduced