McMule for electron scattering at MAMI and MESA

Matteo Ronchi
Johannes Gutenberg University

CRC 1660: Annual Graduate School

22nd-26th September 2025

What is McMule?

MCMULE

Monte Carlo for MUons and other LEptons
https://mule-tools.gitlab.io

P. Banerjee, G. Billis, A. Coutinho, Y. Fang, S. Gündogdu, F. Hagelstein, S. Kollatzsch, D. Moreno, D. Radic, M. Rocco, M. Ronchi, N. Schalch, V. Sharkovska, A. Signer, Y. Ulrich

- Framework for fully-differential higher-order QED calculations

What is McMule?

McMule

Monte Carlo for MUons and other LEptons
https://mule-tools.gitlab.io

P. Banerjee, G. Billis, A. Coutinho, Y. Fang, S. Gündogdu, F. Hagelstein, S. Kollatzsch, D. Moreno, D. Radic, M. Rocco, M. Ronchi, N. Schalch, V. Sharkovska, A. Signer, Y. Ulrich

- Framework for fully-differential higher-order QED calculations
- Why should we improve radiative corrections by including, e.g., NNLO QED corrections for lepton scattering

- ➢ Advanced detectors (new accelerators) → increased experimental precision (new kinematics)
- ➢ Advanced experimental setups → e.g., recoil proton detection, ISR, …
- ➢ Dark-sector searches → crucial to control background

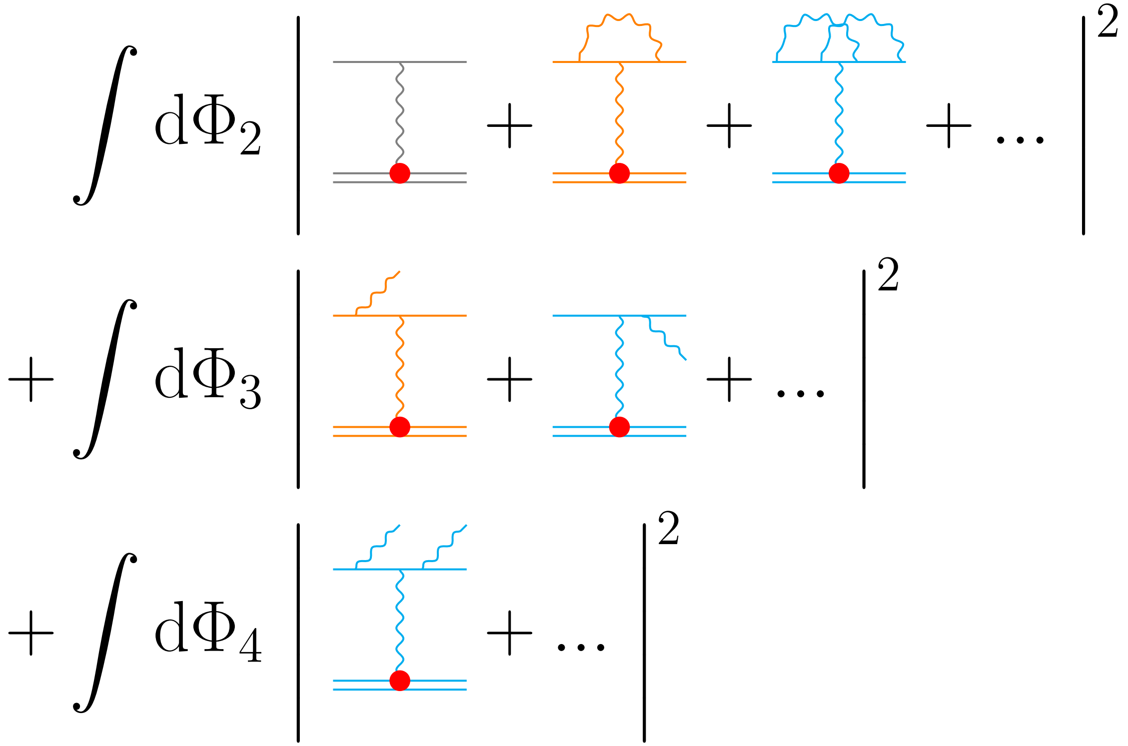
Experiments are improving → theory needs to follow!

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$eN \to eN$	PRad, ULQ2	background	+
$e^-e^- \to e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio	NNLO±
	Belle	τ properties	
$ee \to \gamma\gamma$	Daphne	dark searches	NNLO−
	any e^+e^- collider	luminosity measurement	
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

process	experir	vation	order
$e\mu \to e\mu$	MUonl	$2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Mı	s and weak charge	NNLO
$eN \to eN$	PRad,		+
$e^-e^- \to e^-e^-$	Prad 2	ı	NNLO
	MOLL	w Q^2	
$e^+e^- \to e^+e^-$	any e^+	easurement	NNLO
$ee \to \ell\ell$	VEPP,		NNLO±
	Belle		
$ee \to \gamma\gamma$	Daphn	s	NNLO−
	any e^+	easurement	
$e\nu \to e\nu$	DUNE	W	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG	s	NNLO+
	DUNE	ofiling	
$\mu \to \nu\bar{\nu}e\gamma$	MEG,		NLO
$\mu \to \nu\bar{\nu}eee$	MEG,		NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

Goal:
world domination

How does McMule work?



- **input:** matrix elements by us or others
- **McMule:** integrator (generator WIP)
- **output**: differential physical cross section for any physical quantity at fixed order through phase-space Monte Carlo integration

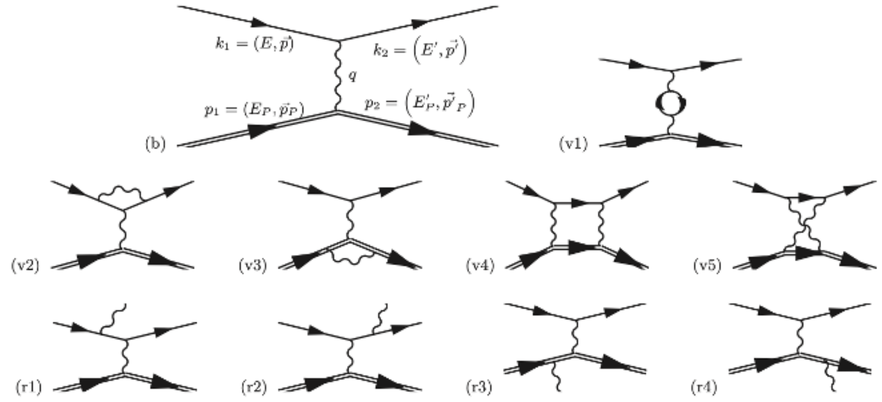
A1 analysis of electron scattering

Most extensive data set for proton electromagnetic form factors
[J.C. Bernauer et al. (2014) https://arxiv.org/abs/1307.6227]

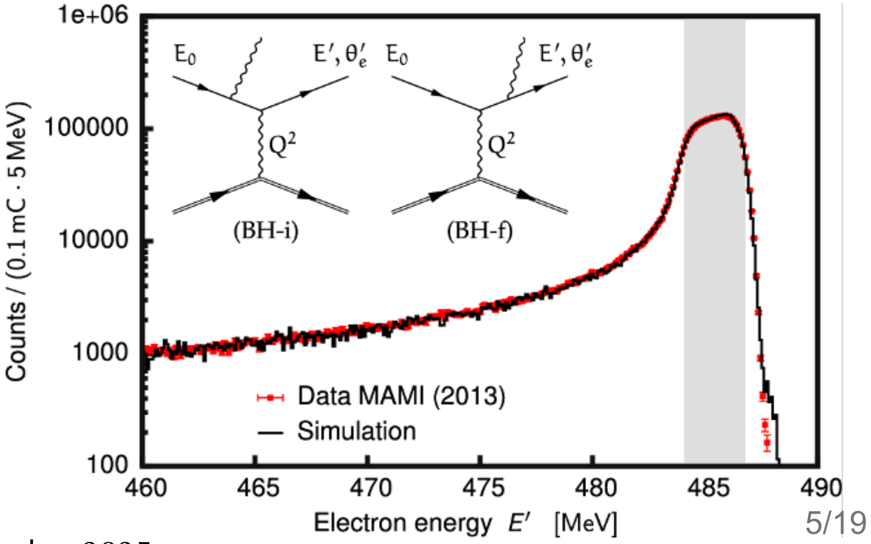
Proton charge radius extraction from initial-state radiation (ISR) experiment at MAMI: $E_0 = 495\,\text{MeV}$
[M. Mihovilovič et al. (2021) https://arxiv.org/abs/1905.11182]

(*) (1969) https://inspirehep.net/literature/52657
(**) (2000) https://arxiv.org/abs/nucl-th/0002058
(***) (2000) https://arxiv.org/abs/hep-ph/0001100

NLO corrections considered

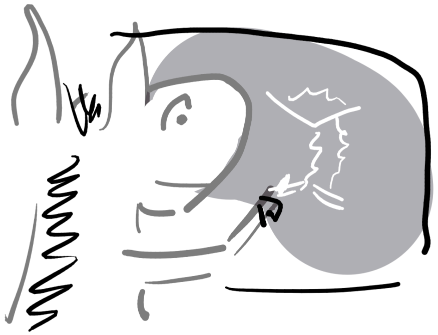


[Mo&Tsai (*), Maximon&Tjon (**), Vanderhaeghen et al. (***)]



Benefits of McMule for lp scattering

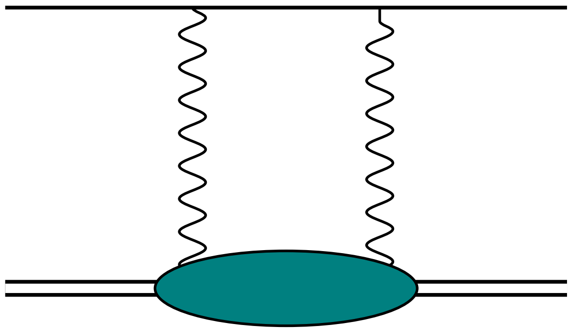
1) NNLO QED corrections
[McMule collaboration (2023)]
https://arxiv.org/abs/2307.16831



4) Lepton mass included

2) Dimensional regularization for IR divergencies

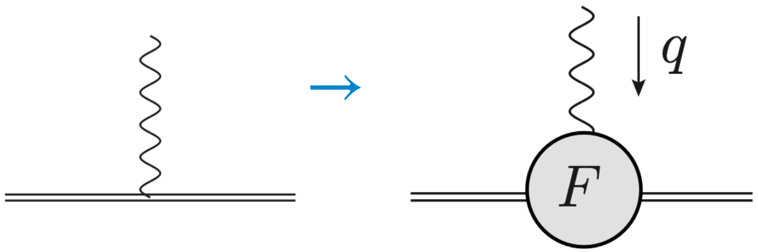
5) Hard TPE can (and will) be included



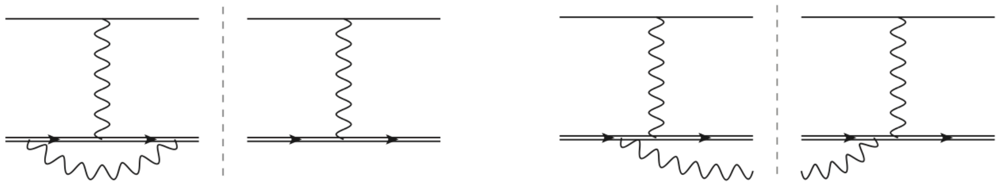
3) No limitation on emitted photon energies

QED and Hadronic Corrections

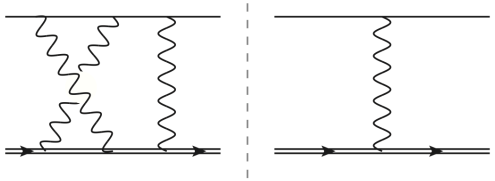
- Natural to extend to lp scattering by including proton form factors



  - ➢ LO+NLO: proton structure included except pure proton-line corrections →



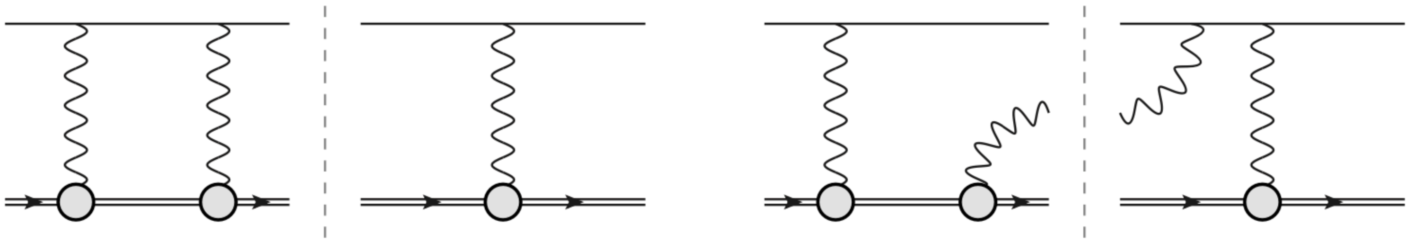
  - ➢ NNLO: proton point-like except electronic corrections →



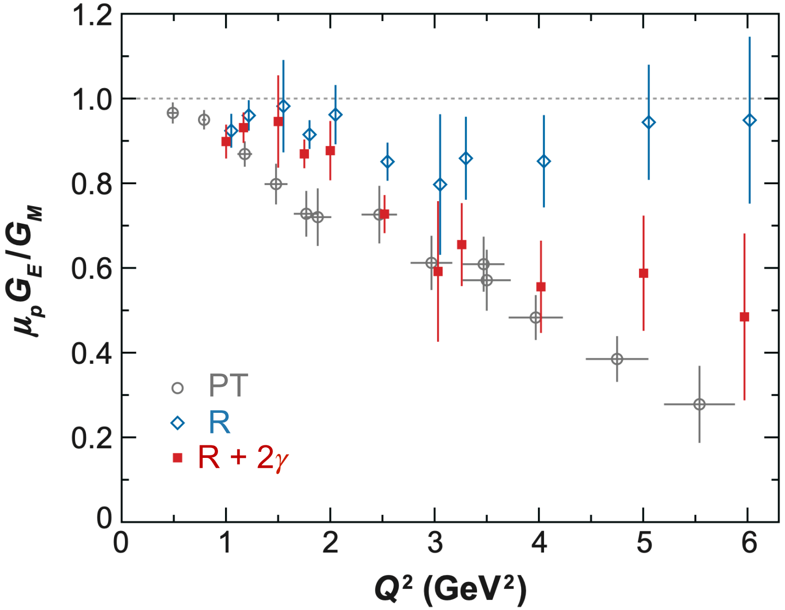
- McMule can connect QED current for the lepton side "in principle" to any current provided for the target side

"Mainz Mules" plans for TPE

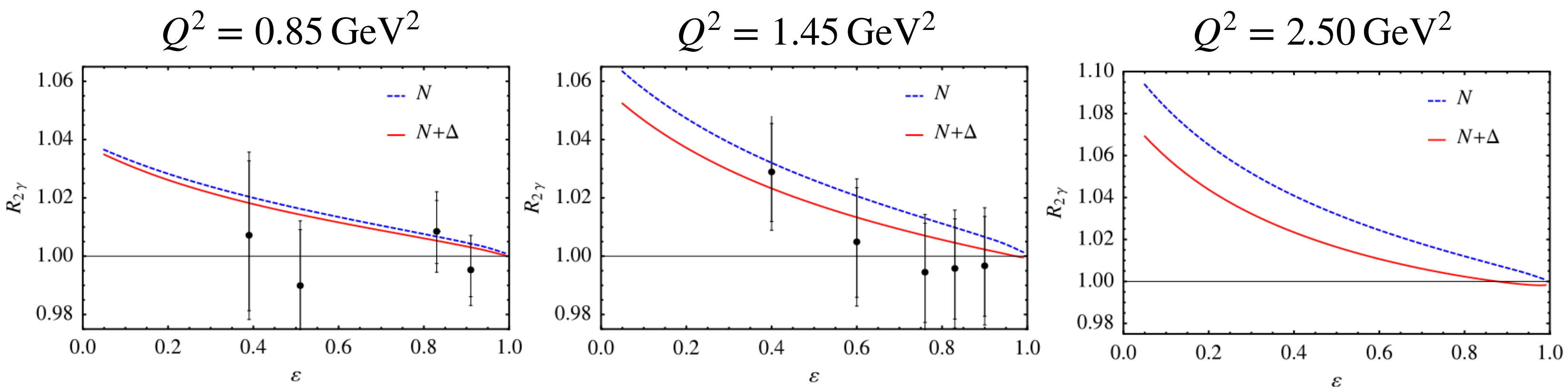
Presently only simple Born TPE model



Importance of TPE corrections strongly depends on experimental kinematics



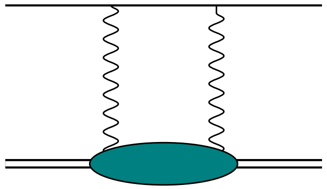
[C E. Carlson, M. Vanderhaeghen (2007)] https://arxiv.org/abs/hep-ph/0701272



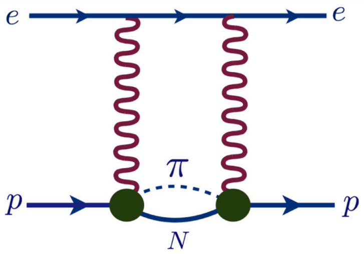
[A. Afanasev, P.G. Blunden, D. Hasell, B.A. Raue (2017)] https://arxiv.org/abs/1703.03874

“Mainz” McMule plans for TPE

Implement State-Of-The-Art TPE models, including inelastic TPE

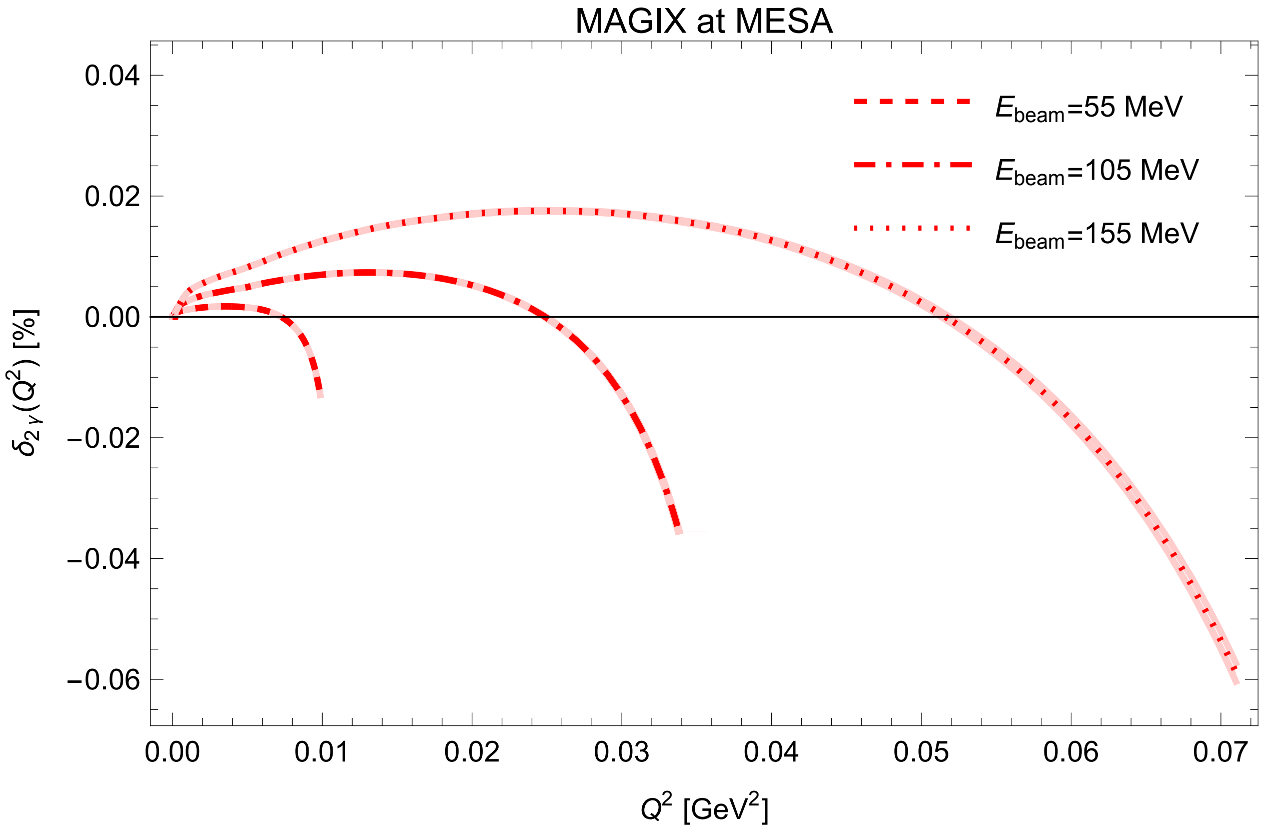


$\lesssim 1\ \mathrm{GeV}^2$ Q^2



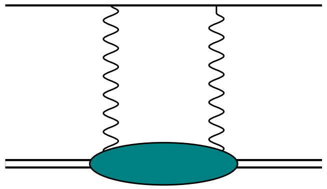
TPE with πN intermediate state

[O. Tomalak, B Pasquini, M. Vandergaeghen (2017)]
https://arxiv.org/abs/1708.03303v2



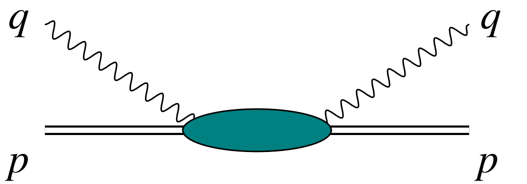
“Mainz Mules” plans for TPE

Implement State-Of-The-Art TPE models, including inelastic TPE

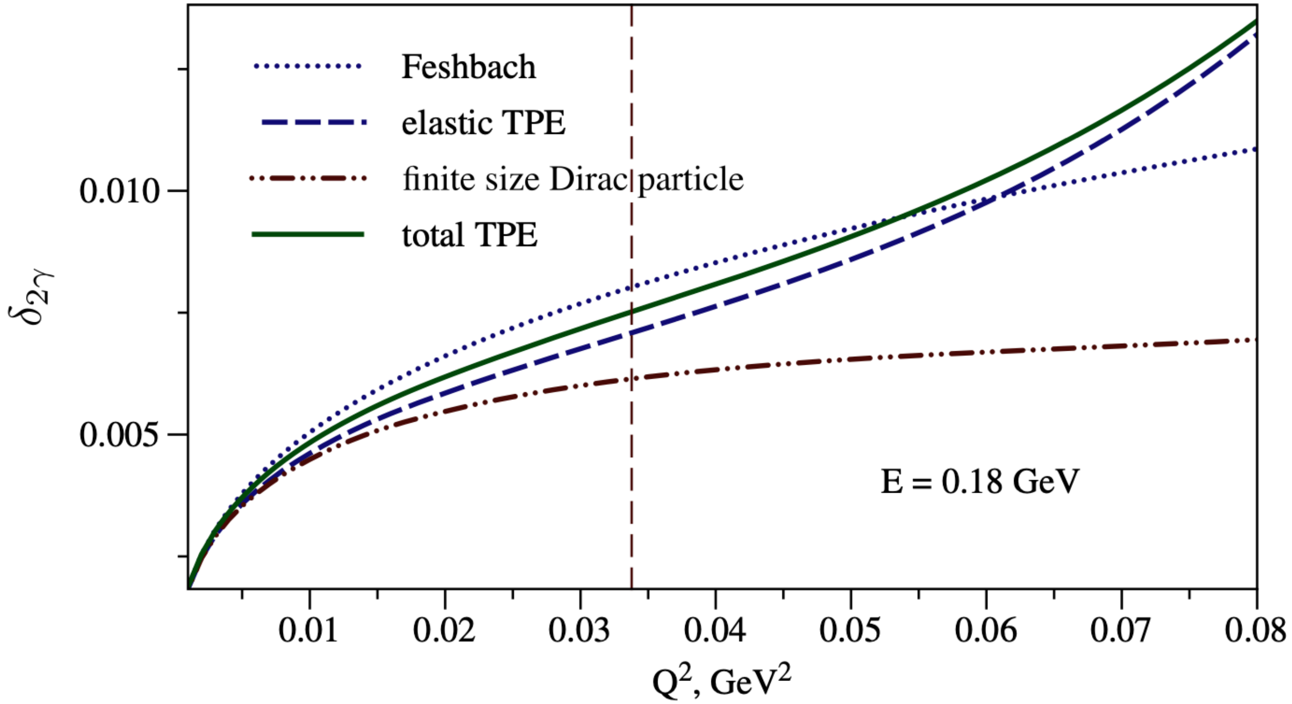


$\lesssim 0.1\ \mathrm{GeV}^2$

Q^2



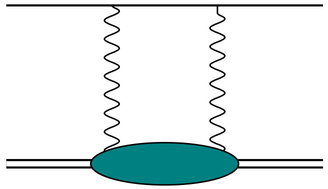
Near-forward approximation through VVCS



[O. Tomalak, M. Vanderhaeghen (2016)] https://arxiv.org/abs/1508.03759

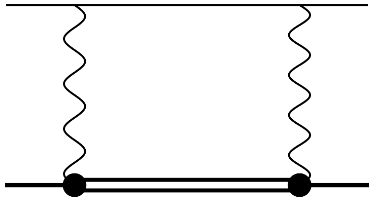
“Mainz” McMule plans for TPE

Implement State-Of-The-Art TPE models, including inelastic TPE

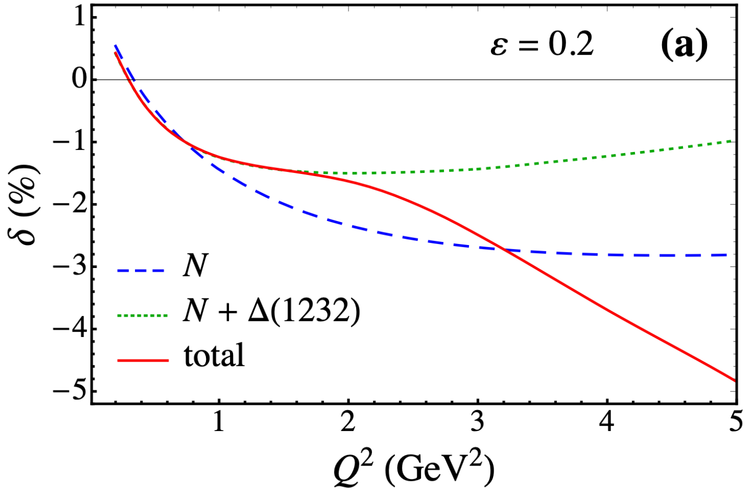


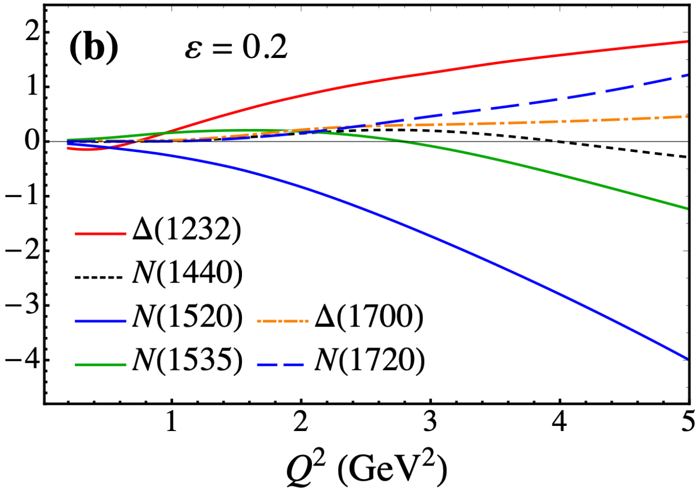
$\lesssim 5\ \mathrm{GeV}^2$

Q^2



TPE from intermediate state resonances

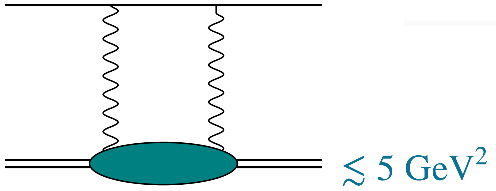


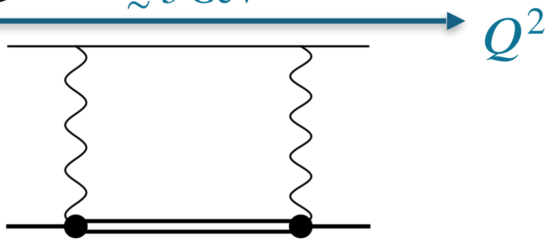


[J. Ahmed, P.G. Blunden, W. Meltnitchouk (2020)]
https://arxiv.org/abs/2006.12543

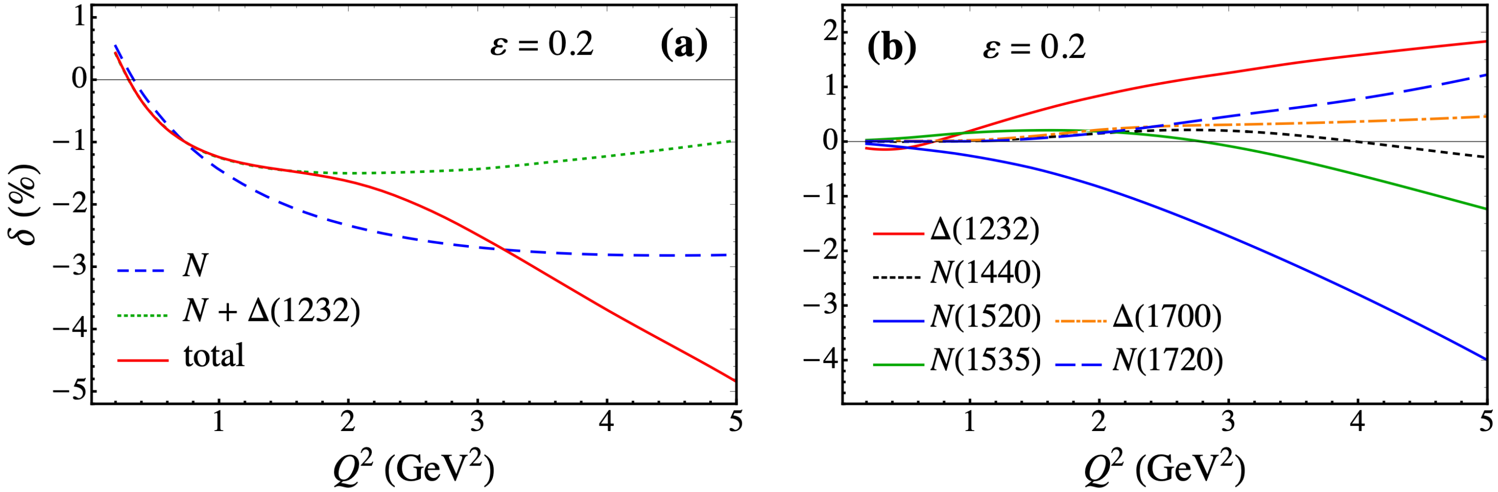
“Mainz” McMule plans for TPE

Implement State-Of-The-Art TPE models, including inelastic TPE





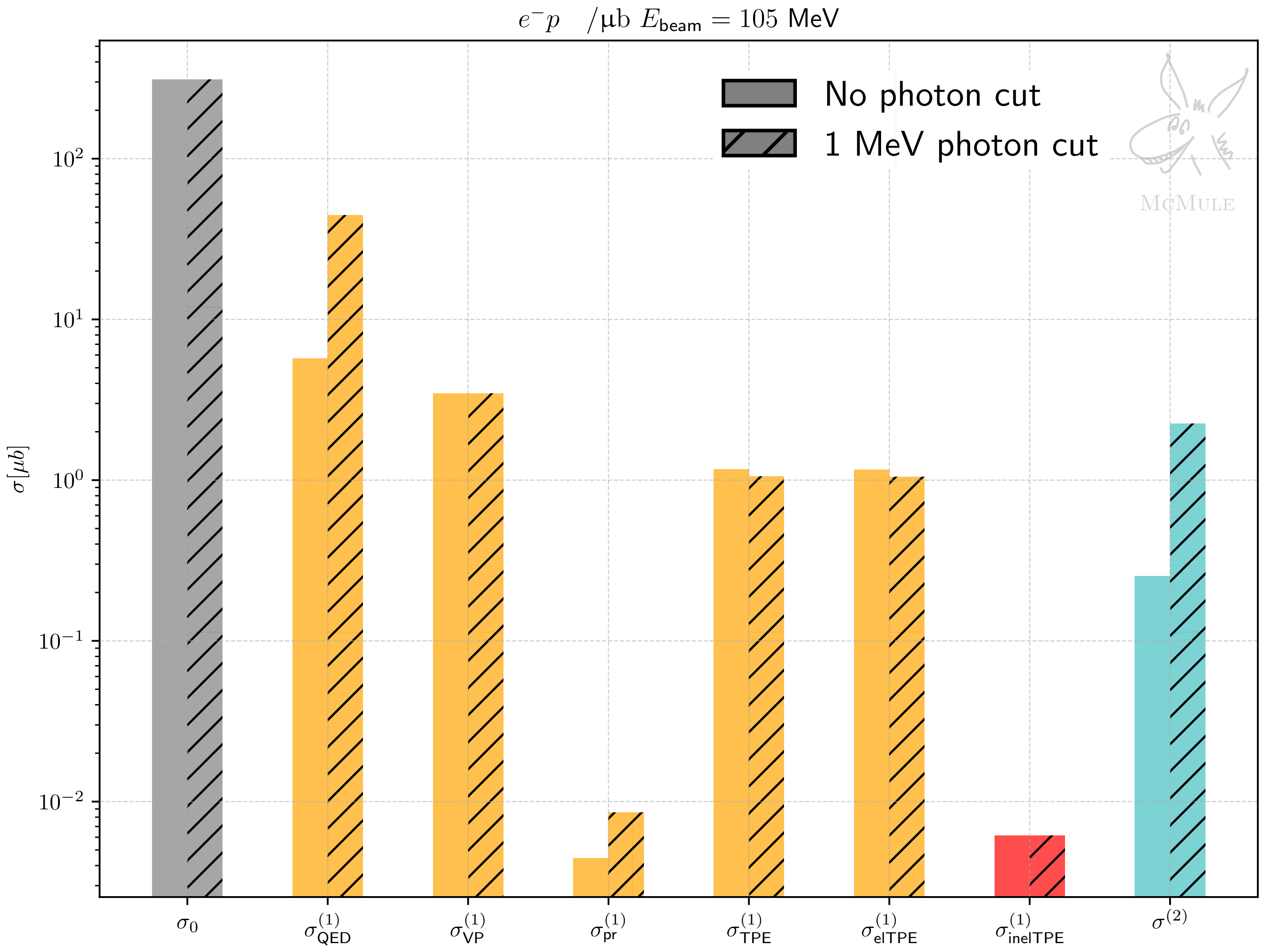
TPE from intermediate state resonances



[J. Ahmed, P.G. Blunden, W. Meltnitchouk (2020)]
https://arxiv.org/abs/2006.12543

CRC plans: Baryon Chiral Perturbation Theory description of inelastic TPE

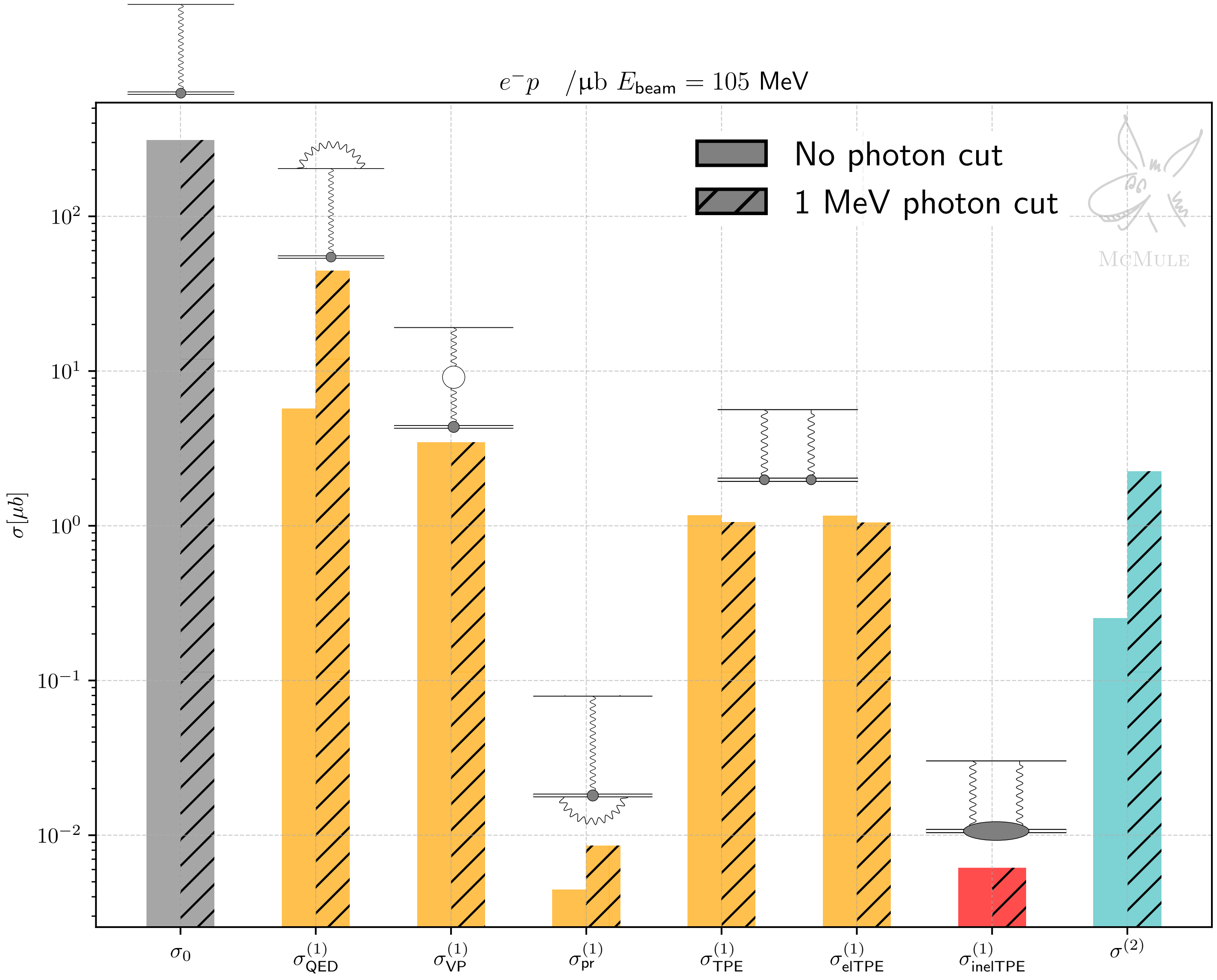
Results for MAGIX: unpolarized cross section



Parameters:

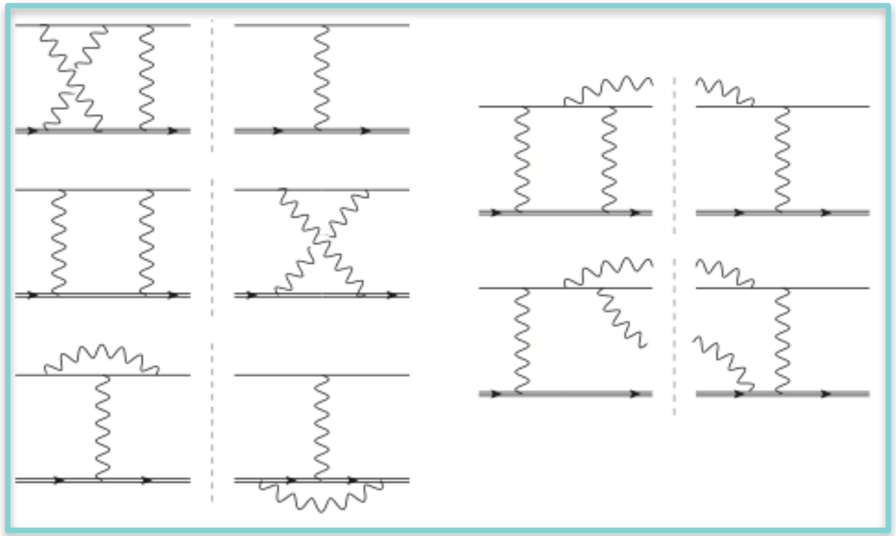
- $E_{\text{beam}} = 105\,\text{MeV}$
- $15^\circ \leq \theta_l \leq 150^\circ$

Results for MAGIX: unpolarized cross section

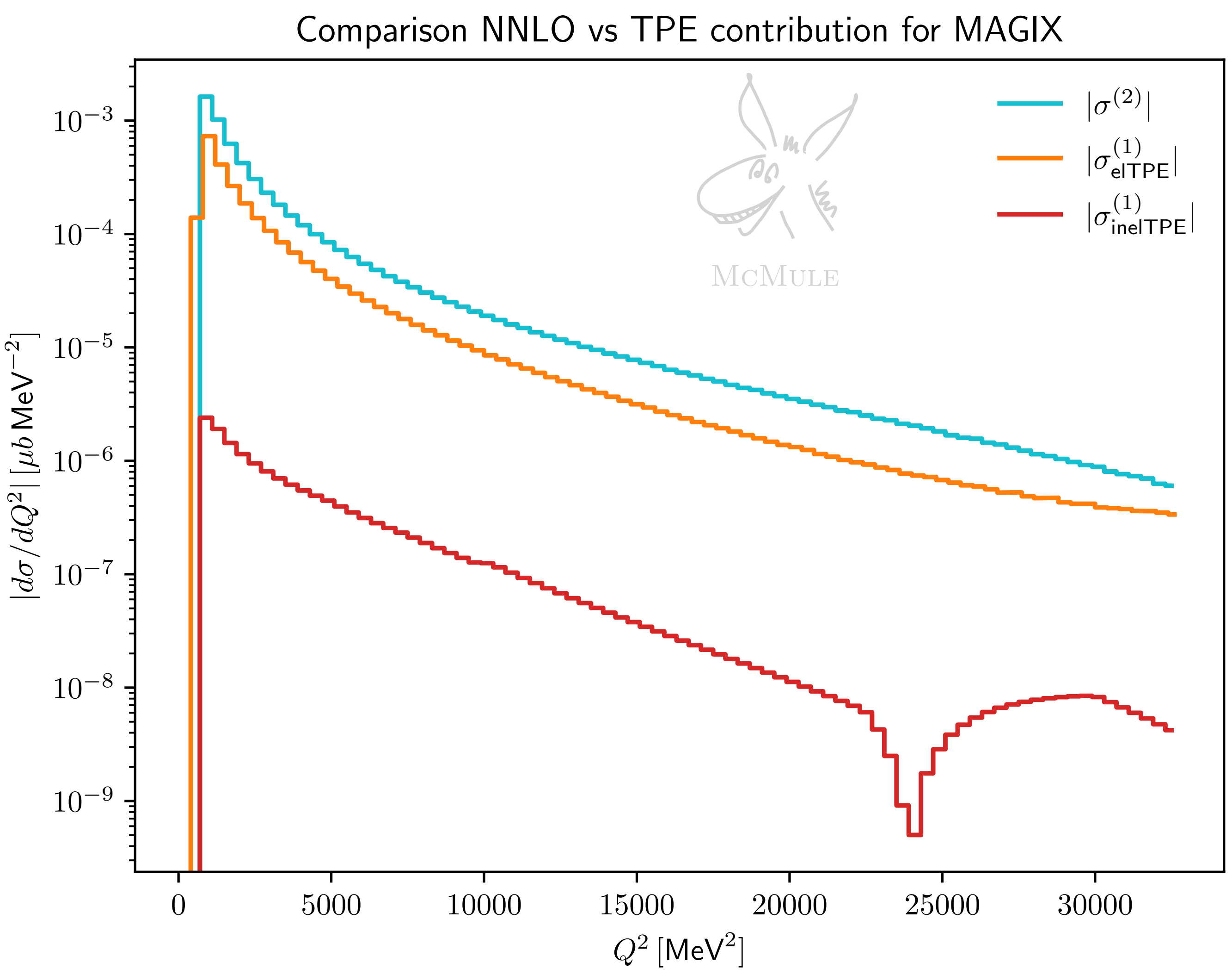


Parameters:

- $E_{\text{beam}} = 105\,\text{MeV}$
- $15^\circ \leq \theta_l \leq 150^\circ$



Results for MAGIX: unpolarized cross section



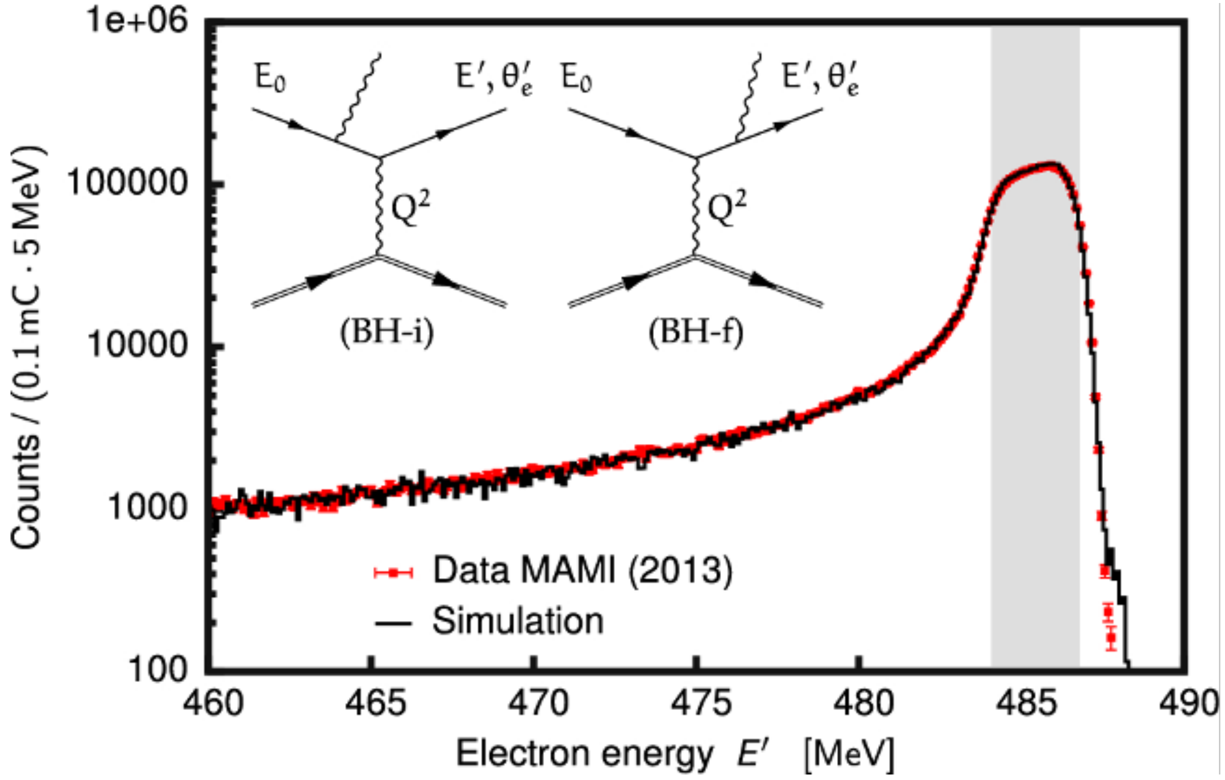
Parameters:

- $E_{\text{beam}} = 105\,\text{MeV}$
- $15^\circ \leq \theta_l \leq 150^\circ$

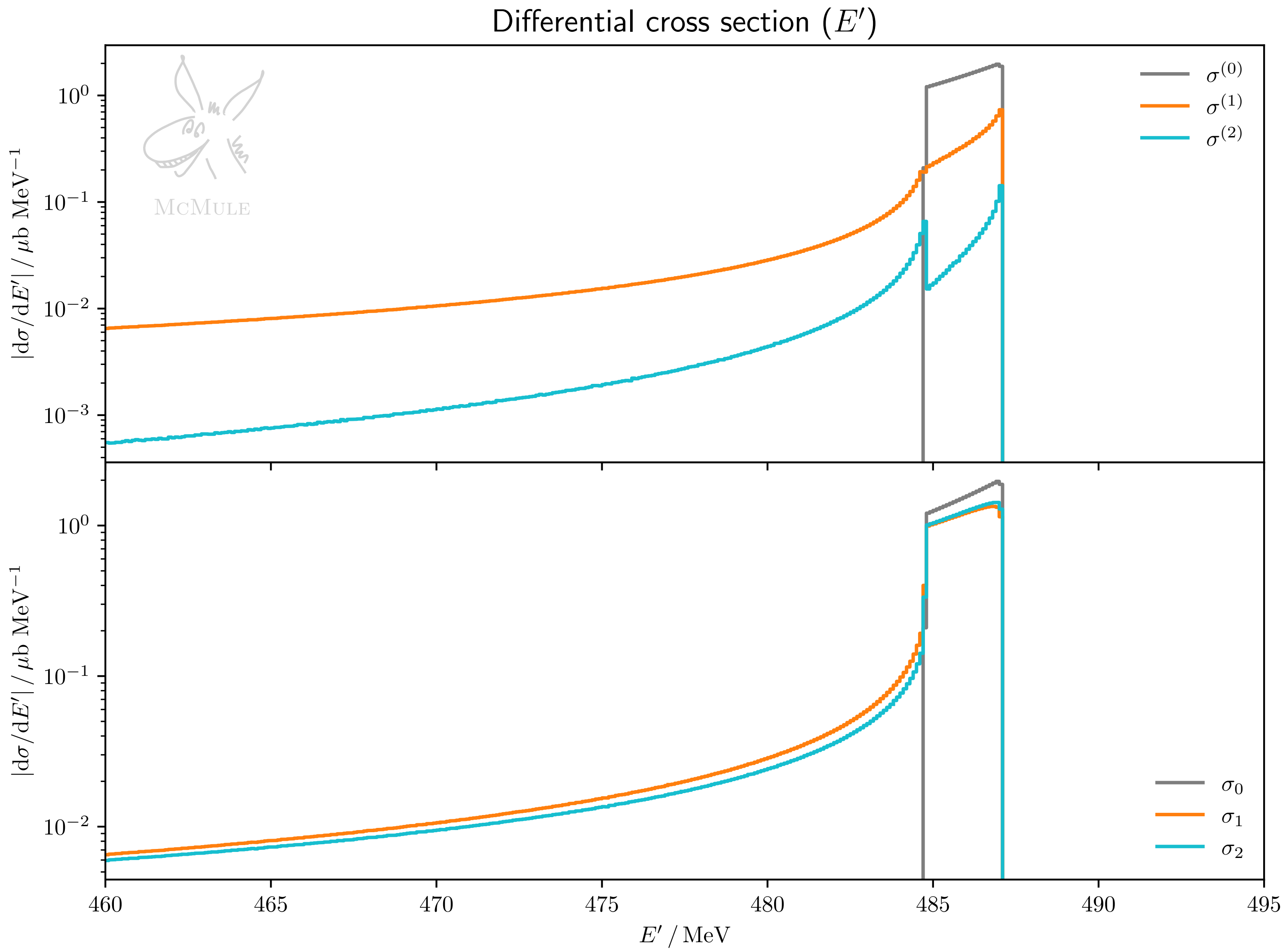
Initial-state-radiation (ISR) technique

Precise reconstruction of the radiative tail, modeled using Monte-Carlo simulation

$$r_p = 0.878 \pm (0.011)(\text{stat.}) \pm (0.031)(\text{sys.}) \pm (0.002)(\text{mod.})$$



Differential cross-section

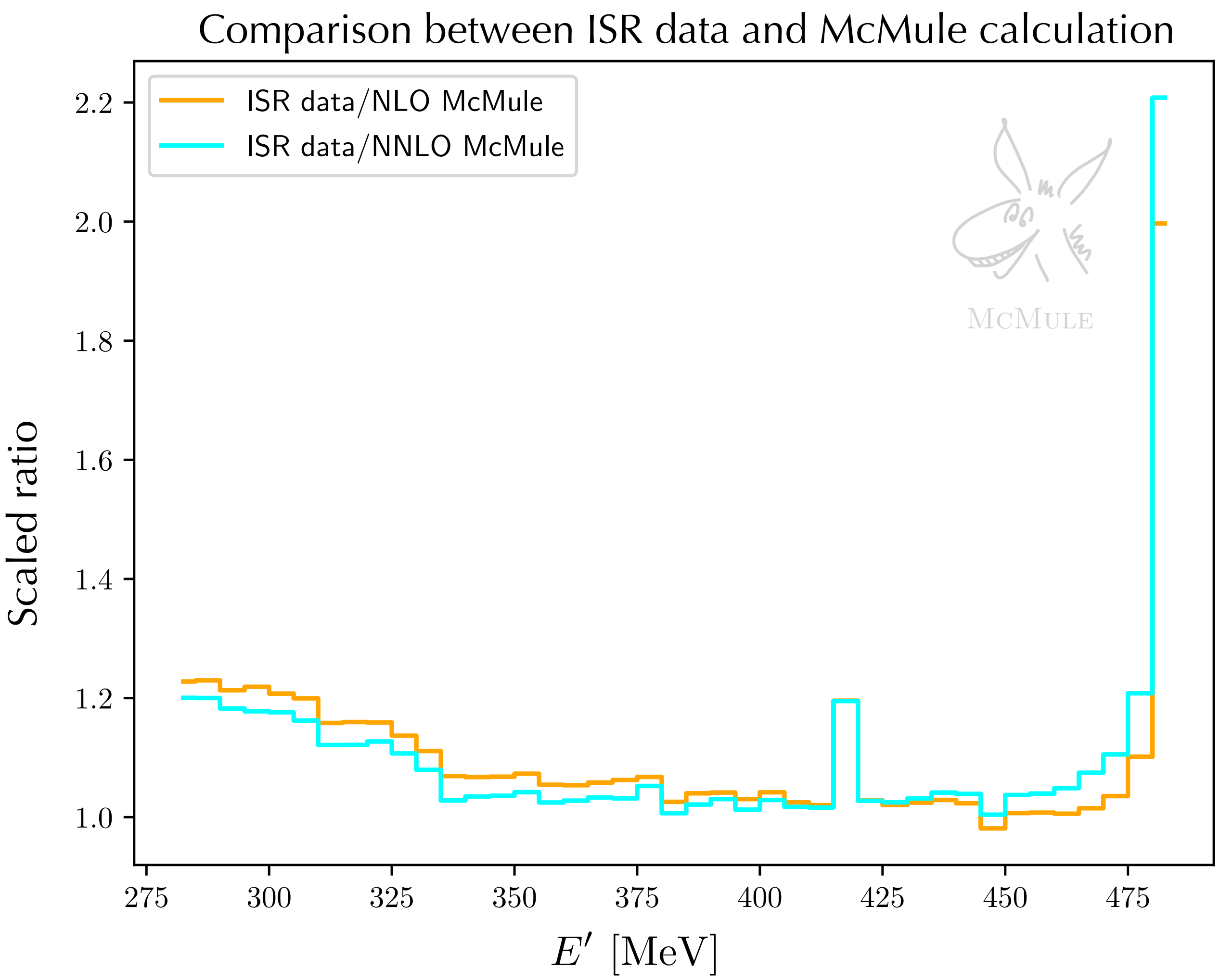


Parameters:

- $E_{\text{beam}} = 495\,\text{MeV}$
- $p_{\text{cut}} = 280\,\text{MeV}$
- $\theta_l = 15° \pm 15\,\text{mrad}$

$$\sigma_k = \sum_{i=0}^{k} \sigma^{(i)}$$

Differential cross-section



Parameters:

- $E_{\text{beam}} = 495\,\text{MeV}$
- $p_{\text{cut}} = 280\,\text{MeV}$
- $\theta_l = 15^\circ \pm 15\,\text{mrad}$

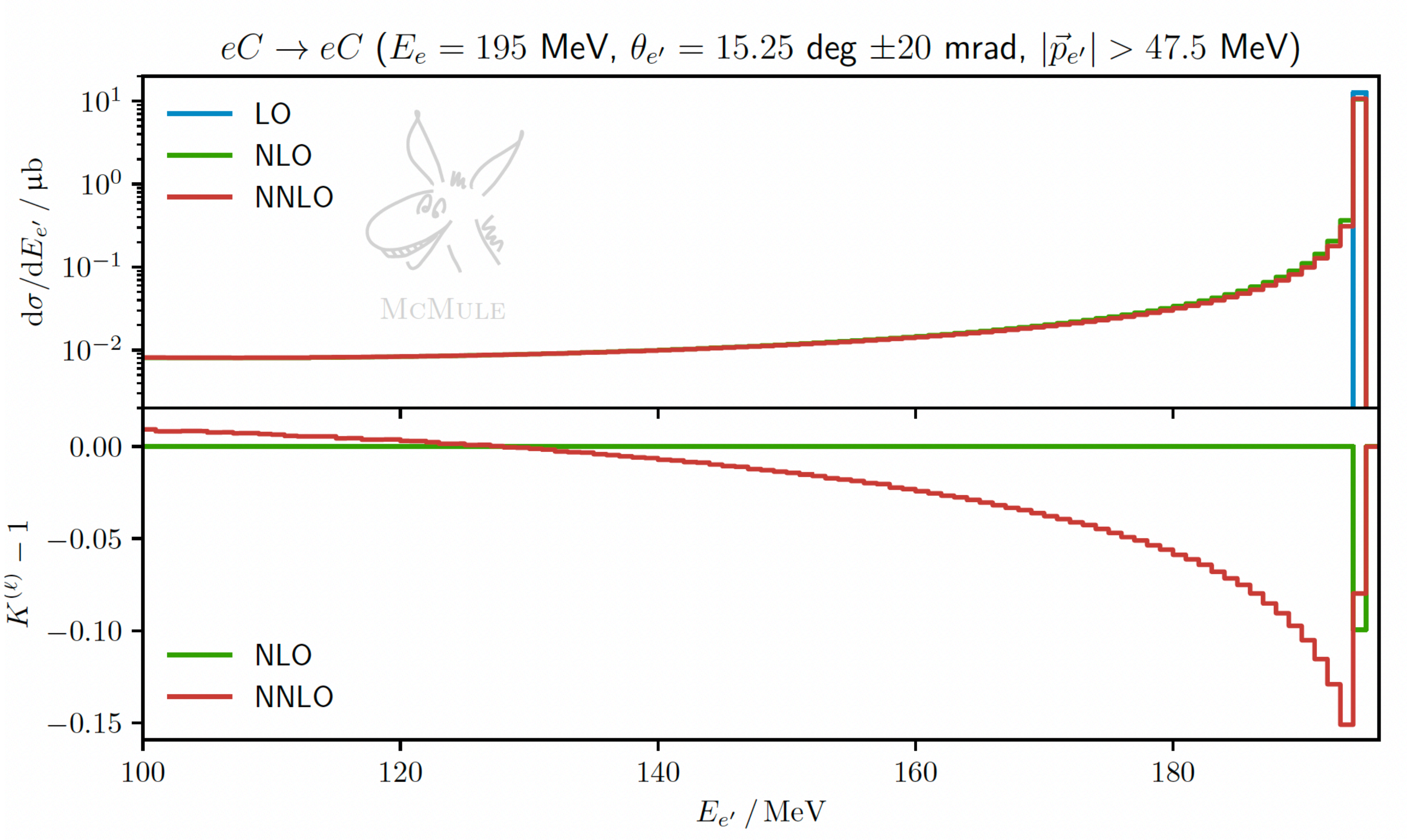
$$\frac{d\sigma^{\text{exp}}(E')}{d\sigma^{(0)}\left[1 + \sum_{i=1}^{N} \delta^{(i)}(E')\right]}$$

E': scattered electron energy

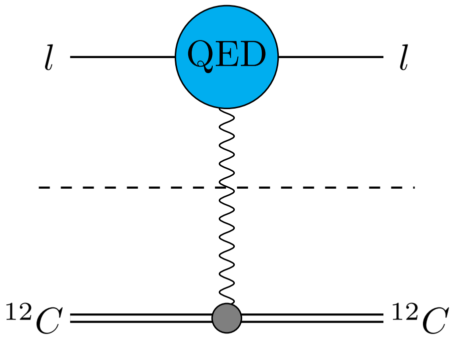
$i = 1$: next-to-leading order (NLO)

$i = 2$: NNLO

Outlook for any target: we can do e.g. ^{12}C



[M. Rocco's plot]



Outlook and conclusions

- Which targets can be included?

  - Arbitrary spin extension implemented for "elastic" contributions → form factor input needed

  - Proton and deuterium targets can be understood by "Mainz Mules"

  - Anything possible (heavier nuclei, excited waves, …), given hadronic/nuclear input!

Need input from other groups (Bacca, Gorshteyn, … ?)

- Extension to polarized observables?

Any questions?

Generator level https://arxiv.org/abs/2501.03703

- If detector simulation (histogramming & cuts) is fast → don't need a generator
- Otherwise: we want a generator
- Problem: at NLO and beyond, weights might be negative → reduce efficiency

  If $r \times N$ of N weights are negative, we would need more $\propto 1/(1-2r)^2$ more events

- McMule use CRES to eliminate negative weights [Anderson, Maier (2021) https://arxiv.org/abs/2109.07851] [Anderson, Maier, Maitre (2023) https://arxiv.org/abs/2303.15246]
- Generator still needs to know some detail of experiment!
- NOTE: future cutting-edge theory improvements unlikely to be available as generators due to the limited power of our computers
  → Fast simulations are needed!